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Influence of Triaxial Loading Systems Properties on Non-Elastic Deformation and Failure of Granular Composites

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ABSTRACT: The primary objective of this paper is to describe structural damage evolution that depends on the loading system properties, which characteristics are taken into account by addition of the boundary value problem to the third kind boundary conditions. Macroscopically failure of the granular composite can be considered as final stage of the structural damage nucleation, localization and formation of failure cluster. When the loading system stiffness is sufficient the damage accumulation can be occur in equilibrium regime. The failure of the composite can take place in any point of the descending branch of the stress-strain diagram in correspondence with loading system stiffness if the stability conditions is non carried out. The lager loading system stiffness the lager failure strains. During structural damage accompanied by dissipation energy, post-critical deformation stage realisation is the accommodation mechanism of the heterogeneous body to loading conditions.

Introduction

The macroscopic failure of the composite materials is preceded by complete multilevel processes, accompanied by accumulation and localization of damaged centres and formation of failure cluster. Therefore, the research of this mechanisms is one of the actual tasks for a modern composite materials, applied in aerospace engineering.

The non-linear character of relation between macro-stresses and macro-strains is a consequence not only plastic deformation of the structural elements. The appearance of non-elastic properties of heterogeneous medium can be effected by the structural damage accumulation. Stable evolution of the process under certain conditions results in implementation of the post-critical deformation stage (negative hardening or work-softening behaviour), that followed the attainment of strength and appeared as a descending branch on the stress-strain diagram. The provision of conditions of equilibrium damage accumulation conditions and the post-critical deformation stage is the tool of the strength reserves and a rise of carrying capacity of constructions. The information of the mechanical behaviour at the work-softening stage at the various complete triaxial stress-strain states is necessary for the extension of the physical base of being available mechanics of heterogeneous media models, description of deformation as a continuous process of the damage accumulation and optimal designing of composite constructions.

The observable difference in character of structural damage evolution of the composites and constructions can not always be explained only by the mechanical properties or by presence of porosity and structural imperfections. Special experimental researches of the composite materials deformation are testified, that the resistance to damage by an essential image depends on the loading system stiffness too.

We named the set of solid, liquid and (or) gaseous bodies, deformation of which take place in result of the load transmission to a body or construction as a whole or to a separate part as the loading system. The replacement of one material state to another in considered volume due to the replacement of an external load depending on properties and constructive device of the loading system, that characteristics at the boundary value problems of non-elastic deformation and damage solution are taken into account by the third kind boundary conditions [1]. The increase of the loading system stiffness allows to stabilise the structural damage evolution and results to work-softening behaviour of the body.

The main mechanisms of non-elastic deformation effected by structural damage of the granular composites are described. The matrixes of the modern carbon-carbon composites, ceramics and received by the powder metallurgy techniques composites can be referred to the indicated type of materials. Two-level structurally-phenomenological model of the deformation and damage accumulation of heterogeneous medium consisting of structural elements with identical elastic and random strength properties is developed. The obtained relation of macro-stress tensor invariants from macro-strain tensor invariants at the different schemes of proportional displacement-controlled loading mode are a basis for the construction of the constitution equations for the work-softening media. The research of the loading system stiffness influence on stability of deformation, damage accumulation and failure at monotone and non-monotone triaxial loading is spent.

Structurally-phenomenological model

The investigation of deformation and damage accumulation processes in a heterogeneous body is carried out with the help of two-level structurally-phenomenological model [2]. We suppose the structural elements of the granular composite are homogeneous and firmly connected with each other. Geometry and mutual disposition of the elements are given and do not vary during loading and damage accumulation. The mechanical properties of the medium are the discreetly constant functions of co-ordinate $\mathbf{r}$. The phenomenological equations and classically mechanical relationships are valid on the structural level. Non-elastic deformation and structural damage accumulation during quasi-static loading of the composite body Ω with closed surface Σ at space R^3 are described by simultaneous difference equations in incremental form as a united process:

$$\nabla \cdot \mathbf{d\sigma}(\mathbf{r}) = 0, \quad \mathbf{d\varepsilon}(\mathbf{r}) = 1/2\left[\nabla \otimes \mathbf{du}(\mathbf{r}) + \mathbf{du}(\mathbf{r}) \otimes \nabla\right], \tag{1}$$

$$\mathbf{d\sigma}(\mathbf{r}) = \tilde{\mathbf{C}}(\mathbf{r}) \cdot\cdot\, \mathbf{d\varepsilon}(\mathbf{r}) = \left[3K(\mathbf{r})(1-\kappa)\mathbf{V} + 2G(\mathbf{r})(1-g)\mathbf{D}\right] \cdot\cdot\, \mathbf{d\varepsilon}(\mathbf{r}), \tag{2}$$

where $\mathbf{du}(\mathbf{r})$, $\mathbf{d\sigma}(\mathbf{r})$, $\mathbf{d\varepsilon}(\mathbf{r})$ are increments of structural displacement vector, structural stress and structural strain tensors; $\mathbf{V}$ and $\mathbf{D}$ are hydrostatics part and deviator of identity 4-th rank tensor; $K(\mathbf{r})$ and $G(\mathbf{r})$ are bulk elastic modulus and shear elastic modulus of scleronomous isotropic components; κ and g are independent damage functions.

