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Calculation of Stress Intensity Factors for Cracks Subjected to Arbitrary Non-Linear Stress Fields

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ABSTRACT: *Fatigue cracks in shot peened and case hardened notched machine components are subjected to the notch tip stress field induced by the load and the residual stress resulting from the surface treatment. Both stress fields are highly non-linear and appropriate handbook stress intensity solutions are unavailable for such configurations, especially in the case of planar surface breaking cracks. The method presented in the paper is based on the generalized weight function technique enabling the stress intensity factors to be calculated for any Mode I loading. Both the general weight functions and the calculated stress intensity factors are validated against various numerical and analytical data. The numerical procedure for calculating stress intensity factors for arbitrary non-linear stress distributions is briefly discussed as well. The method is particularly suitable for modeling fatigue crack growth of single buried elliptical, surface semi-elliptical and multiple cracks.*

Notation

a - depth of a semi-elliptical, elliptical (minor semi-axis) or edge crack
A - the deepest point of surface, semi-elliptical crack
B - the surface point of semi-elliptical crack
c - half length of semi-elliptical or elliptical crack (major semi-axis)
K_I - mode I stress intensity factor (general)
K_{IA} - mode I stress intensity factor at the deepest point A
K_{IB} - mode I stress intensity factor at the surface point B
M_i - coefficients of weight functions (i= 1, 2, 3)
M_{iA} - coefficients of the weight functions for the deepest point A (i= 1, 2, 3)
M_{iB} - coefficients of the weight functions for the surface point B (i= 1, 2, 3)
m(x,a) - weight function (general)
$m_A(x,a)$ - weight function for the deepest point A of a semi-elliptical surface crack
$m_B(x,a)$ - weight function for the surface point B of a semi-elliptical crack
Q - elliptical crack shape factor
S - external (applied) load
s - shortest distance between the point load and the crack front

t - thickness
Φ - angle co-ordinate for parametric representation of an ellipse
Ω - crack area
ρ - distance between the point load and any point A on the crack front
$\rho_1, \rho_2, \rho_3, \rho_4$ - geometrical parameters of a planar crack
$\sigma(x)$ - a stress distribution over the crack surfaces
σ_0 - nominal or reference stress (usually the maximum value of $\sigma(x)$)
x - the local, through the thickness co-ordinate
Y - geometric stress intensity correction factor

Introduction

Fatigue durability, damage tolerance and strength evaluation of notched and cracked structural elements require calculation of stress intensity factors for cracks located in regions characterized by complex stress fields. This is particularly true for cracks emanating form notches or other stress concentration regions that are frequently found in mechanical and structural components. In the case of engine components, complex stress distributions are often due to temperature, geometry and surface finish resulting in superposition of applied, thermal and residual stresses. In the case of welded or riveted structural components, it is often necessary to deal with cracked components repaired by overlapping patches. Such components require fatigue analysis of cracks propagating through a variety of interacting stress fields. Moreover, these are often planar two-dimensional surface or buried cracks with irregular shapes. The existing handbook stress intensity factor solutions are not sufficient in such cases due to the fact that most of them have been derived for simple geometry and load configurations. The variety of notch and crack configurations, and the complexity of stress fields occurring in engineering components require more versatile tools for calculating stress intensity factors than the currently available ready made solutions, obtained for a range of specific geometry and load combinations.

Therefore, a method for calculating stress intensity factors for one- and two-dimensional cracks subjected to two-dimensional stress fields is discussed below. The method is based on the use of the weight function technique.

Stress Intensity Factors And Weight Functions

Most of the existing methods of calculating stress intensity factors require separate analysis of each load and geometry configuration. Fortunately, the weight function method developed by Bueckner [1] and Rice [2] simplifies considerably the determination of stress intensity factors. The important feature of the weight function is that it depends only on the geometry of the cracked body. If the weight function is known for a given cracked body, the stress intensity factor due to any load system applied to the body can be determined by using the same weight function. The success of the weight function technique for calculating stress intensity factors lies in the possibility of using superposition. It can be shown, [3], that the stress intensity factor for a cracked body (Fig. 1) subjected to the external loading, S, is the same as the stress intensity factor in a geometrically identical body with the local stress field $\sigma(x)$ applied to the crack faces. The local stress field, $\sigma(x)$, induced in the prospective crack plane by the external load, S, is determined for *uncracked* body which makes the stress analysis relatively simple.

