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Applicability of Shakedown Theory to Power Plant Pipelines

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ABSTRACT: *The paper presents the application of an incremental collapse criterion in the case when the yield limit value is temperature-dependent. The Young's modulus and thermal expansion coefficient values are assumed to be independent of temperature. The criterion is illustrated by the incremental collapse analysis of a thick-walled tube subjected to variations of internal pressure and temperature field. Both linear and logarithmic temperature distributions across the pipe wall thickness are found suitable for the analysis. The computations are performed for two different temperatures at the external wall side (293 and 473K). The extreme values of shakedown parameters are determined. The computational results are compared with those obtained from a lot of 9 actual pipes in service.*

Introduction

Structural components subject to cyclic loading are assumed to undergo either incremental collapse, alternating plasticity or plastic shakedown. The shakedown theory is an obvious extension of the limit analysis to the variable load case. A principal goal of the shakedown theory is to determine allowable variations of loads that will not induce plastic strains during the cycles following the first one or a few first ones producing residual stresses. If loads vary over a sufficiently large range then a structure will collapse due to increasing plastic strains of a given sign (incremental collapse) or due to alternating plastic strains (alternating plasticity).

The two principal theorems of the shakedown theory were given by Melan (1) and Koiter (2). The Melan theorem is a generalized form of the theorem dealing with statically permissible stress fields for time-dependent loads. With the Melan theorem applied to elastic–plastic structures there is no way of knowing whether the actual conditions will lead to incremental collapse or alternating plasticity. It is possible to differentiate between those conditions if a kinematically permissible solution is considered. This solution makes use of properties of the permissible rate field and the Koiter theorem is again a generalized form of the theorem pertaining to kinematically allowable rate fields for time-dependent loads.

A practically important case of loading involves a combination of variable mechanical loads and time-dependent temperatures. The following effects can then be specified (3):

- thermal deformations affect stress fields,
- yield point stress value varies with temperature,
- elastic constants vary with temperature.

Kinematic shakedown theorem

The shakedown theorems have been derived initially accounting only for mechanical loads (2, 4). Their extensions to thermal actions (5–8) took into consideration not only thermal stresses but also the fact that material constants such as yield point stress vary with temperature. In the case of a static approach (5, 6, 8) this effect as well as the temperature dependence of elastic moduli can be incorporated relatively easily. However, more complicated boundary-value problems are to be solved by means of the kinematic approach, especially if incremental collapse is considered. The methods developed (9–12) allow to find out the critical loads which may cause divergent increments of plastic deformations simply from the analysis of possible mechanisms of those increments, without tedious integration with respect to time as the original theorem required.

The kinematic shakedown theorem accounting for both thermal and mechanical actions may be formulated as follows (13):

A given structure will not shake down over a certain load–temperature path if there exists over a certain time period (t_1, t_2) a load–temperature path and a plastic strain rate cycle resulting in compatible increments of plastic strain

$$\Delta\bar{\varepsilon}_{ij} = \int_{t_1}^{t_2} \dot{\bar{\varepsilon}}_{ij} dt = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right), \tag{1}$$

and such that

$$\int_{t_1}^{t_2} \left\{ \sum_{s=1}^{r} \beta_s(t) \left[\int_V F_i^s \dot{u}_i dV + \int_{S_P} P_i^s \dot{u}_i dS \right] + \int_V M_{ij} T \dot{\hat{\rho}}_{ij} dV \right\} dt > \int_{t_1}^{t_2} \int_V D\left(\dot{\bar{\varepsilon}}_{ij}, T\right) dV dt, \tag{2}$$

where M_{ij} – tensor of the thermal expansion coefficients, D – dissipation function, $\dot{\hat{\rho}}_{ij}$ denotes the residual stress field associated with the plastic strain field $\bar{\varepsilon}_{ij}$. The external actions resulting in some mechanical loads as well as in temperature fields are controlled by a set of load–temperature factors β_s, $s = 1,\ldots,r$, referring to each one of the actions, respectively:

$$P_i(x,t) = \sum_{s=1}^{r} \beta_s(t) P_i^s(x), \ F_i(x,t) = \sum_{s=1}^{r} \beta_s(t) F_i^s(x), \ T(x,t) = \sum_{s=1}^{r} \beta_s(t) T^s(x). \tag{3}$$

Here P_i – surface tractions, F_i – body forces, T – temperature measured from the natural state.

