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A New Treatment of Fracture in Uniaxial Fields

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Keywords: uniaxial, mixed cracks, directional fracture approach DFA, crack path, directional maximum circumferrential tensile stress criterion ($\sigma_{\theta\ max}$).

ABSTRACT: The macroscopic mixed mode cracks under both remotely uniaxial tension and compression loading are studied in LEFM to predict the initial fracture direction (θ_{cr}), fracture load (σ_{cr}), threshold fracture factors (K_I-K_{II}) and the final crack path during fracture process. The used fracture theory is the Directional Maximum Circumeferential Tensile Stress Criterion ($\sigma_{\theta max}$) [1] which is newly introduced and based on the Directional Fracture Approach (DFA) [1]. In case of mixed mode crack under compression loading, the effect of friction between crack surfaces is considered taking various values of the coefficient of friction (μ). Also, the effect of the negative value of stress intensity factor for mode (I) (K_I) is taken into consideration. The results of this study are fracture angle -crack angle relations (β-θ), normalized fracture load-crack angle relations ($\sigma_{cr}^{\beta}/\sigma_{cr}^{90}$-$\beta$), critical envelopes of stress intensity factors (K_I-K_{II}) and the final crack path during fracture process. Crack trajectories are developed according the new directional fracture technique of crack path [1]. The obtained results are compared with the previous theoretical and experimental work beside the experimental work of the author [1] recording a good agreement.

Notation

θ	Fracture angle at crack tip.	K_{Icr}	Critical stress intensity factor for mode I..
θ_o	Initial fracture angle.	K_{II}	Stress intensity factor for mode II.
θ_{cr}	Critical fracture angle.	K_{IIcr}	Critical stress intensity factor for mode II.
θ_I	Fracture angle for mode I.	σ^{β}_{cr}	Critical fracture load for mixed mode.
θ_{Icr}	Critical fracture angle for mode I.	σ^{90}_{cr}	Critical fracture load for mode I.
θ_{II}	Fracture angle for mode II	σ_{cr}	Fracture load.
θ_{IIcr}	Critical fracture angle for mode II	σ	Applied load
K_I	Stress intensity factor for mode I	β	Crack angle.

$\sigma_\theta, \sigma_r, \tau_{r\theta}$	Stresses at crack tip in polar representation.
$\sigma_{\theta cr}$	Critical value of circumferential tensile stress.
$\sigma_{\theta I}$	Circumferential tensile stress of mod I.
$\sigma_{\theta II}$	Circumferential tensile stress of mod II.
$\sigma^{\beta}_{\theta I}$	Circumferential tensile stress of mod I at crack angle (β).
$\sigma^{90}_{\theta I}$	Circumferential tensile stress of mod I at crack angle (90).
r	Crack tip radius or fracture increment length .

Introduction

The previous studies of fracture analysis were based mainely on two fracture approaches; " Scalar approach " and " Vector-Like quantity approach " . Scalar approach considered the fracture parameters in a scalar shape and dealt only with tension-shear cracks [2] . Vector -Like quantity approach considered the direction of the fracture factors [3] . It was applied for tension-shear cracks but for mixed modes under compression , it considered that the crack to be under shear stress neglecting the effect of normal stress and the friction between crack surfaces. It assumed thae the crack to extend in a negative direction with respect to the original crack direction for mixed modes under tension loading and in a positive direction for mixed modes under compression . They considered that the parameters of mode (I) crack are the only material intrinsic parameters neglecting mode (II) and mixed modes parameters .

Trying to overcome the difficulties in previous work , the present study is based on a new concept named Directional Fracture Approach (DFA) which consider the value and direction of the fracture factors [1] .

So,in this paper , the crack behaviour is studied in macroscopic scale to predict the initial fracture direction (θ_{cr}), fracture load (σ_{cr}) , stress intensity factors (K_I-K_{II}) and the final crack path during fracture process . The study is performed for slant cracks to the load direction with crack angle (β) having various values (from 0° to 90°) under remotely loading conditions for both uniaxial tension and copression for brittle materials in LEFM . The used criterion is anew criterion named Directional Circumferential Tensile Stress criterion (σ_θ) or (D σ_θ) . It is a new formulation of the (σ_θ) criterion [5] with a new treatment based on the Directional Fracture Approach (DFA).This new approach considers the value and direction of the fracture factors of (σ_θ) , while the previous studies considered only their scalar values .

Directional Maximum Circumferential Tensile Stress Criterion (σ_{θ} max)

Formulation and Hypotheses :

The new formulation and hypotheses of this criterion depends on the directional treatment for its parameters and their variations at the crack tip according to the hypotheses of DFA. These new hypotheses include mode (I) , mode (II) , mixed mode under tension and mixed mode under compression including the effect of friction between crack surfaces figures (1,2,3,4) .