The boundary conditions of third kind

$$\left[\mathbf{d\sigma}^* \cdot \mathbf{n}(\mathbf{r}) + \mathbf{R}(\mathbf{u},\mathbf{r}) \cdot \mathbf{du}(\mathbf{r})\right]_{\Sigma_S} = \mathbf{dS}^0(\mathbf{r}) \tag{3}$$

or

$$\left[\mathbf{d\varepsilon}^* \cdot \mathbf{r} + \mathbf{Q}(\mathbf{S},\mathbf{r}) \cdot \mathbf{d\sigma}(\mathbf{r}) \cdot \mathbf{n}(\mathbf{r})\right]_{\Sigma_U} = \mathbf{du}^0(\mathbf{r}), \tag{4}$$

where $\mathbf{du}^0(\mathbf{r})$ is real increment of displacement vector of boundary points with normal $\mathbf{n}(\mathbf{r})$, $\mathbf{d\sigma}^*$ and $\mathbf{d\varepsilon}^*$ are real increments of macro-stress and macro-strain tensors provide the macro-homogeneous stress or strain states. Further all macroscopical level values will be marked by the index (*).

Symmetric positively definite second rank tensors $\mathbf{R}(\mathbf{u},\mathbf{r}) = -\partial S_i/\partial u_j \mathbf{e}_i \otimes \mathbf{e}_j$ and $\mathbf{Q}(\mathbf{S},\mathbf{r}) = -\partial u_i/\partial S_j \mathbf{e}_i \otimes \mathbf{e}_j$ of loading systems stiffness and compliance

$$\forall \mathbf{a} \neq \mathbf{0} : \quad \mathbf{a} \cdot \mathbf{R}(\mathbf{u},\mathbf{r}) \cdot \mathbf{a} > 0, \quad \mathbf{a} \cdot \mathbf{Q}(\mathbf{S},\mathbf{r}) \cdot \mathbf{a} > 0$$

connect the increments vectors of external forces $\mathbf{dS}^0(\mathbf{r})$ and displacements $\mathbf{du}^0(\mathbf{r})$ by

$$\mathbf{dS}^0(\mathbf{r}) = \mathbf{R}(\mathbf{u},\mathbf{r}) \cdot \mathbf{du}^0(\mathbf{r}), \quad \mathbf{du}^0(\mathbf{r}) = \mathbf{Q}(\mathbf{S},\mathbf{r}) \cdot \mathbf{dS}^0(\mathbf{r}), \quad \mathbf{R} \cdot \mathbf{Q} = \mathbf{I}, \tag{5}$$

where $\mathbf{e}_i$ are basis vectors and $\mathbf{I}$ is identity second rank tensor. The vectors are given by loading or deformation processes on the parts $\Sigma_S \subset \Sigma$ and (or) $\Sigma_U = \Sigma \setminus \Sigma_S$ of the composite body surface. The eq. (3) and eq. (4) are interinverted and so the boundary conditions of one type can be applied for the whole solid closed surface $\Sigma = \Sigma_S \cup \Sigma_U$. In the extreme cases of $\mathbf{R} = \mathbf{0}$ or $\mathbf{Q} = \mathbf{0}$ the boundary conditions (3) and (4) corresponds to stress-controlled or displacement-controlled loading processes (extreme soft or extreme hard loading modes).

The complete macroscopically failure of the heterogeneous body is resulted from the structural elements carrying capacity. Each structural damage act is accompanied by stresses redistribution leads to continuing or discontinuing of damage accumulation at given level of external loading. The structural damage results in non-linear macroscopic behaviour of the medium even in the case of elastic-brittle properties of components. The main problems of structurally-phenomenological models construction can be formulated as the decision of structural strength criterion and the investigation of material properties after satisfaction of one or another damage condition. The structural element can be damaged from different mechanisms.

The tensor-linear constitution equations (2) contain independent damage functions κ and g, which arguments are only linear and quadratic independent invariants of strain

$$j_\varepsilon^{(1)} = \mathbf{tr}(\varepsilon), \quad j_\varepsilon^{(2)} = \left[\mathbf{tr}(\breve{\varepsilon} \cdot \breve{\varepsilon})\right]^{1/2}, \quad \breve{\varepsilon} = \varepsilon - 1/3\,\mathbf{tr}(\varepsilon)\,\mathbf{I} \tag{6}$$

or stress tensors

$$j_\sigma^{(1)} = 1/3\,\mathbf{tr}(\sigma), \quad j_\sigma^{(2)} = \left[\mathbf{tr}(\breve{\sigma} \cdot \breve{\sigma})\right]^{1/2}, \quad \breve{\sigma} = \sigma - 1/3\,\mathbf{tr}(\sigma)\,\mathbf{I}. \tag{7}$$

Forming is considered as a basic cause of damage and as effect of structural elastic properties reduction. Independent damage functions can be represented by

$$g\left(j_{\varepsilon}^{(2)}\right)=\begin{cases}0, \ j_{\varepsilon}^{(2)}(\mathbf{r})<j_{\varepsilon\ cr}^{(2)}(\mathbf{r});\\ 1, \ j_{\varepsilon}^{(2)}(\mathbf{r})\geq j_{\varepsilon\ cr}^{(2)}(\mathbf{r})\end{cases}, \tag{8}$$

$$\kappa\left(j_{\varepsilon}^{(1)}, j_{\varepsilon}^{(2)}\right)=\begin{cases}0, \ \left(j_{\varepsilon}^{(2)}(\mathbf{r})<j_{\varepsilon\ cr}^{(2)}(\mathbf{r})\right)\vee\left(j_{\varepsilon}^{(2)}(\mathbf{r})\geq j_{\varepsilon\ cr}^{(2)}(\mathbf{r})\wedge j_{\varepsilon}^{(1)}(\mathbf{r})\leq 0\right);\\ 1, \ j_{\varepsilon}^{(2)}(\mathbf{r})\geq j_{\varepsilon\ cr}^{(2)}(\mathbf{r})\wedge j_{\varepsilon}^{(1)}(\mathbf{r})>0\end{cases},$$