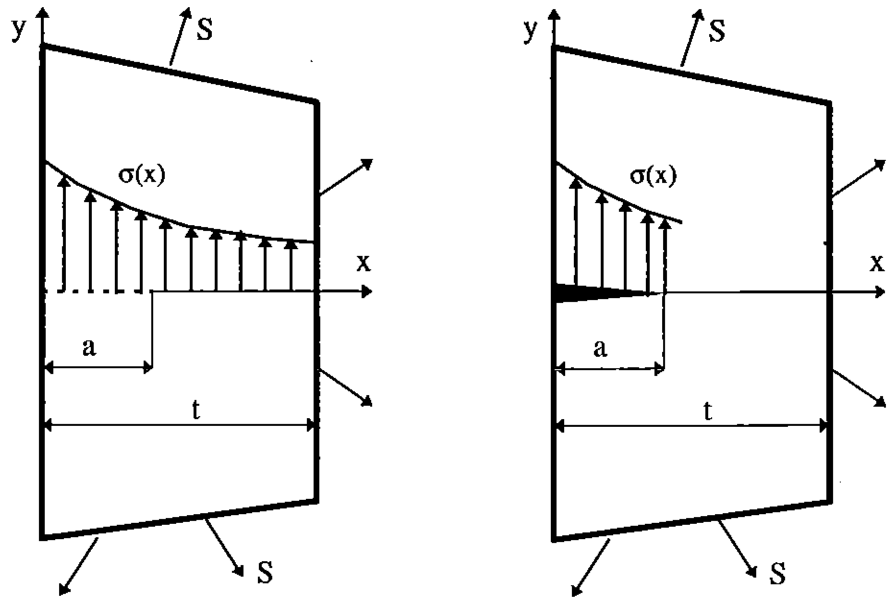

Fig.1. Nomenclature and the concept of superposition

Therefore, if the weight function is known there is no need to derive ready made stress intensity factor expressions for each load system and associated internal stress distribution.

The stress intensity factor for a one dimensional crack can be obtained by multiplying the weight function, m(x,a), and the internal stress distribution, $\sigma(x)$, in the prospective crack plane, and integrating the product along the crack length 'a'.

$$K = \int_0^a \sigma(x) m(x,a) dx \qquad (1)$$

The weight function, m(x,a), can be interpreted (Fig.2) as the stress intensity factor that results from a pair of splitting forces, P, applied to the crack face at position x.

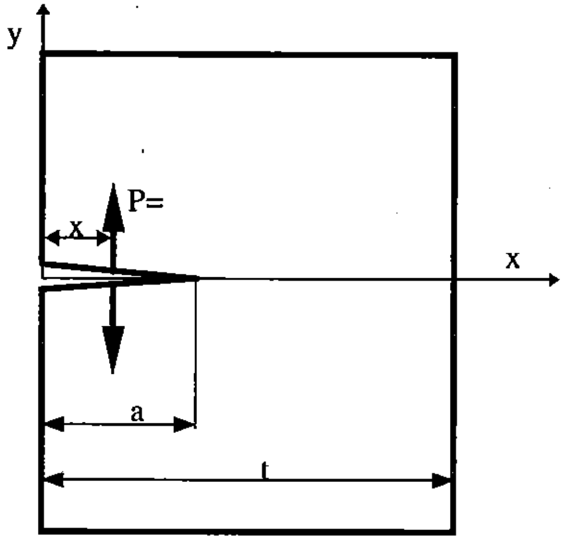

Fig. 2. Weight function for an edge crack in a finite width plate; nomenclature

Since the stress intensity factors are linearly dependent on the applied loads, the contributions from multiple splitting forces applied along the crack surface can be superposed and the resultant stress intensity factor can be calculated as the sum of all individual load contributions. This results in the integral, (1), of the product of the weight function, m(x,a), and the stress function, $\sigma(x)$, for a continuously distributed stress field. A variety of one dimensional (line-load) weight functions can be found in references [4,5,6]. However, their mathematical forms vary from case to case and therefore they are not easy to use. Therefore, Shen and Glinka [7] have proposed one general weight function form which can be used for a wide variety of Mode I cracks.