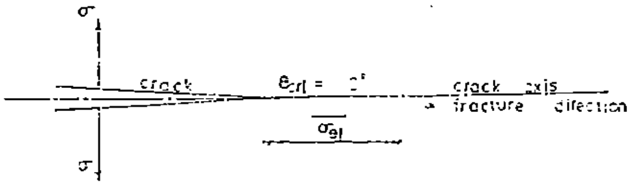

Fig. 1 Mode (I) behaviour

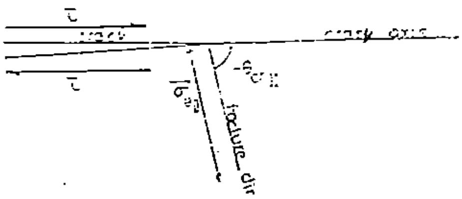

Fig. 2 Mode (II) behaviour

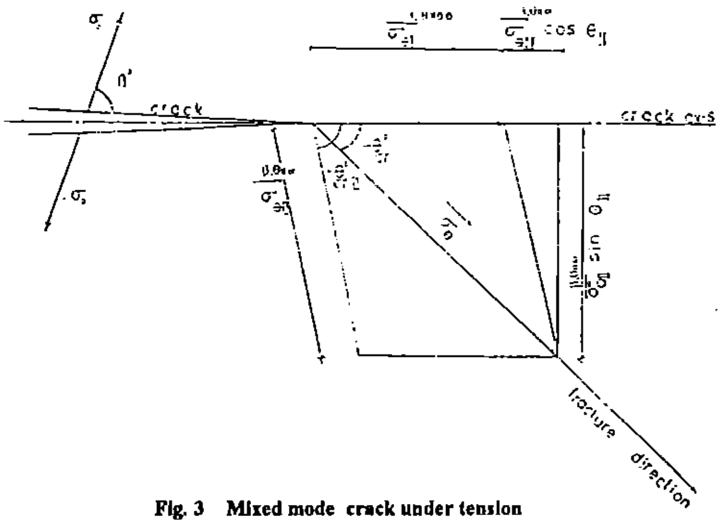

Fig. 3 Mixed mode crack under tension

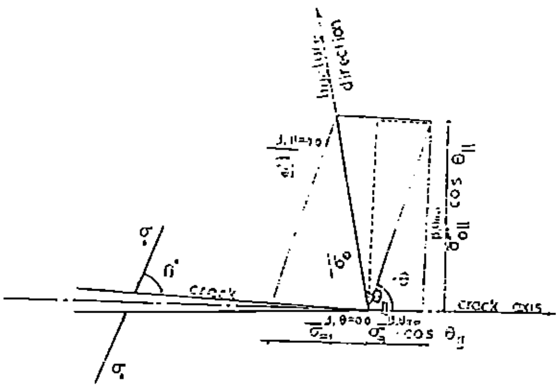

Fig. 4 Mixed mode crack under compression

The general hypotheses of this criterion for various crack modes are :

a) Cracks Under Single Mode Conditions (I) or (II)

1- Crack under pure mode (I) or mode (II) will extend in the direction of the maximum value of the applied fracture factors $(\sigma_{\theta I})_{max}$ or $(\sigma_{\theta II})_{max}$.

2- Crack will start to propagate when the applied fracture factor reachs its critical value .

b)Mixed Mode Cracks :

1- Crack under mixed mode (tension-shear) or (compression-shear) will extend in the direction of vectorial summation of the applied fracture factors of mode (I) and mode (II) for the material under consideration :

$$\vec{(\sigma_\theta)} = \vec{(\sigma_{\theta I})} + \vec{(\sigma_{\theta II})} \qquad (1)$$

2- Crack will start to propagate when the value of the above summation reachs its critical value :

$$\vec{(\sigma_\theta)} \geq \vec{(\sigma_\theta)}_{cr} \qquad (2)$$

Fundamental Hypotheses :

This criterion postulates that :

i) Crack extension starts in the direction normal to the maximum (σ_θ) at a distance $r = r_o$ from crack tip , (where $r_o = 0.025\ a$) .

ii) Crack will begin to extend when the circumferential tensile stress $(\sigma_{\theta\ max})$ reachs a material critical value .

Where :

$$\sigma_\theta = [1/(2\pi r)]^{1/2} \cos\theta/2\ [K_I \cos^2\theta - 1.5 K_{II} \sin\theta] + \dots\dots\dots \quad (3)$$

a) Mode (I):

i) Crack extends in the direction normal to the direction of : $(\sigma_{\theta I})_{max}$

$$\partial \sigma_{\theta I} / \partial \theta = 0 \quad (4)$$

&

$$\partial^2 \sigma_{\theta I} / \partial \theta^2 < 0 \quad (5)$$

Where :

$$[\sigma_{\theta I}] = K_I \cos^3(\theta/2) / (2\pi r)^{1/2} \quad (6)$$

ii) Crack will extend when :

$$[\sigma_{\theta I}]_{max} \geq [\sigma_{\theta I}]_{cr} \quad (7)$$

Where :

$$\sigma_{\theta I\ cr} = K_{Ic} / [2\pi r_o]^{1/2}$$

And : $r_o = 0.025\ a$ (a is the half crack length)

b) Mode (II):

i) Crack extends in the direction which is perpendicular to the direction of $[\sigma_{\theta II}]_{max}$

$$\partial [\sigma_{\theta II}] / \partial \theta = 0 \quad (8)$$

$$\partial^2 [\sigma_{\theta II}] / \partial \theta^2 < 0 \quad (9)$$

where :

$$[\sigma_{\theta II}] = -1.5\ K_{II} \cos\theta/2\ \sin\theta / (2\pi r)^2 \quad (10)$$

ii) Crack will extend when :