where $j_{\varepsilon\ cr}^{(2)}(\mathbf{r})$ is strength constant of structural element on forming. If the strength criterion $j_{\varepsilon}^{(2)}(\mathbf{r})\geq j_{\varepsilon\ cr}^{(2)}(\mathbf{r})$ is satisfied possible variants of the material property reduction in the damaged volume takes place: the complete loss of carrying capacity in the case of $j_{\varepsilon}^{(1)}(\mathbf{r})>0$ and remaining of the capacity to resist only hydrostatic compressible loading in the case of $j_{\varepsilon}^{(1)}(\mathbf{r})\leq 0$.

The mechanical behaviour of macro-homogeneous and macro-isotropic media could be described on macroscopical level by relationships:

$$J_{\sigma}^{(1)}=3K^{*}\left(1-\kappa^{*}\right)J_{\varepsilon}^{(1)}, \quad J_{\sigma}^{(2)}=2G^{*}\left(1-g^{*}\right)J_{\varepsilon}^{(2)}. \tag{9}$$

The invariants of $J_{\sigma}^{(\bullet)}$ and $J_{\varepsilon}^{(\bullet)}$ can be also calculated on eq. (6) and eq. (7). The values of macro-stress σ_{ij}^{*} and macro-strain ε_{ij}^{*} tensors components are applied.

On numerically solution of boundary value problem

We pay attention on the satisfaction of the third kind boundary conditions eq. (3) and eq. (4) at numerically solution of the boundary value problem by the finite element method. We carry out the discretization of deformable body Ω on N finite elements. Further all values related to finite elements will be marked by index e. Let us assume that the symmetrical matrix $[\mathrm{R}]$ of loading system properties are defined in each surface Σ points. The nodal forces vector $\left\{\mathrm{dS}^{(e)}\right\}_{S}$ due to increments of distributed loads $\mathbf{dS^0}$ on the part of

Σ_S body surface are represented by:

$$\left\{dS^{(e)}\right\}_S = \int_{\Sigma_S^e} \left[N^{(e)}\right]^T \left\{dS^0\right\} d\Sigma + \int_{\Sigma_S^e} \left[N^{(e)}\right]^T [R] \left[N^{(e)}\right] d\Sigma \left\{du^{(e)}\right\}, \qquad (10)$$

where $\left[N^{(e)}\right]$ is shape functions ma*trix, $\left\{du^{(e)}\right\}$ is nodal displacements vector of the finite element and $\Sigma_S^{(e)} \subset \Sigma_S$. Applying the condition (5) can reduce the relationship (10) for the nodal forces vector $\left\{dS^{(e)}\right\}_U$ due to increment of distributed displacements $\mathbf{du}^0$ on the part Σ_U of body surface ($\Sigma_U^{(e)} \subset \Sigma_U$) to:

$$\left\{dS^{(e)}\right\}_U = \int_{\Sigma_S^e} \left[N^{(e)}\right]^T [R] \left\{du^0\right\} d\Sigma + \int_{\Sigma_S^e} \left[N^{(e)}\right]^T [R] \left[N^{(e)}\right] d\Sigma \left\{du^{(e)}\right\}. \qquad (11)$$

Let integrals in the second items of eq. (10) and eq. (11) be presented as symmetrical loading system stiffness matrix $\left[\tilde{R}^{(e)}\right]$ of the finite element. For the elements disposed inside the discretizated body the matrix $\left[\tilde{R}^{(e)}\right]$ is zero-matrix. The matrix has the same dimension as the stiffness matrix $\left[K^{(e)}\right]$ of the finite element. Following traditional sequence of simultaneous resolved equations assembly stage with specifically applying the connection $\left\{du^{(e)}\right\} = \left[L^{(e)}\right]^T \{dU\}$ let one received as

$$\{dS\} = \left[\tilde{K}\right]\{dU\}, \quad \left[\tilde{K}\right] = [K] + [K'] = \sum_{e=1}^{N} \left\{\left[L^{(e)}\right]^T \left(\left[K^{(e)}\right] + \left[\tilde{R}^{(e)}\right]\right)\left[L^{(e)}\right]\right\}, \qquad (12)$$

where $\{dS\}$ and $\{dU\}$ are nodal forces and nodal displacement vectors of the discretizated body.

We indicate matrix $\left[\tilde{K}\right]$ as overall stiffness matrix of the nodal assembly [3]. In the case of stress-controlled loading process the matrix $[K']$ is degenerated to zero-matrix. In the case of displacement-controlled loading process matrix $\left[\tilde{K}\right]$ is the matrix of discretizated body with satisfied kinematic boundary conditions by the 'suppression method'. Simultaneous equation (12) has a single-valued solution, if the overall stiffness matrix is positively defined.