Universal Weight Functions For One-Dimensional Stress Fields

The weight function is dependent on the geometry only and in principle should be derived individually for each geometrical configuration. However, Glinka and Shen [7] have found that one general weight function expression can be used to approximate weight functions for a variety of geometrical crack configurations subjected to one-dimensional stress fields of Mode I.

$$m(x,a)=\frac{2}{\sqrt{2\pi(a-x)}}\left[1+M_1\left(1-\frac{x}{a}\right)^{\frac{1}{2}}+M_2\left(1-\frac{x}{a}\right)^{1}+M_3\left(1-\frac{x}{a}\right)^{\frac{3}{2}}\right] \tag{2}$$

As an example the system of coordinates and the notation for an edge crack are given in Fig. 2. In order to determine the weight function, m(x,a), for a particular cracked body, it is sufficient to determine, [8], the three parameters M_1, M_2, and M_3 in expression (2). Because the mathematical form of the weight function, (2), is the same for all cracks, the same methods can be used for the determination of parameters M_1, M_2, and M_3 and the integration routine for calculating stress intensity factors from eq.(1). The method of finding the M_i parameters has been discussed in reference [8]. Moreover, it has been found that only limited number of generic weight functions is needed to enable the calculation of stress intensity factors for a large number of load and geometry configurations. In the case of 2-D cracks such as the surface breaking semi-elliptical crack in a finite thickness plate or cylinder, the stress intensity factor changes along the crack front. However, in many practical cases the deepest point, A, and the surface point, B, are associated (Fig. 3) with the highest and the lowest value of the stress intensity factor respectively.

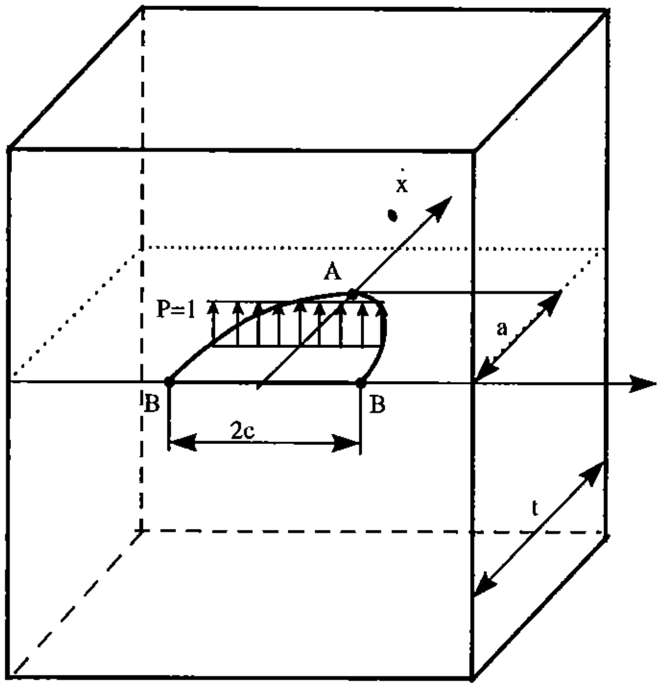

Fig. 3. Semi-elliptical surface crack under the unit line load; weight function notations

Therefore, weight functions for the points A and B of a semi-elliptical crack have been derived, [9], analogously to the universal weight function of eq.(2).

- For point A (Fig. 3)

$$m_A(x,a,a/c,a/t) = \frac{2}{\sqrt{2\pi(a-x)}}\left[1 + M_{1A}\left(1-\frac{x}{a}\right)^{\frac{1}{2}} + M_{2A}\left(1-\frac{x}{a}\right)^{1} + M_{3A}\left(1-\frac{x}{a}\right)^{\frac{3}{2}}\right] \quad (3)$$

- For point B (Fig. 3)

$$m_B(x,a,a/c,a/t) = \frac{2}{\sqrt{\pi x}}\left[1 + M_{1B}\left(\frac{x}{a}\right)^{\frac{1}{2}} + M_{2B}\left(\frac{x}{a}\right)^{1} + M_{3B}\left(\frac{x}{a}\right)^{\frac{3}{2}}\right] \quad (4)$$

The weight functions, $m_A(x,a)$ and $m_B(x,a)$, for the deepest and the surface points, A and B, respectively have been derived for the crack face unit *line loading* making it possible to analyze one-dimensional stress fields (Fig. 3), dependent on one variable , x , only.

A variety of universal line load weight functions [9-13] have been derived and published already. The M_i parameters for the edge (Fig.2) and the semi-elliptical surface crack (Fig.3) in a finite thickness plate are given in the Appendix.