$$[\sigma_{\theta II}]_{max} \geq [\sigma_{\theta II}]_{cr} \quad (11)$$

c) Mixed Modes

i) Crack will extend in the direction normal to the direction of $[\ \sigma_\theta\ ]_{max}$

$$[\vec{\sigma_\theta}] = [\vec{\sigma_{\theta I}}]^{\beta,\theta=0} + [\vec{\sigma_{\theta II}}]^{\beta,\theta IIc} \qquad (12)$$

a) For mixed mode tension -shear :

$$\sigma_\theta = [\ (\sigma_{\theta I}^{\beta,\theta=0} + \sigma_{\theta II}^{\beta,\theta IIc} \cos\theta_{IIc})^2 + (\sigma_{\theta II}^{\beta,\theta IIc} \sin\theta_{IIc})^2\ ]^{1/2} \qquad (13)$$

b) For mixed mode compression-shear :

$$\sigma_\theta = [\ (\sigma_{\theta I}^{\beta,\theta=0} - \sigma_{\theta II}^{\beta,\theta IIc} \cos\theta_{IIc})^2 + (\sigma_{\theta II}^{\beta,\theta IIc} \sin\theta_{IIc})^2\ ]^{1/2} \qquad (14)$$

Where :

$$[\sigma_{\theta I}]^{\beta} = K_I \cos^3(\theta/2)/(2\ \pi\ r)^{1/2} \qquad (15) \quad \text{or}$$

$$[\sigma_{\theta I}]^{\beta,\theta=0} = \sigma\,(a)^{1/2}\,\mathrm{Sin}^2\beta\ /\ (2\ \ r)^{1/2} \qquad (16)$$

$$[\sigma_{\theta II}]^{\beta,\theta} = -\ 1.5\ K_{II}\ \cos\ \theta/2\ \sin\ \theta\ /(\ 2\pi\ r)^{1/2} \qquad (17) \quad \text{or}$$

$$[\sigma_{\theta II}]^{\beta,\theta IIc} = -\ 1.5\ \sigma\,(a)^{1/2} \sin\ \beta \cos\beta\ \cos\ \theta_{IIc}/2\ \sin\ \theta_{IIc}/(\ 2\,r)^{1/2} \qquad (18)$$

ii) Crack starts to extend when :

$$[\ \sigma_\theta]_{max} \geq [\ \sigma_\theta\ ]_{cr} \qquad (19)$$

Prediction of Crack Extension Direction :

a) Mode (I):

$$\partial[\sigma_{\theta I}]/\partial\theta = 0$$

$$\partial[\sigma_{\theta I}]/\partial\theta = -1.5\ \cos^2\theta/2\ \ \sin\ \theta/2 \qquad (20)$$

Then , $\theta = 90^o$ or $\theta = 0.0^o$ But ,

$$\partial^2[\sigma_{\theta I}]/\partial\theta^2 = -1.5\ K_I[\,0.5\cos^3\theta/2 - \cos\theta/2\ \sin2\ \theta/2\,]/(2\,\pi\,r)^{1/2} \qquad (21)$$

, at $\theta = 90^o$

$$\partial^2[\sigma_{\theta I}]/\partial\theta 2 = 0.265\ K_I\ /(\ 2\ \pi r\ )^{1/2} > 0 \qquad (22) \qquad \text{not o.k.}$$

and at $\theta = 0.0^\circ$

$$\partial^2[\sigma_{\theta I}]\ /\partial\theta^2 = -0.75\ K_I\ /\ (\ 2\ \pi\ r\ )^{1/2} < 0 \qquad (23) \qquad \text{o.k.}$$

Then $\theta_{Icr} = 0.0^\circ$

b) Mode (II):

$$\partial[\sigma_{\theta II}] / \partial\theta = 0$$

$$\partial[\sigma_{\theta II}] / \partial\theta = -1.5\ K_{II}[\cos\theta/2 \cos\theta - 0.5 \sin\theta \sin\theta/2] / (2\pi r)^{1/2} \quad (24)$$

$$\cos\theta/2\ \cos\theta - 0.5\ \sin\theta \sin\theta/2 = 0 \quad (25)$$

$$2 - \tan\theta \tan\theta/2 = 0 \quad (26)$$

Then the fracture angle may be , $\theta_{IIcr} = \pm\ 70.54°$

And ,

$$\partial^2[\sigma_{\theta II}]/\partial\theta^2 = -1.5\ K_{II}[-\cos\theta/2 \sin\theta - 0.5 \cos\theta \sin\theta/2 - 0.25 \sin\theta \cos\theta/2 - 0.5\ \sin\theta/2\ \cos\theta] \quad (27)$$

At $\theta = \pm 70.54°$

$$\partial^2[\sigma_{\theta II}]/\partial\theta^2 = -1.73\ K_{II}/(2\ \pi\ r)^{1/2} < 0 \quad (28) \quad \text{o.k.}$$

Then $\theta_{IIcr} = \pm\ 70.54°$

Influence of Fracture Factors ($\sigma_{\theta I}$, $\sigma_{\theta II}$):

Figures (5,6) show the variations of the directional fracture factors ($\sigma_{\theta I}$, $\sigma_{\theta II}$) at crack tips.They prove that the fracture angle of mode (I) lies at [θ_{Icr}=0.0°] and fracture angle of mode (II) lies at [θ_{IIcr}=±70.54 °]