Stability conditions of deformation and damage accumulation

Applying Druccer's approach [4] to total combination of deformable and loading systems the sufficient conditions of deformation and damage accumulation stability we could represent by

$$\int_{\Omega} \delta\varepsilon(\mathbf{r}) \cdot\cdot \tilde{\mathbf{C}}(\mathbf{r}) \cdot\cdot \delta\varepsilon(\mathbf{r})\, d\Omega + \int_{\Sigma} \delta\mathbf{u}(\mathbf{r}) \cdot \mathbf{R}(\mathbf{u},\mathbf{r}) \cdot \delta\mathbf{u}(\mathbf{r})\, d\Sigma > 0 \qquad (13)$$

for the virtual increments of strains $\delta\varepsilon(\mathbf{r})$ and displacements $\delta\mathbf{u}(\mathbf{r})$ provided the boundary conditions are constant ($\mathbf{du}^0 = \mathbf{0}$). The condition (13) can be written by

$$\delta\varepsilon^* \cdot\cdot \tilde{\mathbf{C}}^* \cdot\cdot \delta\varepsilon^* \Omega + \sum_{m=1}^{3} \delta\varepsilon^* \cdot \mathbf{r}^{(m)} \cdot \mathbf{R}^{(m)} \cdot \delta\varepsilon^* \cdot \mathbf{r}^{(m)} \Sigma^{(m)} > 0\,, \qquad (14)$$

where $\tilde{\mathbf{C}}^*$ is tensor of effective tangent elastic modulus and $\delta\varepsilon^*$ is kinematically feasible variations of macro-strains, for discovered at the macro-homogeneous stress-strain state rectangular representative volume of heterogeneous medium, on that surface the coefficients of loading system stiffness $R_{ij}(\mathbf{u},\mathbf{r})$ are uniformly distributed. The body faces with the normal $\mathbf{n}^{(m)}$ and area $\Sigma^{(m)}$ are marked by index m.

During to research of the structural damage accumulation of the heterogeneous body can be extracted external (loading system) and internal sources of supplied mechanical energy. The internal energy source is connected with the release of elastic deformation potential energy when the partial or complete unloading of undamaged structural elements is carried out. According to inequation (13) the spontaneous damage propagation without increasing an external loading is impossible, if for damage work is insufficient supplied external and release mechanical energy. Non-fulfilment of stability conditions (13) corresponds to unstability macroscopically failure.

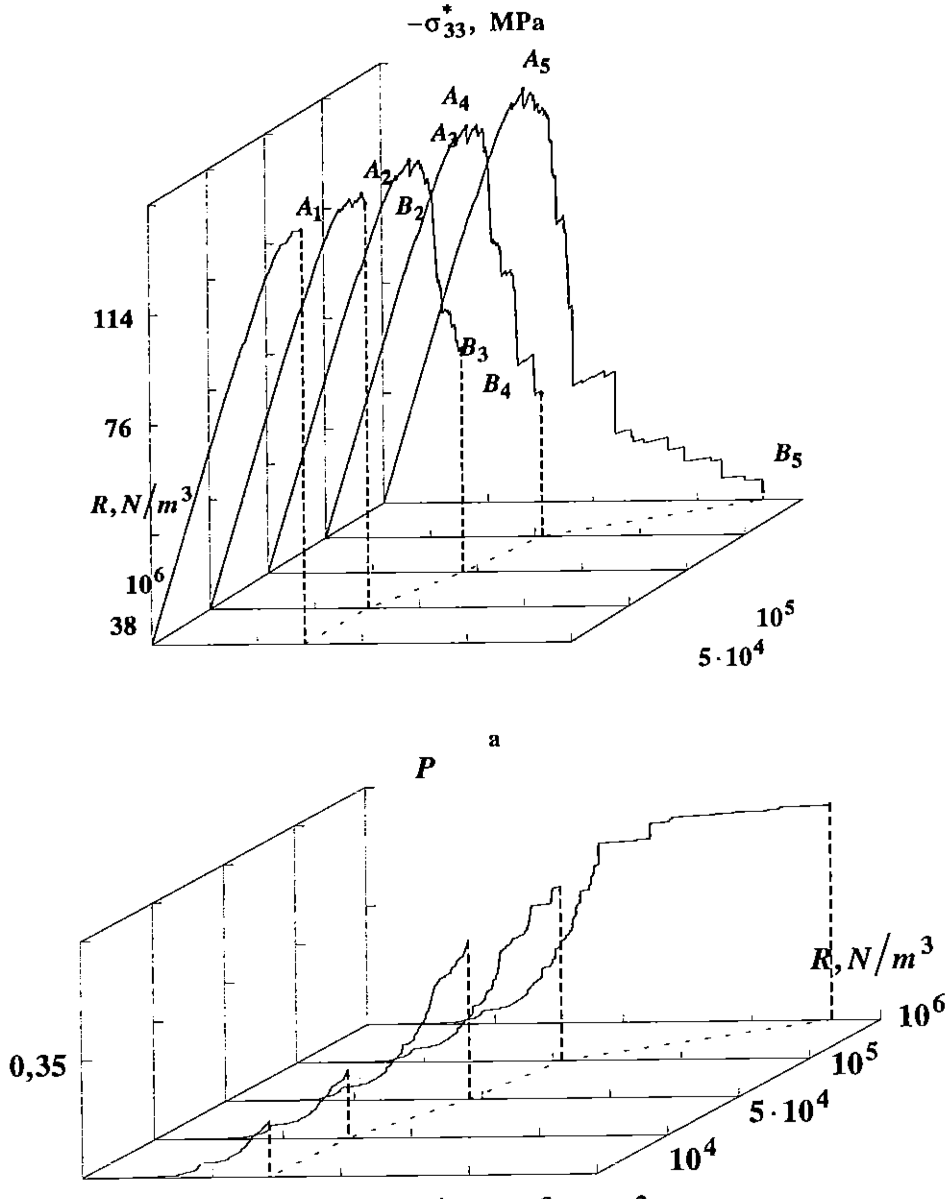

Fig . 1. One-axial compression stress-strain diagrams of the granular composite (a) and damage accumulation curves (b)