Sequence of Steps for Calculating Stress Intensity Factors Using Weight Functions

In order to calculate stress intensity factors using the weight function technique the following tasks need to be carried out:

- Determine stress distribution, $\sigma(x)$, in the prospective crack plane using linear elastic analysis of uncracked body (Fig. 1a), i.e. perform the stress analysis ignoring the crack and determine the stress distribution $\sigma(x) = \sigma_0 f(S,x)$;
- Apply the "uncracked" stress distribution, $\sigma(x)$, to the crack surfaces (Fig. 1b) as traction
- Choose appropriate generic weight function
- Integrate the product of the stress function $\sigma(x)$ and the weight function, $m(x,a)$, over the entire crack length or crack surface, eq.(1).

The weight function (3) for the deepest point A was used to calculate stress intensity factor for the non-linear stress field (5) acting in the crack plane.

$$\sigma(x) = \sigma_0 \left(1 - \frac{x}{a}\right)^2 \quad (5)$$

The M_{iA} parameters for the weight function (3) are given in the Appendix (eq. A4-A6). The accuracy and the versatility of the weight function (3) for the semi-elliptical crack in a finite plate (Fig. 3) is illustrated in Figs. 4 and 5, showing the comparison with the Finite Element results of Wang [12] and Shiratori [14]. It can be seen that the agreement is good over the entire range of parameters for which the weight function parameters (A4-A6) have been derived. The parameters M_i of the weight function (3) were derived using the reference data for $0<a/t<0.9$ and $0<a/c<1$. The accuracy of the weight function while compared with the finite element data was better than 3%.

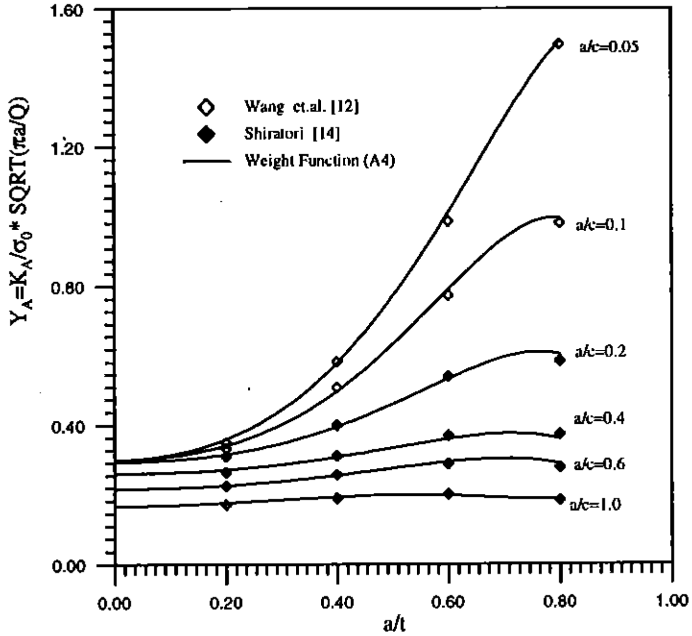

Fig. 4. Comparison of the weight function based stress intensity factor and FEM data [14] for quadratic stress distribution; the deepest point A

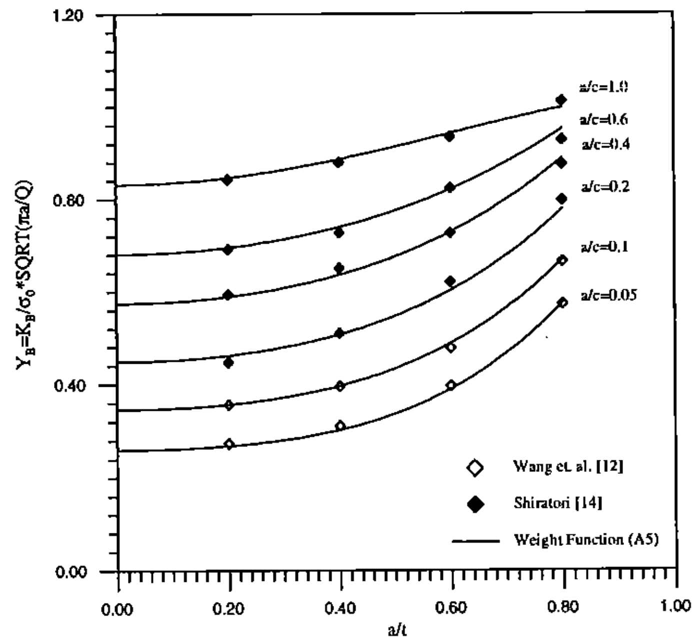

Fig. 5. Comparison of the weight function based stress intensity factor and FEM data [12, 14] for quadratic stress distribution; the surface point B

Weight Functions for Two-Dimensional Stress Fields

In spite of the efficiency and great usefulness of the line load weight functions (Figs. 2 & 3), they cannot be used in many practical cases where the stress fields is of two-dimensional nature, i.e. where the stress field σ(x,y) in the crack plane depends on the x and y coordinates. Therefore, a weight function for the unit point load (Fig. 6) is needed in order to calculate stress intensity factors for cracks subjected to two-dimensional stress fields.