The values of the factors β_s belong to a certain set Ω in the r-dimensional space of those parameters. The set Ω defines the range of their prescribed variations.

The inequality (2) can be easily rearranged by applying the principle of virtual work:

$$\int_{t_1}^{t_2} \int_V \sigma_{ij}^E(x,t) \dot{\bar{\varepsilon}}_{ij}(x,t) dV dt > \int_{t_1}^{t_2} \int_V D\left(\dot{\bar{\varepsilon}}_{ij}, T\right) dV dt, \tag{4}$$

where the thermoelastic stress σ_{ij}^E can be presented as follows:

$$\sigma_{ij}^E(x,t) = \sum_{s=1}^{r} \beta_s(t) \sigma_{ij}^{Es} = \sum_{s=1}^{r} \beta_s(t) \left[\sigma_{ij}^{EEs}(x) + \rho_{ij}^{Ts}(x) \right]. \tag{5}$$

Here σ_{ij}^{Es}, ρ_{ij}^{Ts} are respective thermoelastic and thermal stress fields associated with unit external actions, σ_{ij}^{EEs} denoting respective mechanical stresses.

Incremental collapse criterion

Let us consider the case in which the temperature variations of yield stress cannot be neglected. Then the dissipation function depends not only on the plastic strain rate $\dot{\varepsilon}_{ij}^{p}$ but also on the instantaneous temperature:

$$D = \sigma_{ij}\dot{\varepsilon}_{ij}^{P} = D\left(\dot{\varepsilon}_{ij}^{P}, T\right), \tag{6}$$

and is proportional to the increase in the yield point stress *k*:

$$D = D_0\left(\dot{\varepsilon}_{ij}^{P}\right)g(T), \tag{7}$$

where *g(T)* defines the temperature dependence of *k*:

$$k(T) = k_0 g(T), \quad g(0) = 1, \tag{8}$$

and D_0 is the value of the dissipation at zero temperature, determined uniquely by the plastic strain rate $\dot{\varepsilon}_{ij}^{P}$.

In further considerations the function *g(T)* will be linearized:

$$g(T) = 1 - AT, \tag{9}$$

A being a non-negative material constant.

After some rearrangements we can present formula (4) in the following form:

$$\int_{t_1}^{t_2}\int_V \sum_{s=1}^{r} \beta_s(t)\left[\sigma_{ij}^{Es}(x)\dot{\bar{\varepsilon}}_{ij}(x,t) + AT^s(x)D_0\left(\dot{\bar{\varepsilon}}_{ij}\right)\right]dVdt > \int_{t_1}^{t_2}\int_V D_0\left(\dot{\bar{\varepsilon}}_{ij}\right)dVdt. \tag{10}$$

König (13, 14) showed that the incremental collapse criterion assumes finally the form:

$$\int_V L(x)dV = \int_V D_0\left(\Delta\bar{\varepsilon}_{ij}\right)dV, \tag{11}$$

where

$$L(x) = \max_{\beta_s \in \Omega} \sum_{s=1}^{r} \beta_s\left[\sigma_{ij}^{Es}(x)\Delta\bar{\varepsilon}_{ij}(x) + AT^s(x)D_0\left(\Delta\bar{\varepsilon}_{ij}\right)\right]. \tag{12}$$

If the domain Ω is defined by the set of inequalities

$$\beta_s^- \le \beta_s \le \beta_s^+, \tag{13}$$

then equation (11) can be written as below:

$$\int_V \sum_{s=1}^{r} a_s(x) J_s(x) dV = \int_V D_0\left(\Delta\bar{\varepsilon}_{ij}\right) dV, \tag{14}$$

where $a_s(x)$, $J_s(x)$ are given by the formulas:

$$a_s(x) = \begin{cases} \beta_s^+ & \text{if } J_s(x) > 0 \\ \beta_s^- & \text{if } J_s(x) < 0 \end{cases}, \tag{15}$$

$$J_s(x) = \sigma_{ij}^{Es}(x)\Delta\bar{\varepsilon}_{ij}(x) + AT^s(x) D_0\left(\Delta\bar{\varepsilon}_{ij}(x)\right). \tag{16}$$

Application of the kinematic method to analysis of a thick-walled tube

The incremental collapse analysis of a thick-walled tube, closed with rigid decks, subjected to variations of internal pressure and temperature field is considered.