Replacing inequation (13) or inequation (14) by the discrete analogue we could demonstrate, that condition of deformation and damage accumulation stability is equivalent to positively definition of discretizated body overall stiffness matrix $[\tilde{K}]$. Thus, the equilibrium structural damage of the damage-loosed material is possible only in the special condition defined by the loading system stiffness. Even in the case, when the loading system is extreme hard in two directions $\mathbf{e}_1$ and $\mathbf{e}_2$, for example, and extreme soft in $\mathbf{e}_3$ direction, the damage accumulation can proceed in an equilibrium mode, that leads to the work-softening behaviour of the medium [5].

Structural damage evolution and failure of heterogeneous body

As an example we can consider results of the boundary value problem solution with boundary conditions (3) and stability conditions (13) for one of the representative volume of the granular composite realization, that fills a unit cub and contains 3072 homogeneous tetrahedron elastic-brittle structural elements. Weibull distribution parameters of the random strength constants $j_{\varepsilon\ cr}^{(2)}(\mathbf{r})$ and the values of the determined elastic modula are presented in the article [6]. The accounts by the finite element method is spent at step proportional increasing of the macro-stress tensor components.

The stress-strain diagrams plotted as a result of computing tests at one-axial compression in the $\mathbf{e}_3$ direction for one of the granular composite representative volume random structure realization are shown on fig. 1, a. The hypothesis of the loading system stiffness coefficients $R_{ij}(\mathbf{u},\mathbf{r})$ uniformity distribution on the close surface of the heterogeneous body and possibility of representation

$$\mathbf{R} = R_1\mathbf{e}_1 \otimes \mathbf{e}_1 + R_2\mathbf{e}_2 \otimes \mathbf{e}_2 + R_3\mathbf{e}_3 \otimes \mathbf{e}_3.$$

are accepted. The loading system is supposed extreme soft in directions $\mathbf{e}_1$, $\mathbf{e}_2$ and have a finite stiffness in $\mathbf{e}_3$ direction.

The volumetric fraction P of the structural elements damaged by forming during deformation of the granular composite as a function of ε_{33}^{*} is shown on fig. 1, b. To the maximal points A_i on the stress-strain diagrams corresponds damage level 16,7 %. The macroscopic failure of the composite as the effect of loss of damage-loosed body deformation stability was determined by absence of the boundary value problem solution at a violation of positive definition of the discretizated body overall stiffness matrix $[\tilde{K}]$. The equilibrium states of the composite directly prior to failure are marked by points B_i on fig. 1, a. It was established, that the coefficient of variation for maximum in modulus macro-stress and failure strains values does not exceed 3 % as the result of averaging on 20 realization of the composite random structure.

The stages of the structural damage, which replace each other are observed at loading of the composite. While the damage level does not exceed 7 % the process proceeds uniformly in all representative volume. Further increase of external loading lead to defects nucleation, local and general damage localization. The development of the localized damage centres is observed at the attainment moment of strength limit formation. In a mode proportional soft loading ($R \equiv R_3 \leq 10^2 \ N/m^3$) the diagram is broken off in the peak point A_5 at $\sigma_{33}^* = -143{,}4$ MPa and $\varepsilon_{33}^* = -1{,}7 \cdot 10^{-3}$. On the ascending branches of the stress-strain diagrams the value of the coefficient R has no significant influence on the mechanical behaviour of the heterogeneous body.

The negative hardening of the granular composite is accompanied by the uniform accumulation of localized damaged centres (stage of secondary dispersible damage accumulation) and some decreasing up to 35 % of volume fraction of partially unloaded structural elements. Continuation of loading effects to modification of the structural damage mechanism (secondary localization) connected with the beginning of the failure cluster formation. Violent increasing up to 60 % volume fraction of unloaded structural elements and increasing of descending branch angle of inclination took place.

At one-axial compression of the composite with $R = 10^4 \ N/m^3$ equilibrium states on a section A_4B_4 is registered, and with $R = 5{,}0 \cdot 10^4 \ N/m^3$ — on a section A_3B_3. Failure of the medium occurs in the first case as result of unstable evolution of localized damage clusters at $\varepsilon_{33}^* = -1{,}8 \cdot 10^{-3}$ ($\sigma_{33}^* = -140{,}1$ MPa, $P = 20{,}3$ %), and in the second one owing to relative stabilisation of the process only at $\varepsilon_{33}^{*} = -2{,}3 \cdot 10^{-3}$ ($\sigma_{33}^{*} = -77{,}5\,\text{MPa}$, $P = 42{,}2\,\%$).

At $R = 10^5$ N/m^3 the structural damage proceeds as a whole in an equilibrium mode before $\varepsilon_{33}^{*} = -2{,}5 \cdot 10^{-3}$ and $P = 51{,}6\,\%$. The failure cluster formation is completed by unstable development and as a consequence of the composite failure at $\sigma_{33}^{*} = -49{,}8\,\text{MPa}$. Plotted at $R = 10^6$ N/m^3 the stress-strain diagrams does not differ from a curve registered in the displacement-controlled mode. In this case the macroscopically failure of the body occurs owing to stability evolution of the failure cluster at $\sigma_{33}^{*} = -3{,}0\,\text{MPa}$, $\varepsilon_{33}^{*} = -4{,}3 \cdot 10^{-3}$ and $P = 64{,}3\,\%$.