A two-dimensional point-load weight function, $m_A(x,y)$, represents the stress intensity factor at point, A, on the crack front (Fig. 6), induced by a pair of unit forces attached to the crack surface at point P(x,y). If the weight function is given in a closed mathematical form, it allows to calculate the stress intensity factor at any point along the crack front. In order to determine the stress intensity factor at any point on the crack front induced by a two-dimensional stress field, σ(x,y), the product of the stress and the weight function needs to be integrated over the entire crack surface area Ω .

$$K_A = \int_{\Omega} \sigma(x,y) m_A(x,y,P) dx dy \tag{6}$$

There are only a few point load (2-D) weight functions available. Among them the weight function for an embedded elliptical crack in an infinite body is the most often discussed in the literature [15-17]. Unfortunately, most of the existing point-load weight functions are given in the form of complex mathematical expressions difficult to use in practice. Therefore an attempt was made to present all the existing point-load weight functions in an uniform form making them easier for comparisons and numerical analysis. It has been found that all the existing point load weight functions for cracks in an infinite three dimensional bodies can be expressed using the s, ρ, ρ_1, ρ_2, ρ_3 and ρ_4 parameters shown in Fig. 6.

It was found that the weight function for an elliptical crack in an infinite 3-D body can be written in the form of eq.(7).

$$m_A(x,y,P) = \frac{P\sqrt{s}}{\pi^{3/2}\rho^2}\sqrt{2 - \frac{s}{4\rho_1} - \frac{s}{4\rho_2} - \frac{s}{4\rho_3} - \frac{s}{4\rho_4}} \tag{7}$$

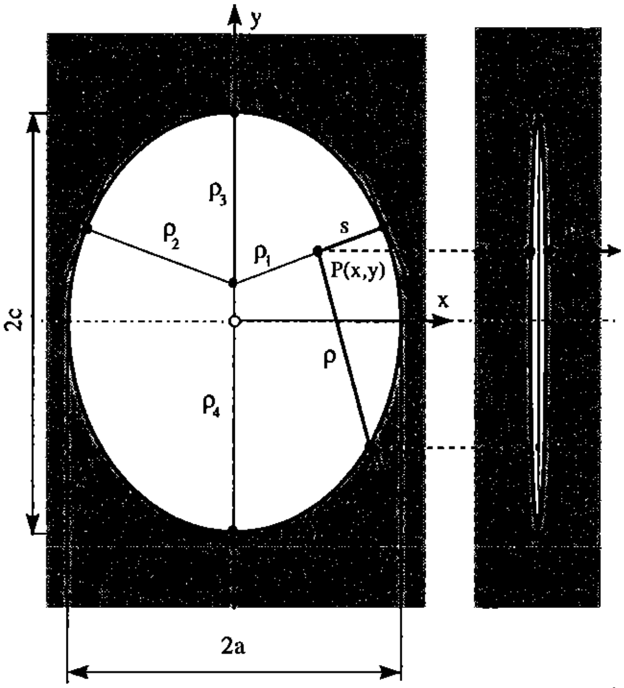

Fig. 6. Elliptical crack in infinite three-dimensional body: point load weight function notation