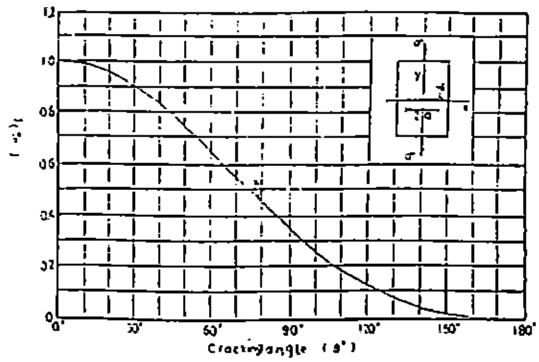

Fig. 5 Variation of circumferential stress at crack tip for pure mode (I)

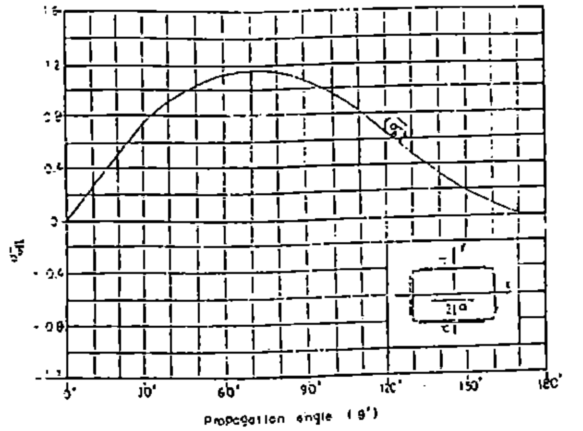

Fig. 6 Variation of circumferential stress at crack tip for pure mode (II)

Crack Angle - Fracture Angle Relations [β–θ] :

a) Mixed Mode (Tension - Shear) :

From figure (3) and the criterion hypothesis, the following relations are derived .

$$\theta_{cr} = \tan^{-1} \left[\sigma_{\theta II}^{\beta,\theta IIc} \sin \theta_{IIc} / \left(\sigma_{\theta I}^{\beta,\theta=0} + \sigma_{\theta II}^{\beta,\theta IIc} \cos \theta_{IIc} \right) \right] \quad (29)$$

Where :

$$[\sigma_{\theta I}]^{\beta,\theta=0} = \sigma (a)^{1/2} \sin^{2} \beta / (2 \quad r)^{1/2}$$

$$[\sigma_{\theta II}]^{\beta,\theta IIc} = - 1.5 \ \sigma (a)^{1/2} \sin \beta \cos \beta \cos \theta_{IIc}/2 \ \sin \theta_{IIc} / (2 r)^{1/2}$$

Then:

$$[\theta]_{cr} = \tan^{-1}[1.088 \cos \beta / (\sin \beta + 0.3846 \cos \beta)] \quad (30)$$

Figure (7) represents the comparison between this criterion , the previous criteria and experimental results.The comparison records that $[\sigma_{\theta}]$-criterion is satisfactory .

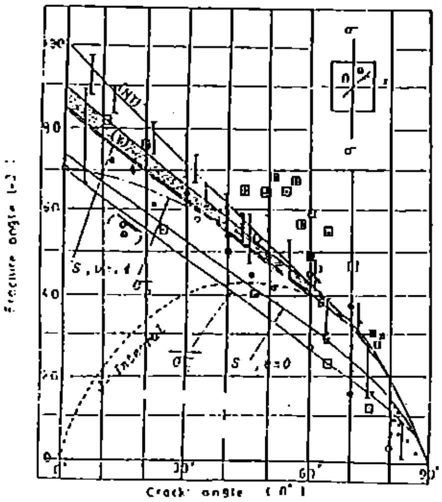

Fig. 7 Crack angle -fracture angle relation for mixed mode under tension

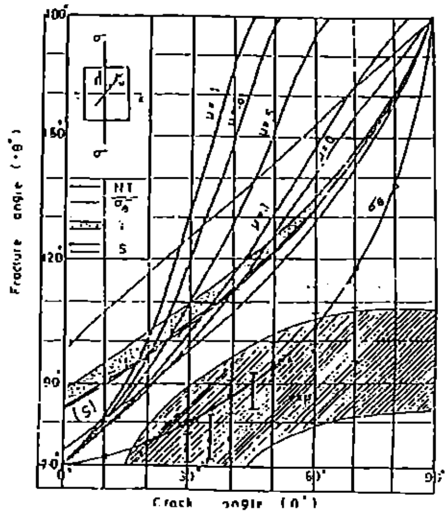

Fig. 8 Crack angle -fracture angle relation for mixed mode under compression

b) Mixed Mode (Compression - Shear) Including Friction :

From figure (4) and the criterion hypothesis, the following relations are derived .

$$\theta_{cr} = 90 + \tan^{-1} \left[\left(\sigma_{\theta I}^{\beta,\theta=0} - \sigma_{\theta II}^{\beta,\theta IIc} \cos \theta_{IIc} \right) / \left(\sigma_{\theta II}^{\beta,\theta IIc} \sin \theta_{IIc} \right) \right] \quad (31)$$

$$\theta_{cr} = 900+ \tan^{-1}[\sin \beta - 0.385(\cos \beta - \mu \sin \beta) / 1.088(\cos \beta - \mu \sin \beta)] \quad (32)$$

Figure (8) shows the above relation for (μ = 0.0 , 0.1 ,0.5 ,0.8 ,1.0) . It also contains the relations for other criteria and available experimental data. The figures show that [σ_θ] curves are near from the experimental results and define the maximum value of crack angle at which the crack will be closed [$\beta_{max} = \tan^{-1} 1/\mu$] . These curves are newly introduced .