Thus, the description of deformation and damage within the framework of a considered structurally-phenomenological model of heterogeneous media allows to register and to investigate the phenomenon of growth of failure strains at the increase of the loading system stiffness.

Unequal resistance of heterogeneous body phenomenon

For the constitutive equations construction of the work-softening media is of interest the mechanical behaviour of the heterogeneous body at different relations between hydrostatics part and deviator or first and second invariants of macro-strain tensor. In the conditions of proportional strain-controlled loading mode this relations are constantly.

The influence of macro-homogeneous stress-strain type on the relations between the second invariant of macro-stress tensor and the second invariant of macro-strain tensor are shown on fig 2, a. The work-softening behaviour at weak decreasing the composite resistance is observed after the attainment moment of strength. The range of deformation at the considered stage depends on the hydrostatics part value of macro-strain tensor. Unequal resistance of heterogeneous body phenomenon is displayed as not observation of the uniform stress-strain diagram on the post-critical deformation stage.

According to loading conditions more intensive increasing of $J_{\varepsilon}^{(1)}$ positive value during increasing of $J_{\varepsilon}^{(2)}$ steeper decreasing of descending branch of the diagrams. In the case of unit forming ($\varepsilon_{11}^{*} = \varepsilon_{22}^{*} = -0{,}5 \cdot \varepsilon_{33}^{*}$, $\varepsilon_{33}^{*} > 0$) streamline passage from positive to negative hardening are observed (curve 4 on fig. 2, a). On the other hand the influence on the strength limit is not so essential. Maximal values of $J_{\sigma}^{(2)}$ are varied within 10%.

The relations between the first invariants of the macro-stress and macro-strain tensors are non-linear. Corresponding to the maximal values of $J_{\sigma}^{(2)}$ the stable states of heterogeneous body on the curves $J_{\sigma}^{(1)} \sim J_{\varepsilon}^{(1)}$ are marked by symbols A_i and attained practically with the maximal values of $J_{\sigma}^{(1)}$ (fig. 2, b). Hence, the passage to work-softening stage is determined by the maximal or critical value of $J_{\sigma}^{(2)} = J_{\sigma\,cr}^{(2)}$, that is a constant of material. The descending branch of the relation $J_{\sigma}^{(1)} \sim J_{\varepsilon}^{(1)}$ is resulted by attainment of the critical damaged state, which induced to realisation of the descending branch of $J_{\sigma}^{(2)} \sim J_{\varepsilon}^{(2)}$ diagram. The critical value of $J_{\sigma}^{(1)}$ are not a constant of material

$$J_{\sigma\,cr}^{(1)} = J_{\sigma}^{(1)}\left(J_{\varepsilon}^{(1)}, J_{\varepsilon}^{(2)}, J_{\sigma\,cr}^{(2)}\right) = J_{\sigma}^{(1)}\Big|_{J_{\sigma}^{(2)} = J_{\sigma\,cr}^{(2)}}$$

and cold be the most various depending on the macro-deformation scheme.
According to eq. (9) the relations are uniquely determined by material functions:

$$\kappa^{*} = \kappa^{*}\left(J_{\varepsilon}^{(1)}, J_{\varepsilon\,cr}^{(2)}\right), \quad g^{*} = g^{*}\left(J_{\varepsilon}^{(1)}, J_{\varepsilon}^{(2)}, J_{\varepsilon\,cr}^{(2)}\right).$$

Results of computing tests have shown, that the value of $J_{\varepsilon}^{(1)}$ more influence on descending branch angle of inclination. However, the character of relations $J_{\sigma}^{(1)} \sim J_{\varepsilon}^{(1)}$ from $J_{\varepsilon}^{(2)}$ has not practically depended.