The following simplifying assumptions are adopted:

- rheological effects are neglected,
- yield limit value is linearly dependent on temperature,
- elastic moduli and thermal expansion coefficient are independent of temperature,
- a simplified (regular–cyclic) pattern of temperature variation is adopted,
- effect of thermal insulation is accounted for in a simplified manner,
- both linear and logarithmic temperature distributions across the pipe wall thickness are taken into account.

The pipe is loaded with:

1. internal pressure p varying over the range $0 \le p \le p_{max}$,
2. internal temperature Θ varying over the range $0 \le \Theta \le \Theta_{max}$. $\Theta = T(a) - T(b)$, provided $T(b) = 0$, where $T(a)$ and $T(b)$ are temperatures of the internal and external pipe surface, respectively.

The stresses due to pressure p are:

$$\sigma_{\phi} = \frac{pa^2}{b^2 - a^2}\left(1 + \frac{b^2}{r^2}\right),$$

$$\sigma_{r} = \frac{pa^2}{b^2 - a^2}\left(1 - \frac{b^2}{r^2}\right), \tag{17}$$

$$\sigma_{z} = \frac{pa^2}{b^2 - a^2},$$

where r - current radius, a - internal pipe radius, b - external pipe radius.

The thermal stresses may be in accordance with **(15)** determined as:

$$\sigma_{\phi} = \frac{E}{1-\nu}\left[\frac{1}{r^2}\int_a^r \alpha T(r) r dr + \frac{r^2 + a^2}{r^2(b^2 - a^2)}\int_a^b \alpha T(r) r dr - \alpha T(r)\right],$$

$$\sigma_{r} = \frac{E}{1-\nu}\left[-\frac{1}{r^2}\int_a^r \alpha T(r) r dr + \frac{r^2 - a^2}{r^2(b^2 - a^2)}\int_a^b \alpha T(r) r dr\right], \tag{18}$$

$$\sigma_{z} = \frac{E}{1-\nu}\left[\frac{2}{b^2 - a^2}\int_a^b \alpha T(r) r dr - \alpha T(r)\right],$$

where $T(r)$ – temperature as a function of the pipe radius r, σ_{ϕ} – circumferential stress, σ_r – radial stress, σ_z – axial stress, E – Young's modulus, ν – Poisson's ratio, α – thermal expansion coefficient. The values of E, ν, α were assumed to be temperature–independent material constants.

It can be inferred from the axial symmetry of the pipe that incremental collapse is the only possible mechanism of failure, hence **(14)**:

$$\dot{u}(r) = \dot{C}/r, \ \Delta\varepsilon_{\phi} = \Delta C/r^2, \ \Delta\varepsilon_{r} = -\Delta C/r^2. \tag{19}$$

Tresca yield criterion was adopted:

$$\left|\sigma_{\phi} - \sigma_{r}\right| \le 2k(T), \ k(T) = k_0(1 - AT), \tag{20}$$

and therefore

$$D_0 = \sigma_r\left(-\frac{\Delta C}{r^2}\right) + \sigma_{\phi}\left(\frac{\Delta C}{r^2}\right) = 2k_0\frac{\Delta C}{r^2} \tag{21}$$

Prior to computations temperature changes across the pipe wall thickness as obtained from the linear and logarithmic temperature distributions were compared:

$$T(r) = \Theta \frac{\ln(b/r)}{\ln(b/a)}, \tag{22}$$

$$T(r) = \Theta \frac{b - r}{a - r}. \tag{23}$$

The computed temperature distributions across the pipe wall thickness are shown in Fig. 1. Two actual tubes were selected with the largest and the smallest value of $k = a/b$.

Assuming $T(b) = 293$ K we arrive at the following form of equations (22) and (23):

$$T(r) = T(b) + [T(a) - T(b)] \frac{\ln(r/b)}{\ln(a/b)}, \tag{22a}$$

for the logarithmic distribution, and

$$T(r) = T(b) + [T(a) - T(b)] \frac{b - r}{b - a}, \tag{23a}$$

for the linear distribution.