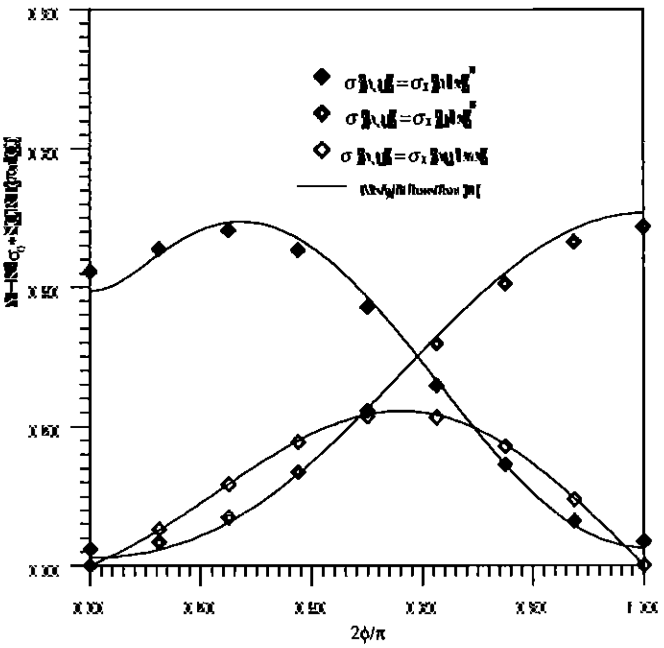

Fig. 7. Comparison of the analytical [19[and the weight function [18] based stress intensity factors for an elliptical embedded crack subjected to nonlinear stress fields; a/c=0.2

The weight function (7) was verified [18] by comparison of the weight function (7) based stress intensity factors with the semi-analytical data of Shah and Kobayashi [19]. The stress intensity factor results shown in Fig. 7 were calculated using eqns.(6) and (7) and three different stress fields given by expressions (8-10).

$$\sigma(x,y)=\sigma_0\left(\frac{x}{a}\right)^2 \tag{8}$$

$$\sigma(x,y)=\sigma_0\left(\frac{x}{c}\right)^2 \tag{9}$$

$$\sigma(x,y)=\sigma_0\frac{xy}{ac} \tag{10}$$

The agreement between the weight function based calculations [18] and the data obtained by Shah and Kobayashi [19] was very good for a wide range of ellipse aspect ratios a/c. The data shown in Fig. 7 were obtained for a/c=0.2. The weight function (7) can be used to derive weight functions already known in the literature.

By assuming that parameters ρ_1, ρ_2, ρ_3 and ρ_4 tend to infinity the weight function (11) for an infinite edge crack in an infinite [20] body can be derived (Fig. 8)

$$m_A(x,y,P)=\frac{P\sqrt{s}}{\pi^{3/2}\rho^2}\sqrt{2} \tag{11}$$

By setting all the parameters $\rho_1=\rho_2=\rho_3=\rho_4=a$ the well known weight function (12) for a penny shape crack [21] in infinite body (Fig. 9) is derived.