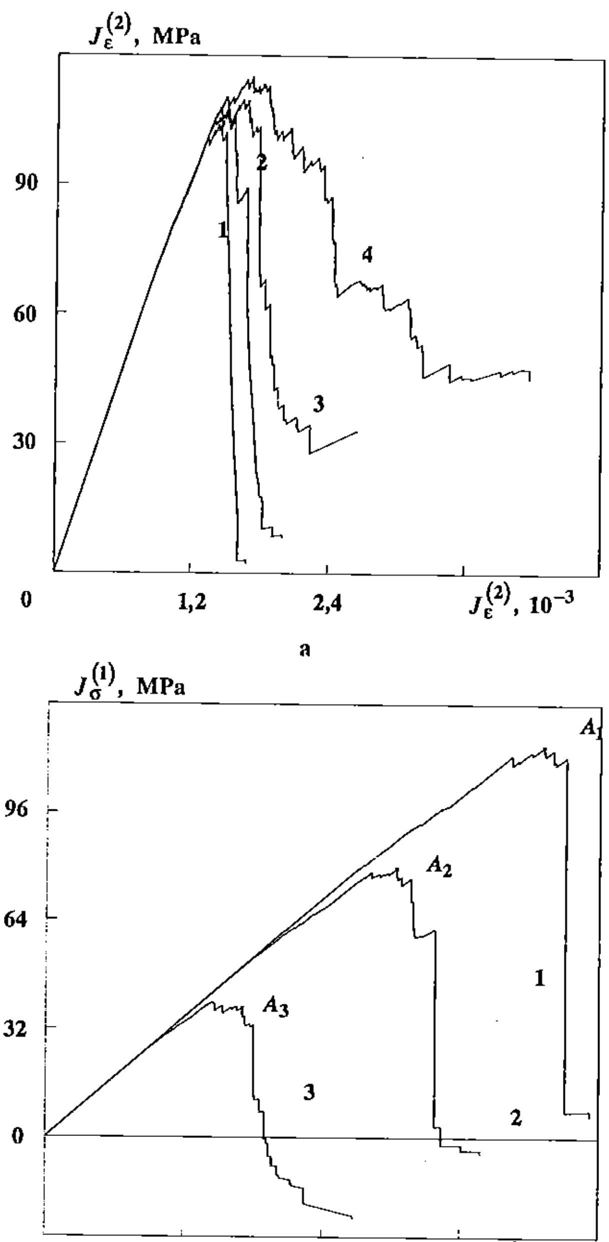

Fig. 2. Stress-strain diagrams of the granular composite in invariant form:
1 — $\varepsilon_{33}^* = 1{,}6 \cdot \varepsilon$, $\varepsilon_{11}^* = \varepsilon_{22}^* = 0{,}1 \cdot \varepsilon$, $\varepsilon > 0$; 2 — $\varepsilon_{33}^* = 1{,}4 \cdot \varepsilon$, $\varepsilon_{11}^* = \varepsilon_{22}^* = -0{,}1 \cdot \varepsilon$, $\varepsilon > 0$;
3 — $\varepsilon_{33}^* = 1{,}2 \cdot \varepsilon$, $\varepsilon_{11}^* = \varepsilon_{22}^* = -0{,}3 \cdot \varepsilon$, $\varepsilon > 0$; 4 — $\varepsilon_{33}^* = \varepsilon$, $\varepsilon_{11}^* = \varepsilon_{22}^* = -0{,}5 \cdot \varepsilon$, $\varepsilon > 0$.

Equilibrium states of damaged heterogeneous medium in conditions of non-monotone loading

At the research of deformation and structural damage of the granular composite local non-stability of the damage accumulation process was marked. In the composite structure local volumes are detected, non-equilibrium damage of which did not affected neither loading system stiffness nor loading increasing steps. Similar discrete energy dissipation appeared as separate more or less expanded sudden changes on the stress-strain diagrams. This phenomenon occured because the damage evolution was defined mainly by internal source of supplied mechanical energy. Therefore, we did not success controlling the character of the damage accumulation even in displacement-controlled monotone loading mode on the structural level completely. The observable stages of stable and unstable damage can be investigated by the modelling of non-monotone loading carried out with the help of servo-controlled testing machines.

The presence of expanded non-equilibrium sudden changes on the relations of macro-stresses from macro-strains is characteristic for materials having tendency to self-supported damage accumulation on the post-critical deformation stage at the given macro-homogeneous stress-strain state. The mechanical behaviour allows to assume the possibility of existence of additional equilibrium states of the medium, that cannot be realised within the framework of the displacement-controlled monotone loading mode.

The preventive unloading method is offered for mathematical modelling of the damage accumulation during the test of materials on the servo-controlled systems. The method assumes maintenance of balance between resistance forces of the body and external forces, that is attained by realization of a number of cycles unloading and active loading at appearance of uncontrolled character of damage accumulation. The excess of critical volume fraction of damaged elements in the result of the stress redistribution after the next damage act was considered as a condition of the preventive elastic unloading. The cycles of unloading and active loading are accompanied by the structural damage of the material. Therefore, the heterogeneous medium cannot attained the stress-strain state took place at the beginning of preventive unloading.