Differences between temperatures at the middle point of the wall obtained from the two formulas are small. For a pipe with $k = 0.70$ the discrepancy is about 23 K and for $k = 0.90$ it does not exceed 6.5 K.

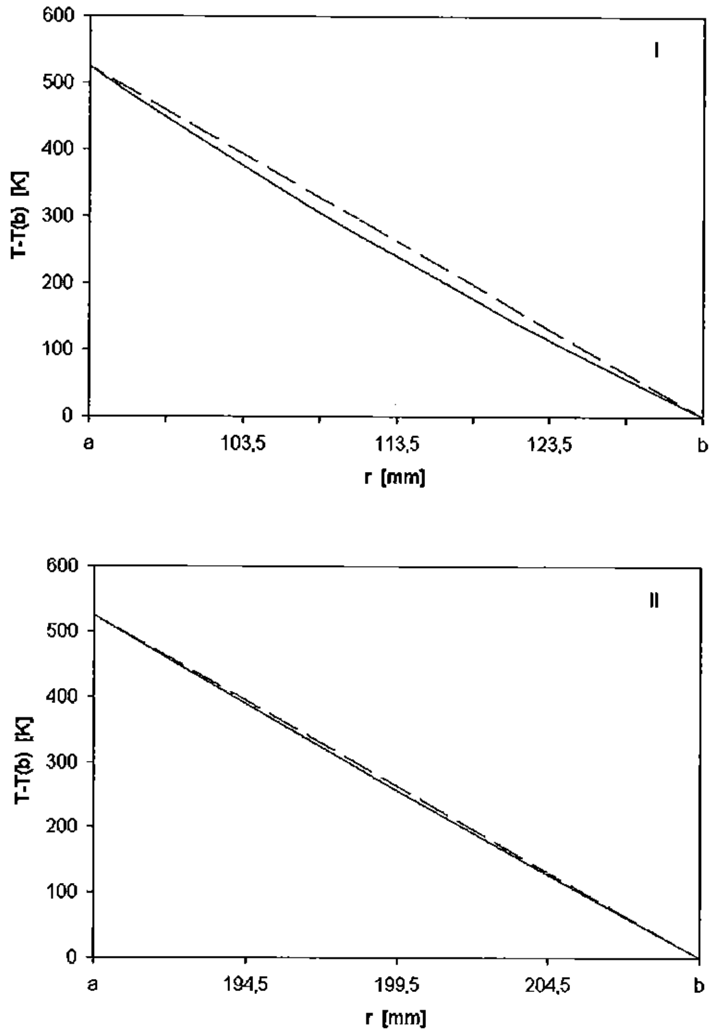

Fig. 1 **Steel 10H2M. Calculated temperature distribution (linear - dashed line, logarithmic - solid line) for two pipes: I) - k = 0.70, II) - k = 0.90, where k = a/b, a and b are internal and external radius of the pipe, respectively. T(a) = 818K, T(b) = 293K**

Substituting (22) into (18) and taking account of (17) we get total values of stress due to pressure and temperature:

$$\sigma_\phi = \frac{pa^2}{b^2 - a^2}\left(1 + \frac{b^2}{r^2}\right) - \frac{E\alpha a^2 \Theta}{2(1-\nu)\left(b^2 - a^2\right)}\left\{1 + \frac{b^2}{r^2} + \frac{\left(b^2 - a^2\right)\left[1 + \ln(r/b)\right]}{a^2 \ln(a/b)}\right\},$$

$$\sigma_r = \frac{pa^2}{b^2 - a^2}\left(1 - \frac{b^2}{r^2}\right) - \frac{E\alpha a^2 \Theta}{2(1-\nu)\left(b^2 - a^2\right)}\left[1 - \frac{b^2}{r^2} + \frac{\left(b^2 - a^2\right)\ln(r/b)}{a^2 \ln(a/b)}\right],$$