Normalized Fracture Load-Crack Angle Relation [$\sigma^{\beta}_{c}/\sigma^{90}_{c}$- β] :

From equations (6) for mode I ,(10) for mode II and (13 , 14) for mixed mode tension-shear and compression-shear resbectively and figures (3 , 4) , the criterion hypothesis are applied to develop the relations between crack angle and fracture load for both mixed mode crack under tension and under compression including friction effect as follows :

relations between crack angle and fracture load for both mixed mode crack under tension and under compression including friction effect as follows :

a) Mixed Mode Tension - Shear :

$$\sigma^{\beta}_{cr} = [\,\sigma_{\theta c}\,(2r/a)^{1/2}\,]\,[\,1/[\sin^4\beta + 1.33\sin^2\beta\cos^2\beta\,]\,]^{1/2} \qquad (33)$$

$$\sigma^{90}_{cr} = \sigma_{\theta lc}\,(2r/a)^{1/2} \qquad (34)$$

$$[\sigma^{\beta}_{c}/\sigma^{90}_{c}] = 1/[\,\sin^4\beta + 1.332\sin^2\beta\cos^2\beta\,]^{1/2} \qquad (35)$$

The comparison between the above relation , previous plane strain results and other plane stress cases are shown from figures (9),(10) resbectively . They are compared to the experimental work recording a good agreement .

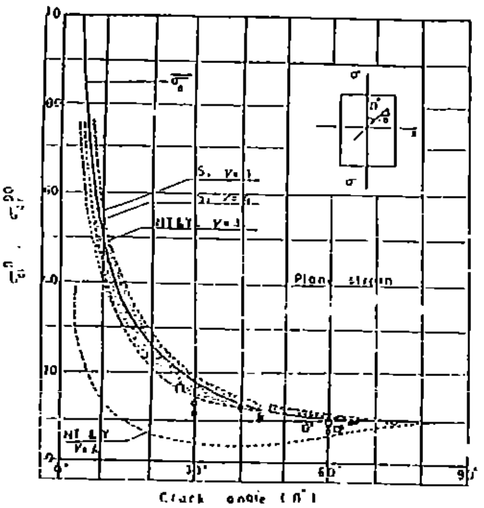

Fig. 9 Normalized fracture load -crack angle relation for tension and plane strain results

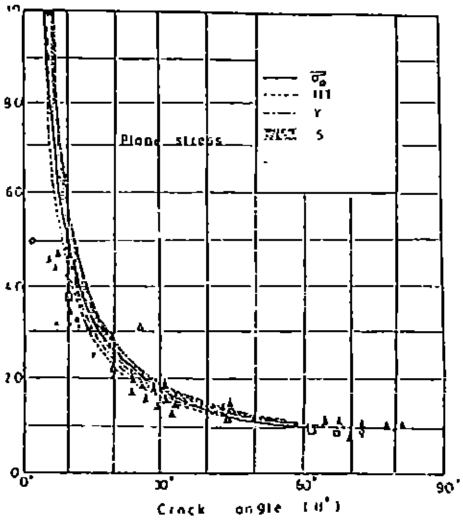

Fig. 10 Normalized fracture load -crack angle relation for tension and plane stress results

b) Mixed Mode (Compression - Shear) Considering Negative (-KI) Factor :

$$\sigma^{\beta}_{cr} = \sigma_{\theta c}(2r/a)^{1/2}[1/[-\sin^4\beta + 1.3\,(\sin\beta\cos\beta - \mu\sin^2\beta)^2\,]^{1/2} \qquad (36)$$

$$\sigma^{90}_{c} = \sigma_{\theta lc}(2r/a)^{1/2}$$

$$[\sigma^{\beta}_{c}/\sigma^{90}_{c}] = 1/[\,-\sin^4\beta + 1.332\,(\sin\beta\cos\beta - \mu\sin^2\beta)^2\,]^{1/2} \qquad (37)$$

Figures (11,12) show the above relation in comparison with the plane strain and plane stress results of previous criteria resbectively and experimental work recording the best results. They are presented for [μ=0,0.1,0.5,0.8,1.0] determining the maximum crack angle for each (μ).