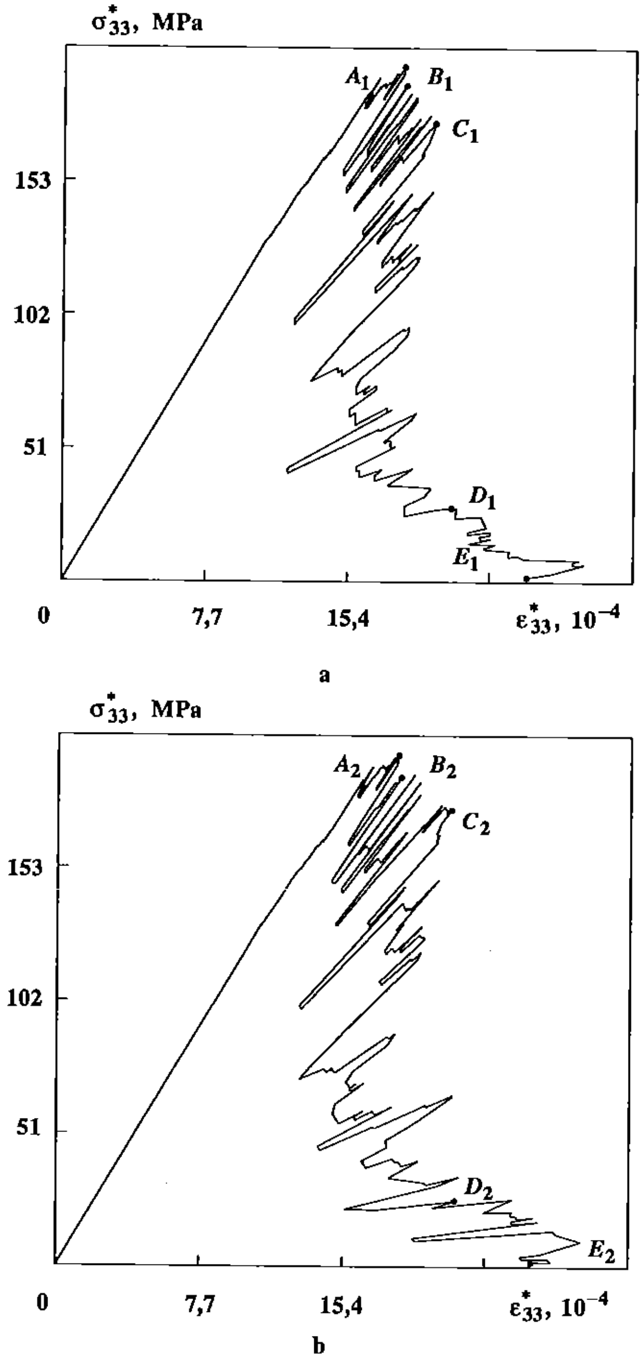

Fig. 3. Stress-strain diagrams one-axial ($\varepsilon^*_{33} > 0$, $\varepsilon^*_{11} = \varepsilon^*_{22} = 0$) deformation of the granular composite plotted by preventive unloading method: extreme hard loading (a) and loading with $R = 10^4 \; N/m^3$ (b).

The geometrical place of extreme equilibrium states registered by the preventive unloading method is represented on fig. 3. The maximal points of the curves are corresponded to the same stress-strain states with the damage level of 8,1 %. The stages of damage development occured without increasing an external load in result of stress redistributions but at interaction of localized damage centres with failure cluster were detected at a research of damage accumulation processes in the monotone loading mode. Thus, the level is varied from 15,9 % (points B_i) up to 33,6 % (points C_i).

The critical volume fraction of damage in the condition of preventive unloading corresponds to damage by forming only one structural element. The failure of the granular composite occured at decreasing of σ_{33}^* practically up to zero (points E_i). The damage level of the medium at the moment of loss of carrying capacity made 39,3 %. The ascending branches of the relations $\sigma_{33}^* \sim \varepsilon_{33}^*$ (fig. 3) plotted by the preventive unloading method does not practically differ from the sections plotted in the displacement-controlled monotone mode [2].

The possibility of the descending branch plotting on the testing system with rather small loading system stiffness $R = 10^4 \ N/m^3$ for the considered case by preventive unloading method at one-axial deformation of the granular composite is illustrated on fig. 3, b. The loading system is supposed extreme hard in directions $\mathbf{e}_1$ and $\mathbf{e}_2$, having a finite stiffness in $\mathbf{e}_3$ direction. We can marked, that the point C_2 corresponded to damage level 17,7% is the last equilibrium point fixed in a monotone loading mode with an indicated stiffness. The implementation of the post-critical deformation stage in this computing test allows to make the conclusion, that the displacement-controlled mode can be imitated by a series of soft loading and unloading cycles.

On the post-critical deformation stage each loading cycle represents the autonomous test of the granular composite with an increased damage level and new strength constant. Therefore, the descending branch of the stress-strain diagram is possible to define as the geometrical places of material strength with a various damage volume fraction owing to continuity of the damage accumulation.

Conclusion

Thus, the developed two-level structurally-phenomenological model of non-elastic deformation and damage accumulation of the heterogeneous medium allows to investigate the dispersible and localized, an equilibrium and non-equilibrium damage stages. It was shown, that the increase of the loading system stiffness promotes stabilisation of the processes. The structural damage accompanied by the composite negative hardening within the framework of the considered model is the mechanism of elastic energy dissipation, that sufficient for the material accommodation to the strain-controlled process at the conditions of the mechanical energy inflow restriction from the side of rather hard loading system. The acts of partial or complete loss of carrying capacity by separate structural elements on the initial deformation stage are developed as random events described within the framework of stochastic representations. On the other hand localization and failure cluster formation stages are defined mainly by energy redistributions between the deformable body and loading system. Mathematical modelling of the heterogeneous media work-softening behaviour with the apply of the preventive unloading method allows to plot complete stress-strain diagrams even at soft loading mode and to detect the equilibrium states, that peculiar of damaged materials with negative slope of the descending branch, not detected neither at displacement-controlled nor at stresses-controlled monotone loading modes. Unequal resistance phenomenon is detected for the indicated type of materials, as the dependency of the mechanical behaviour of the heterogeneous body at the triaxial stress-strain state.

Reference

(1) Wildemann V.E., Sokolkin Yu.V., Tashkinov A.A. (1995) Boundary value problem of deformation and failure mechanics of damaged bodies with work-softening volumes // J. Appl. Mech. Techn. Phis. vol. 36. pp. 122 — 132.

(2) Wildemann V.E., Zaitsev A.V. (1996) Equilibrium fracture processes of granular composites // Mechanics of Composite Materials. vol. 32. pp. 808 — 817.

(3) Wildemann V.E., Zaitsev A.V. (1996) On numerical solutions of boundary value problem of deformation and structural damage mechanics with third kind boundary conditions by the finite element method // Numerical Technologies. vol. 1. pp. 67—79.

(4) Drucker D.C. (1959) A definition of stable inelastic material //Trans. ASME. Ser. E. J. Appl. Mech. vol. 26. pp. 101 — 106.

(5) Ryzhak E.I. (1994) On stability of homogeneous elastic bodies under boundary conditions weaker than displacements conditions // Q. J. Mech appl. Math. vol. 47. pp. 663 — 672.

(6) Wildemann V.E., Zaitsev A.V. (1996) Work-softening behaviour and failure of composite materials with granular structure // Mechanics of Composite Materials and Structures. vol. 2. pp. 117—124.