(24)

$$\sigma_z = \frac{pa^2}{b^2 - a^2} - \frac{E\alpha a^2 \Theta}{(1-\nu)\left(b^2 - a^2\right)}\left\{1 + \frac{\left(b^2 - a^2\right)\left[\frac{1}{2} + \ln(r/b)\right]}{a^2 \ln(a/b)}\right\}.$$

Values of $J_s(r)$ in equation (14) were calculated for $s = p$ and $s = \Theta$ and they are as follows:

$$J_p(r) = \frac{2\Delta C a^2 b^2}{r^4\left(b^2 - a^2\right)}, \tag{25}$$

$$J_\Theta = \frac{\Delta C}{r^2}\left\{\frac{E\alpha a^2}{2(1-\nu)\left(b^2 - a^2\right)}\left[\frac{b^2 - a^2}{a^2 \ln(b/a)} - \frac{2b^2}{r^2}\right] + 2Ak_0 \frac{\ln(b/r)}{\ln(b/a)}\right\}.$$

(26)

Values of J_p and J_Θ are the following functions of radius r:

$$\begin{aligned} &J_p(r) > 0 \quad \text{for} \quad a \le r \le b, \\ &J_\Theta(r) < 0 \quad \text{for} \quad a \le r < r_0, \\ &J_\Theta(r) > 0 \quad \text{for} \quad r_0 < r \le b, \end{aligned} \tag{27}$$

where radius r_0 can be found from the relationship:

$$-\frac{2b^2}{r_0^2}+\frac{b^2-a^2}{a^2\ln(b/a)}+\frac{4Ak_0(1-\nu)(b^2-a^2)\ln(b/r_0)}{E\alpha a^2\ln(b/a)}=0. \quad (28)$$

Values of $a_p(r)$ and $a_\Theta(r)$ from equation (14) are equal to:

$$a_p(r)=p_{max}, \quad a_\Theta(r)=\begin{cases}0 & \text{for } a\le r<r_0\\ \Theta_{max} & \text{for } r_0<r\le b\end{cases} \quad (29)$$

Finally, in view of formulas (25)–(28), the incremental collapse condition can be written as:

$$p_{max}+\Theta_{max}\frac{E\alpha}{2(1-\nu)}\left\{\frac{\ln(\beta/\rho)}{\ln\beta}+\frac{1}{\beta^2-1}-\frac{\beta^2}{\rho^2(\beta^2-1)}+\frac{1}{2}\varepsilon\frac{[\ln(\beta/\rho)]^2}{\ln\beta}\right\}=2k_0\ln\beta \quad (30)$$

where $\beta=\frac{b}{a}$, $\rho=\frac{r_0}{a}$, $\varepsilon=\frac{4Ak_0(1-\nu)}{E\alpha}$.

The value of ρ obtained from an approximated solution of equation (28) if ε is assumed to be small is (14):

$$\rho=\rho_0\left(1-\frac{\varepsilon}{2}\ln\frac{\beta}{\rho_0}\right), \quad \rho_0=\sqrt{\frac{\beta^2\ln\beta^2}{\beta^2-1}} \quad (31)$$

Computation of the shakedown range for the linear temperature distribution

Substituting (23) into (18) and taking account of (17) we get total values of stress due to pressure and temperature:

$$\sigma_\phi=\frac{pa^2}{b^2-a^2}\left(1+\frac{b^2}{r^2}\right)-\frac{E\alpha\Theta}{3(1-\nu)(b^2-a^2)}\frac{1}{r^2}\left[(r^2+a^2)(b^2+ab+a^2)-(b+a)(2r^3+a^3)\right],$$

$$\sigma_r=\frac{pa^2}{b^2-a^2}\left(1-\frac{b^2}{r^2}\right)-\frac{E\alpha\Theta}{3(1-\nu)(b^2-a^2)}\frac{1}{r^2}\left[(r^2-a^2)(b^2+ab+a^2)-(b+a)(r^3-a^3)\right],$$

$$\sigma_z=\frac{pa^2}{b^2-a^2}-\frac{E\alpha\Theta}{3(1-\nu)(b^2-a^2)}\left[2(b^2+ab+a^2)-3(b+a)r\right].$$

(32)