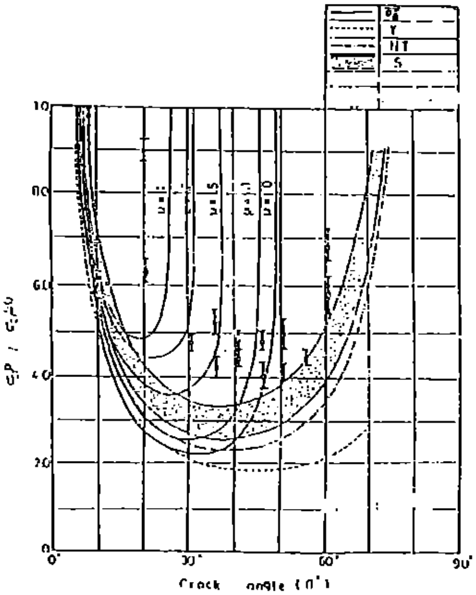

Fig. 11 Normalized fracture load -crack angle relation for compression and plane strain results

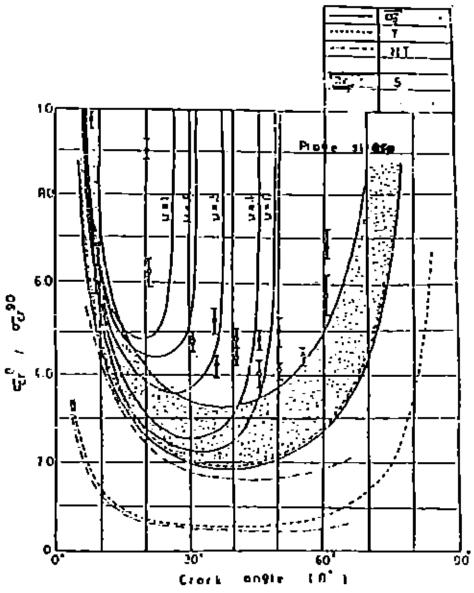

Fig. 12 Normalized fracture load -crack angle relation for compression and plane stress results

Fracture Envelopes of Critical Stress Intensity Factor [K_I-K_{II}] :

The fracture envelopes of stress intensity factors [K_I - K_{II}] are developed for both mixed mode tension -shear and compression - shear including friction as follows :

a) Mixed Mode (Tension - Shear) :

From the following equations :

$$|\sigma_{\theta J}| = [(\sigma_{\theta I}^{\beta,\theta=0} + \sigma_{\theta II}^{\beta,\theta IIc} \cos\theta_{IIc})^2 + (\sigma_{\theta II}^{\beta,\theta IIc} \sin\theta_{IIc})^2]^{1/2}$$

$$\sigma_{\theta Ic} = K_{Ic} / [2\pi r_C]^{1/2}$$

$$\sigma_{\theta I}^{\beta,\theta=0} = K_I / [2\pi r]^{1/2} = K_I / [2\pi r]^{1/2}$$

This relation is plotted in comparison with the scalar one of previous [σ_θ] criterion and the experimental work , figure (13) . It shows that the envelope of directional [σ_θ] has a good agreement with the experimental results.

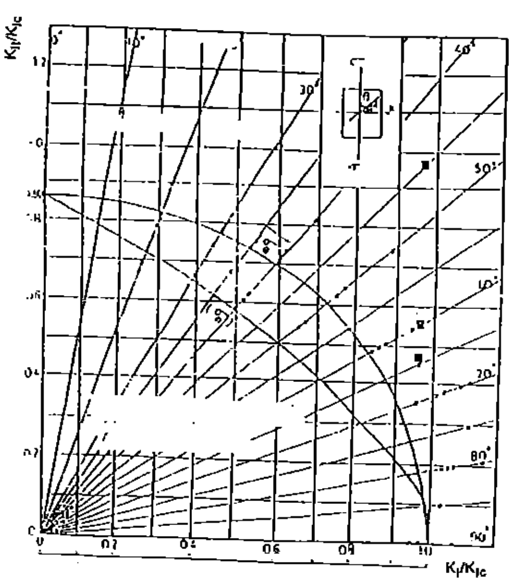

Fig. 13 Fracture envelope (K_I-K_{II}) for tension cracks

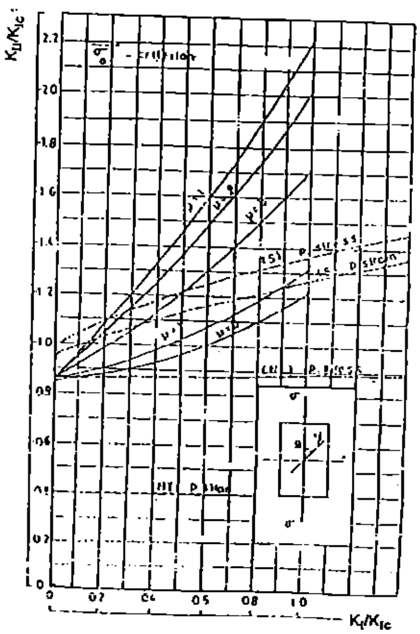

Fig. 14 Fracture envelope (K_I-K_{II}) for compression cracks

b) Mixed Mode (Compression - Shear) Considering Negative ($-K_I$) Factor :

Using the following equations :

$$\sigma_{\theta c} = [(\sigma_{\theta I}^{\beta,\theta=0} + \sigma_{\theta II}^{\beta,\theta IIc} \cos\theta_{IIc})^2 + (\sigma_{\theta II}^{\beta,\theta IIc} \sin\theta_{IIc})^2]^{1/2}$$

$$\sigma_{\theta Ic} = [K_{Ic}] / [2\pi r_C]^{1/2}$$

$$\sigma_{\theta I} = [K_I \cos^3\theta/2] / [2\pi r]^{1/2}$$

$$\sigma_{\theta c} = [(\sigma_{\theta I}^{\beta,\theta=0} + \sigma_{\theta II}^{\beta,\theta IIc} \cos \theta_{IIc})^2 + (\sigma_{\theta II}^{\beta,\theta IIc} \sin \theta_{IIc})^2]^{1/2}$$

$$\sigma_{\theta Ic} = [K_{Ic}] / [2 \pi r_C]^{1/2}$$

$$\sigma_{\theta I} = [K_I \cos^3 \theta /2] / [2\pi r]^{1/2}$$

$$\sigma_{\theta II} = - [1.5 (K_{II} - \mu K_I) \cos \theta_I] \sin \theta_{II}] /[2\pi r]^{1/2}$$