$$m_A(x,y,P)=\frac{P\sqrt{s}}{\pi^{3/2}\rho^2}\sqrt{2-\frac{s}{a}} \tag{12}$$

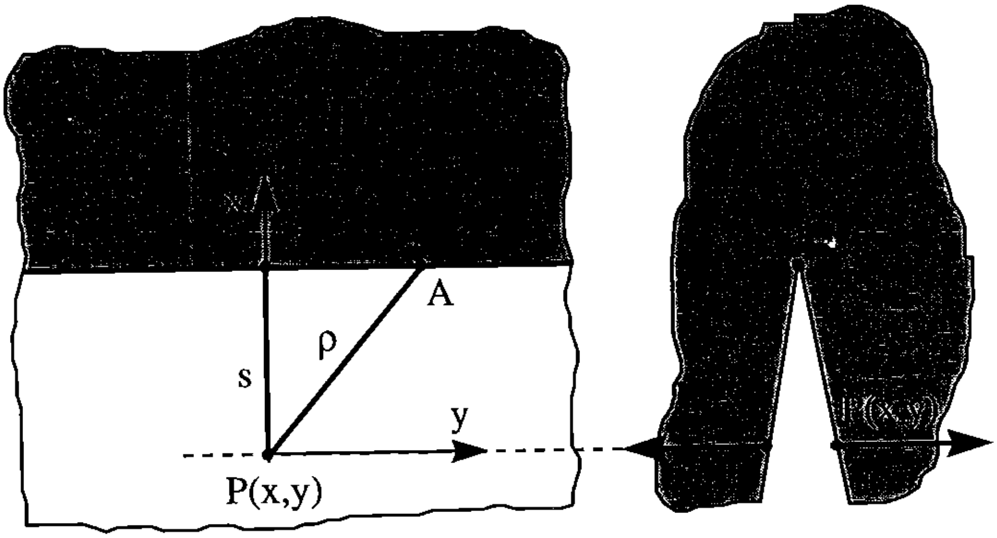

Fig. 8. Infinite edge crack in an infinite body

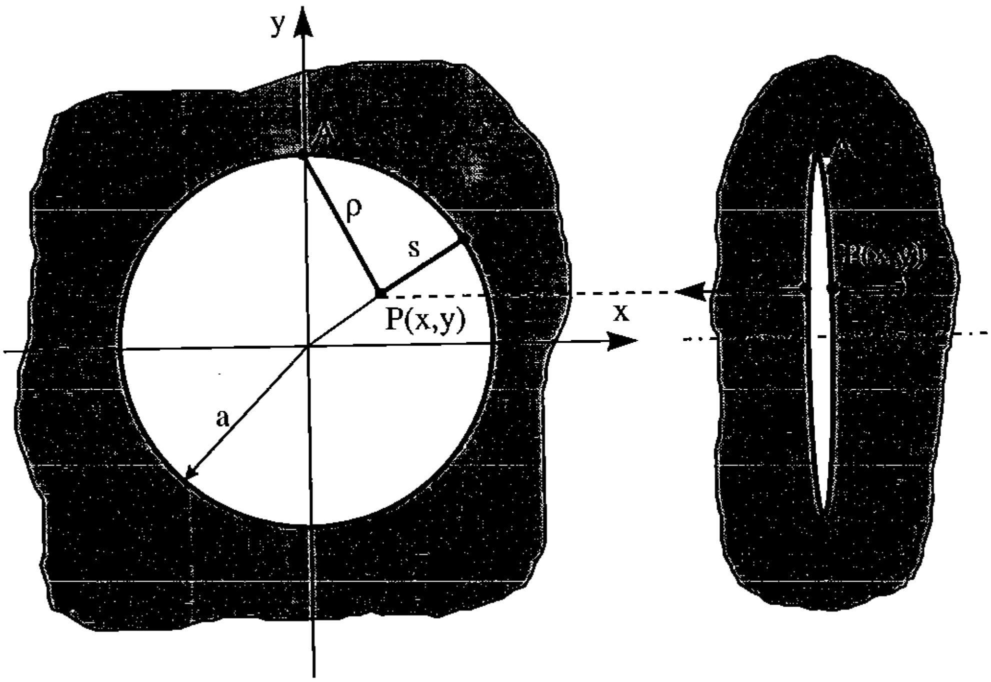

Fig. 9. Penny shape crack in an infinite body

Conclusions

The weight functions for mode I cracks can be approximated by using one general expressions containing the most important geometrical parameters. The knowledge of the general weight function expression makes it possible to determine easily weight functions for particular geometrical configurations and to integrate them using the same numerical procedure. It has been found that the approximate weight functions gave accurate estimation of stress intensity factors for a variety of non-linear stress fields.

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Appendix

a) Parameters M_1 (A1), M_2 (A2) and M_3 (A3) of weight function (2) for an edge crack in a finite thickness plate (Fig. 3)

$$M_1 = \frac{-0.029207+\frac{a}{t}(0.213074+\frac{a}{t}(-3.029553+\frac{a}{t}(5.901933+\frac{a}{t}(-2.657820))))}{1.0+\frac{a}{t}(-1.259723+\frac{a}{t}(-0.048475+\frac{a}{t}(0.481250+\frac{a}{t}(-0.526796+\frac{a}{t}(0.345012)))))}$$

$$M_2 = \frac{0.451116+\frac{a}{t}(3.462425+\frac{a}{t}(-1.078459+\frac{a}{t}(3.558573+\frac{a}{t}(-7.553533))))}{1.0+\frac{a}{t}(-1.496612+\frac{a}{t}(0.764586+\frac{a}{t}(-0.659316+\frac{a}{t}(0.258506+\frac{a}{t}(0.114568)))))}$$

$$M_3 = \frac{0.427195+\frac{a}{t}(-3.730114+\frac{a}{t}(16.276333+\frac{a}{t}(-18.799956+\frac{a}{t}(14.112118))))}{1.0+\frac{a}{t}(-1.129189+\frac{a}{t}(0.033758+\frac{a}{t}(0.192114+\frac{a}{t}(-0.658242+\frac{a}{t}(0.554666)))))}$$