Values of $J_s(r)$ in equation (14) were calculated for $s = p$ and $s = \Theta$ an d they are as follows:

$$J_p(r) = \frac{2\Delta C a^2 b^2}{r^4 \left(b^2 - a^2\right)}, \tag{33}$$

$$J_\Theta(r) = \frac{\Delta C}{r^2}\left\{\frac{E\alpha}{3(1-\nu)\left(b^2 - a^2\right)r^2}\left[-2a^2b^2 + (b+a)r^3\right] + 2Ak_0 \frac{b-r}{b-a}\right\}. \tag{34}$$

In view of relationship (18) radius r_0 can be found from the equation:

$$K_1 r_0^3 + K_2 r_0^2 + K_3 = 0, \tag{35}$$

where

$$K_1 = E\alpha(b+a) - 6Ak_0(1-\nu)(b+a),$$
$$K_2 = 6Ak_0(1-\nu)(b+a)b,$$
$$K_3 = -2a^2b^2E.$$

Relationships (29), (33) and (34) combined give the following incremental collapse criterion:

$$p_{max} + \Theta_{max}\frac{1}{3(1-\nu)\left(b^2 - a^2\right)}\left[E\alpha\left(a^2 + ab + b^2\right) + K_2(\ln b - 1) - \right.$$
$$\left. - E\alpha a^2 b^2 \left(1/r_0^2\right) - K_2 \ln r_0 - K_1 r_0\right] = 2k_0 \ln\frac{b}{a} \tag{36}$$

where K_1, K_2 are to be meant in the same manner as in (35).

Values of r_0 for each pipe were determined from polynomial (35) using the Cardano's formula.

Results

The actual calculations were performed for pipes commonly used in the Polish power plants made of 12HMF, 15HM and 10H2M steels. The external tube surface temperature T(b) was assumed to be constant and equal to 293K or 473K, the latter being valid for insulated pipes.

Pipe dimensions and actual service conditions are presented in Table 1.

Table 1. Actual service conditions

Steel	Dxg [mm]	p [MPa]	T(a) [K]
12HMF	356x36	8.83	773
	273x26	8.83	773
	273x20	8.83	773
15HM	216x32	12.36	753
	216x16	6.87	758
	292x21	7.85	758
10H2M	267x40	12.85	818
	178x26	12.36	818
	419x20	2.94	818

D - pipe diameter, g - pipe wall thickness

Material properties adopted in the calculations may be found in Table 2.

Table 2. Material properties

Steel	R_e^{293} [MPa]	R_e^{473} [MPa]	E [MPa]	ν	α [K^{-1}]	A [K^{-1}]
12HMF	295	256	2.06×10^5	0.3	14.8×10^{-6}	769.8×10^{-6}
15HM	295	275	2.06×10^5	0.3	14.4×10^{-6}	840.4×10^{-6}
10H2M	265	245	2.06×10^5	0.3	14.0×10^{-6}	621.1×10^{-6}

R_e^{293} - yield point at temperature of 293K

R_e^{473} - yield point at temperature of 473K

E - Young's modulus

ν - Poisson's ratio

α - thermal expansion coefficient

A - coefficient to be found in relationship (20)

Equations (30) and (36) corresponding to the logarithmic and linear temperature distributions may be presented in the following generalized form:

$$p_{max} + a_1\Theta_{max} = a_2 \qquad (37)$$

The coefficients of equation (37) for the steels investigated are shown in Tables 3-5. For two selected cases the computational results are presented graphically in Figs 2 and 3.

Table 3. 12HMF. Values of a_1 and a_2 coefficients.

Dimensions Dxg [mm]	T(b) = 293K		T(b) = 473K	
	a_1	a_2	a_1	a_2
Logarithmic temperature distribution				
356×36	0.130	67	0.129	58
273×26	0.122	62	0.121	54
273×20	0.091	47	0.090	41
Linear temperature distribution				
356×36	0.130	67	0.129	58
273×26	0.122	63	0.121	54
273×20	0.091	47	0.091	41

Table 4. 15HM. Values of a_1 and a_2 coefficients.

Dimensions Dxg [mm]	T(b) = 293K		T(b) = 473K	
	a_1	a_2	a_1	a_2
Logarithmic temperature distribution				
216×32	0.198	104	0.197	97
216×16	0.090	47	0.090	44
292×21	0.088	46	0.087	43
Linear temperature distribution				
216×32	0.199	104	0.198	97
216×16	0.091	47	0.090	44
292×21	0.088	46	0.087	43

Table 5. 10H2M. Values of a_1 and a_2 coefficients.