The following equation can be derived :

$$1 = (K_I/K_{Ic} - 0.3846 (K_{II}/K_{Ic} - \mu K_I/K_{Ic})]^2 + [1.0887(K_{II}/K_{Ic} - \mu K_I/K_{Ic})^2]^2 \qquad (39)$$

These envelopes are developed with the consideration of the normal stress effect beside the effect of shear stress. They are presented for various values of coefficient of friction (μ = 0.0,0.1,0.5,0.8,1.0) , figure (14)

Crack Path Examples:

Applying the "Directional Approach " rules for crack path technique.[1] and using the fracture mechanism and fracture increment length for this criterion , some examples of crack path are presented using computer developed computer programs [1] for both mixed modes under tension and compression loading . Figures (15,16,17,18,19,20) show some solved examples od crack path problems under tension loading and figures (21,22,23,24) show the other examples which are presented for cracks under compression loading . They are compared with experimental work results of the tested plane stress samples of PMMA [1] which are carried out by the author realizing a good correlation .

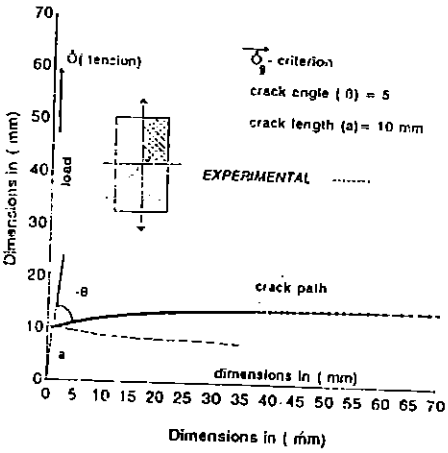

Fig. 15

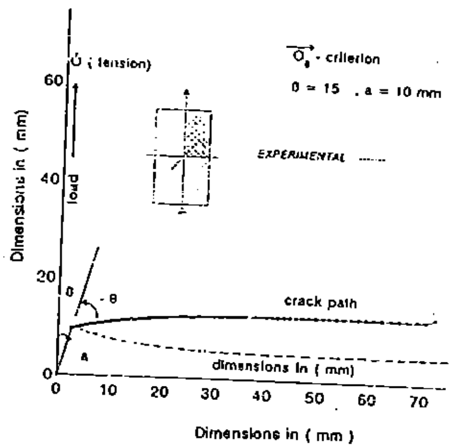

Fig. 16

Crack path examples for mixed mode crack under tension

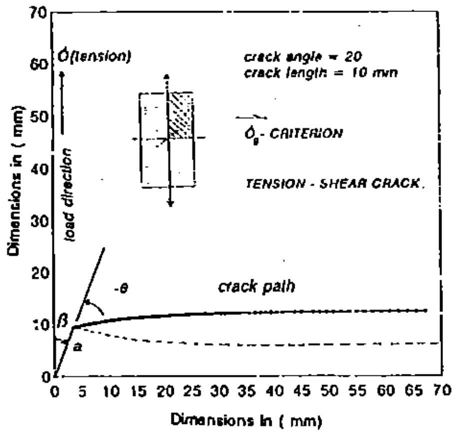

Fig. 17

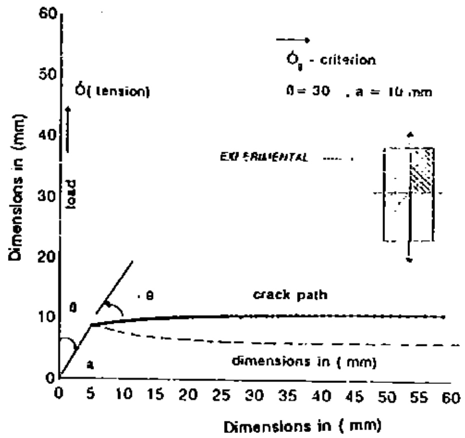

Fig. 18

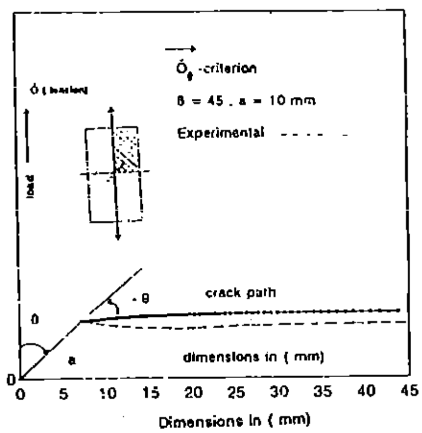

Fig. 19

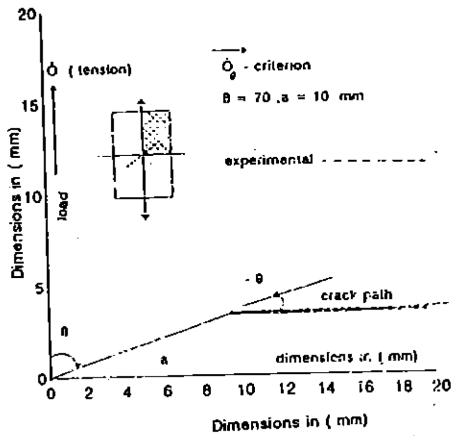

Fig. 20

Crack path examples for mixed mode crack under tension

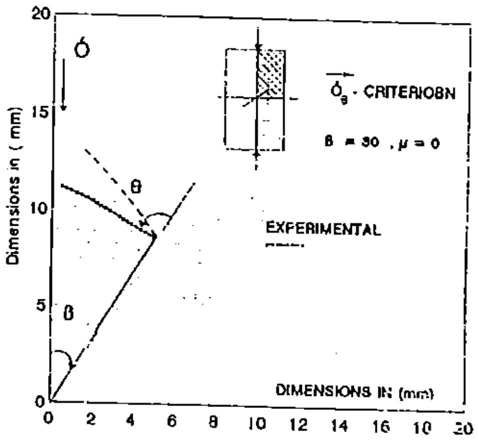

Fig. 21

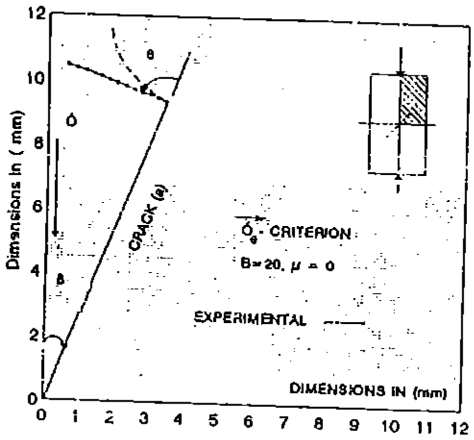

Fig. 22

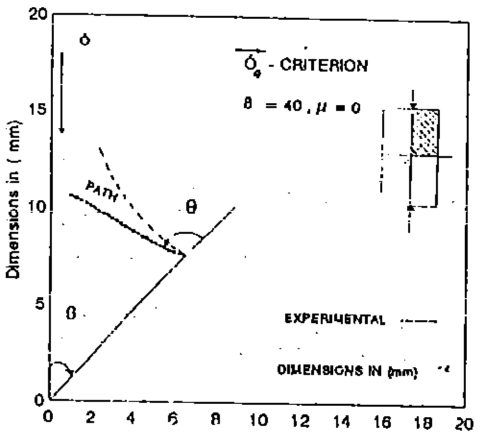

Fig. 23

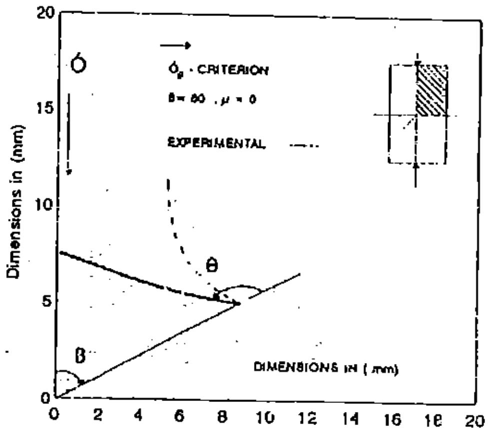

Fig. 24

Crack path examples for mixed mode crack under compression

Conculusions :