Valid for $0 < a/t < 0.9$

b) Parameters M_{iA} of weight function (3) for a semi-elliptical surface crack in a finite thickness plate (Fig. 4) - the deepest point A

$$M_{1A} = \frac{\pi}{\sqrt{2Q}}(4Y_0 - 6Y_1) - \frac{24}{5} \quad \text{(A4)}$$

$$M_{2A} = 3 \quad \text{(A5)}$$

$$M_{3A} = 2\left(\frac{\pi}{\sqrt{2Q}}Y_0 - M_{1A} - 4\right) \quad \text{(A6)}$$

where:

$$Y_0 = B_0 + B_1\left(\frac{a}{t}\right)^2 + B_2\left(\frac{a}{t}\right)^4 + B_3\left(\frac{a}{t}\right)^6$$

$$B_0 = 1.0929 + 0.2581\left(\frac{a}{c}\right) - 0.7703\left(\frac{a}{c}\right)^2 + 0.4394\left(\frac{a}{c}\right)^3$$

$$B_1 = 0.456 - 3.045\left(\frac{a}{c}\right) + 2.007\left(\frac{a}{c}\right)^2 + \frac{1.0}{0.147 + \left(\frac{a}{c}\right)^{0.688}}$$

$$B_2 = 0.995 - \frac{1.0}{0.027 + \frac{a}{c}} + 22.0\left(1 - \frac{a}{c}\right)^{9.953}$$

$$B_3 = -1.459 + \frac{1.0}{0.014 + \frac{a}{c}} - 24.211\left(1 - \frac{a}{c}\right)^{8.071}$$

$$Q = 1.0 + 1.464\left(\frac{a}{c}\right)^{1.65}$$

and

$$Y_1 = A_0 + A_1\left(\frac{a}{t}\right)^2 + A_2\left(\frac{a}{t}\right)^4 + A_3\left(\frac{a}{t}\right)^6$$

$$A_0 = 0.4537 + 0.1231\left(\frac{a}{c}\right) - 0.7412\left(\frac{a}{c}\right)^2 + 0.4600\left(\frac{a}{c}\right)^3$$

$$A_1 = -1.652 + 1.665\left(\frac{a}{c}\right) - 0.534\left(\frac{a}{c}\right)^2 + \frac{1.0}{0.198 + \left(\frac{a}{c}\right)^{0.846}}$$

$$A_2 = 3.418 - 3.126\left(\frac{a}{c}\right) - \frac{1.0}{0.041 + \left(\frac{a}{c}\right)} + 17.259\left(1 - \frac{a}{c}\right)^{9.286}$$

$$A_3 = -4.228 + 3.643\left(\frac{a}{c}\right) + \frac{1.0}{0.020 + \frac{a}{c}} - 21.924\left(1 - \frac{a}{c}\right)^{9.203}$$

c) Parameters M_{iB} of weight function (4) for a semi-elliptical surface crack in a finite thickness plate (Fig. 4) - the surface point B

$$M_{1B} = \frac{\pi}{\sqrt{4Q}}(30F_1 - 18F_0) - 8 \qquad \text{(A7)}$$

$$M_{2B} = \frac{\pi}{\sqrt{4Q}}(60F_0 - 90F_1) + 15 \quad \text{(A8)}$$

$$M_{3B} = -(1 + M_{1B} + M_{1B}) \quad \text{(A9)}$$

where:

$$F_0 = \left[C_0 + C_1\left(\frac{a}{t}\right)^2 + C_2\left(\frac{a}{t}\right)^4\right]\sqrt{\frac{a}{c}}$$

$$C_0 = 1.2972 - 0.1548\left(\frac{a}{c}\right) - 0.0185\left(\frac{a}{c}\right)^2$$

$$C_1 = 1.5083 - 1.3219\left(\frac{a}{c}\right) + 0.5128\left(\frac{a}{c}\right)^2$$

$$C_2 = -1.101 + \frac{0.879}{0.157 + \frac{a}{c}}$$

and

$$F_1 = \left[D_0 + D_1\left(\frac{a}{t}\right)^2 + D_2\left(\frac{a}{t}\right)^4\right]\sqrt{\frac{a}{c}}$$

$$D_0 = 1.2687 - 1.0642\left(\frac{a}{c}\right) + 1.4646\left(\frac{a}{c}\right)^2 - 0.7250\left(\frac{a}{c}\right)^3$$

$$D_1 = 1.1207 - 1.2289\left(\frac{a}{c}\right) + 0.5876\left(\frac{a}{c}\right)^2$$

$$D_2 = 0.190 - 0.608\left(\frac{a}{c}\right) + \frac{0.199}{0.035 + \frac{a}{c}}$$