Dimensions Dxg [mm]	T(b) = 293K		T(b) = 473K	
	a_1	a_2	a_1	a_2
Logarithmic temperature distribution				
267×40	0.191	94	0.190	87
178×26	0.185	92	0.184	85
419×20	0.054	27	0.054	25
Linear temperature distribution				
267×40	0.191	94	0.190	87
178×26	0.185	91	0.185	84
419×20	0.054	26	0.054	24

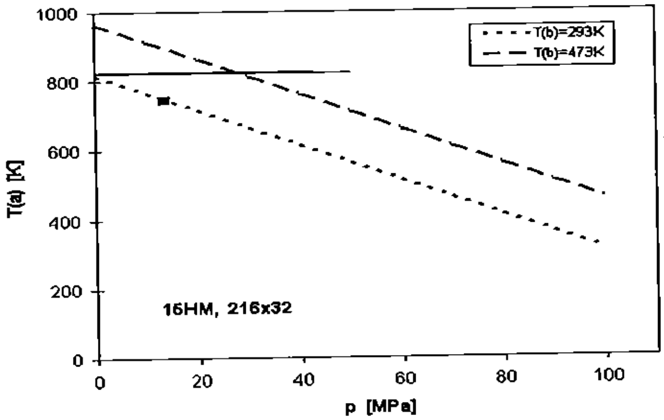

Fig. 2.Steel 15HM.The range of shakedown of the pipe calculated for incremental collapse criterion.

- ■ real parameters of service
- — long-service temperature of steel

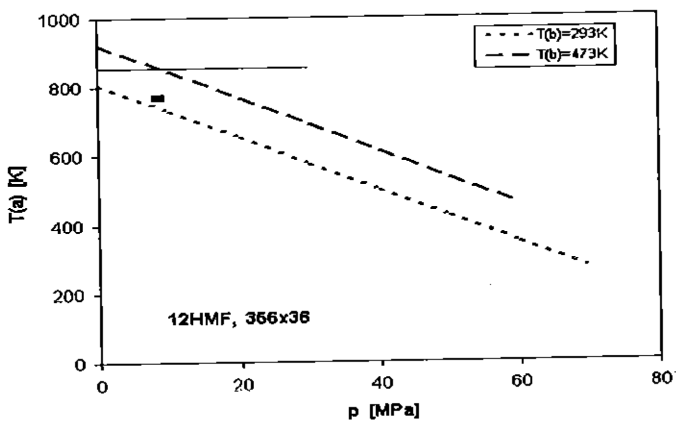

Fig. 3. Steel 12HMF. The range of shakedown of the pipe calculated for incremental collapse criterion.

- ■ real parameters of service
- — long-service temperature of steel

Conclusions

The paper was aimed at evaluating the applicability of the kinematic method (the incremental collapse criterion) to determining the plastic shakedown range in thick-walled tubes operating under cyclic pressure and temperature conditions.

A concept of the method was presented together with a complete solution of the considered case for the logarithmic temperature distribution across the pipe wall thickness with an account taken of the temperature-dependence of material's yield point (**13,14**). A similar solution was found for the linear temperature distribution.

Values of the coefficients $a_1$and a_2 of equation (37) were determined by assuming actual properties of the materials and dimensions of the pipes.

It was found that the discussed differences between the temperature distributions had no significant effect on plastic shakedown parameters, i.e. the corresponding coefficients in equation (37) assumed the same values (see Tables 3-5).

In all cases the real service conditions (pressure and temperature of the internal wall T(a)) were found to be below the parameters of plastic shakedown calculated for insulated pipes with the external wall temperature of T(b) = 473K. In almost all cases the calculated parameters proved, however, to be too low if we assumed T(b) = 293K (see Figs 2 and 3).It is evident therefore that the actual external wall temperature must be taken into account.

By proving that the presented computational method is sufficiently reliable we were in a position to state that service conditions of all nine pipes were properly set and that the plastic shakedown process could take place.

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