As shown from this study , the behaviour of mixed cracks is investigated using the new method of DFA which deals with the cracks in directional forms . It is simple in using , easy in its application with the good efficiency in determination of the fracture load , crack propagation direction ,critical stress intensity factors for all crack modes and the final crack path . The results are good in comparison with the other criteria and experimental work . It is the only approach which can determine the behaviour of the cracks under mixed modes taking into consideration the negative direction of stress intensity factor of mode (I) and the effect of friction between the surfaces of the crack on the determination of the fracture parameters . DFA approch succseeded to determine the angle of crack under compression at which the crack will be closed without propagation . Also , It can determine the value of coefficient of friction (μ) at which crack can be stable and stop to propagate . As obtained from the study , it is clear the effectiveness and intelligency of DFA to investigate the material fracture parameters reaching to make a control and prevention of the cracking process , to know the intelligency of the materials and structures to devot any excess in the stresses without failure to make a good safe structural design against cracking and to make a good analysis of the cracked structures reaching to choose the suitable method of its repair and strenthening .

References:

(1)EL-SHEIKHY Refat. A. I. , (1995), A New Approach for Mixed Mode Fracture Theories , Ph.D. Report , Cairo University .

(2)ERDOGAN F. , and SIH G. C. , (1963), On The Crack Extension in Plates Under Plane Loading and transverse Shear , ASME, J of Basic Eng., Vol. 85, PP.519-527.

(3)SIH G. C. , (1977) , Mechanics of Ductile Fracture , Fracture Mechanics and Technology , Editors: G.C. Sih and C.L. Chow, Vol. II , PP.767-784