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Structural Stresses Evaluation for Damage Modelling of Carbon - Carbon Composite at Multiaxial Loading

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ABSTRACT: *In this study, the so-called zero approximation of homogenisation technique for micro-heterogeneous solids with periodic structure has been applied to investigate structural stresses in spatially reinforced composite material, working in structures, subjected to multiaxial loading and high temperatures. Specifically, the carbon-carbon composite material, 3-D Carb, reinforced with anisotropic fibers at three mutually orthogonal directions in ratio 1:1:1 has been considered. According to the homogenisation technique, local functions, namely the components of elastic moduli tensor and thermoelastic tensor of zero approximation, as well as macrostrain and temperature should be defined to evaluate the structural stresses. The set of periodic problems has been solved to obtain these local functions and effective thermoelastic constants. These constants are used in the solution of homogenised boundary value problem to evaluate macrostrains. The characteristics of spatial distribution of the local functions have been analysed for different cases.*

Notation

$x=(x_1,x_2,x_3)$	relative coordinates in a solid;
α	small length scale ratio;
$\xi = x/\alpha$; $\xi=(\xi_1,\xi_2,\xi_3)$	local coordinates in periodicity cell;
Y	periodicity cell volume; $Y=\sum_{k=1}^{4} Y_k$;
Y_k	cell volume, occupied by fiber of k-direction (k=1,2,3) or matrix (k=4).
$\langle\ldots\rangle$	homogenisation operator, $\langle f \rangle \equiv \frac{1}{Y}\int_Y f(x,\xi)\, d\xi_1\, d\xi_2\, d\xi_3$;
u_i	displacement vector components;
σ_{ij}, ε_{ij}	stress and strain tensor components;
$\Delta T = T - T_0$,	T- current temperature, T_0- temperature of undeformed

	state;
$C_{ijkl}(\xi)$	elastic moduli tensor components;
$\alpha_{kl}(\xi)$	coefficients of linear thermal expansion (CLTE) ;
$I_{ijkl} = \frac{1}{2}\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right)$	components of 4^{th} rank unit tensor;
$e_{ij} = \left\langle \varepsilon_{ij} \right\rangle$	macrostrain tensor components;
$s_{ij} = \left\langle \sigma_{ij} \right\rangle$	macrostress tensor components;
h_{ijkl}	effective elastic moduli;
a_{kl}	effective coefficients of linear thermal expansion (CLTE) ;
E, ν, G , a	effective technical thermoelastic constants for 3- D Carb;
$[[f]]$	the gap of f value while transiting fiber -matrix interface;
v_f	fiber volume ratio;

Local functions for structural stress evaluation:

$C^0_{ijpq}(\xi)$	elastic moduli of zero approximation;
$\beta^0_{ij}(\xi)$	thermoelastic tensor components of zero approximation.

Indexes in thermoelastic constants:
m - matrix;
f - fiber;
L - longitudinal direction in fiber;
T - transversal direction in fiber;

Introduction

Carbon-carbon composite materials have a number of unique properties which make these materials desirable for applications in some aerospace structures, working at high temperatures [1]. One of such materials is 3-D Carb, periodically reinforced with anisotropic carbon fibers at three mutually orthogonal directions in ratio 1:1:1. The space between fibers is filled with weak carbon matrix phase, which is treated as isotropic. The model structure of such a material is obtained by parallel transfer of periodicity cells. Such periodicity cell is shown on Fig.1. Machine components of this material is subjected to high temperature field and multiaxial mechanical loading.

The computer modelling of damage accumulation in carbon matrix, carbon fibers and fiber-matrix interface of composite structure subjected to thermomechanical multiaxial loading demands the evaluation of structural stresses field [2]. The direct evaluation by numerical finite element analysis of stress fields in solid, containing great amount of periodic cells, is, of course, impossible. The homogenisation method for heterogeneous solids with periodic structure was developed to deal with this problem [3].

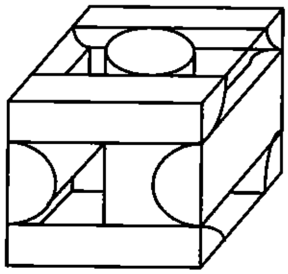

Fig.1 Periodicity cell of 3-D Carb composite.

This technique is based on the two length scale nature of the problem and construction of a series solution of boundary-value heterogeneous problem in powers of small parameter α, that relates the length scale of periodicity cell, the ξ- scale, to the scale natural for describing the solid specimen geometry [4,5]. The so-called zero approximation of homogenisation method for micro-heterogeneous solids with periodic structure allows to obtain rather good estimations for structural stresses if the size of periodic cell is much less then characteristic size of macro-stresses alteration [3].

Firstly, we should solve a set of periodic problems to obtain local functions and their gradients and construct the so-called elastic moduli tensor and thermoelastic tensor of zero approximation. Then we get the effective elastic constants and coefficients of linear thermal expansion (CLTE) by volume averaging of the obtained tensor.

Secondly, we are to solve boundary-value problem of homogeneous anisotropic solid with obtained effective elastic constants and CLTE, and evaluate macrostress and macrostrain tensors. The temperature field is calculated as a solution of thermal conductivity problem. And at least the structural stress tensor can be evaluated by summarising the results of multiplying the elastic moduli tensor of zero approximation by macrostrain tensor and thermoelastic tensor of zero approximation by difference between local and actual state temperatures.

Local Functions and Effective Constants for Thermoelastic Problem.

The equations of thermoelastic boundary-value problem for heterogeneous solid with periodic microstructure are written:

$$\frac{\partial}{\partial x_j}\left(C_{ijkl}(x)\left(\frac{\partial u_k}{\partial x_l}-\alpha_{kl}(x)\cdot\Delta T(x)\right)\right)=0\,, \qquad (1a)$$

$$u_i\Big|_{\Gamma_U}=u_i^o\,;\quad C_{ijkl}\frac{\partial u_k}{\partial x_l}n_j\Big|_{\Gamma_S}=S_i^o.\,, \qquad (1b)$$

with Γ_U and Γ_S denoting parts of solid boundary where boundary conditions of the first and the second mode are given. The first two terms of a series solution of boundary-value problem (1) in powers of small parameter α are written as functions of "slow" coordinates x and "fast" coordinates ξ :

$$u_i(x,\xi)=v_i(x)+\alpha\left(N_{ipq}(\xi)\frac{\partial v_p(x)}{\partial x_q}+\Theta_i(\xi)\,\Delta T(x)\right) \qquad (2)$$

Functions N_{ipq} and Θ_i, depending on ξ, are ξ - periodical. One can obtain strain components by differentiating (2). In the process of differentiating, the variables x and ξ are divided according to:

$$\frac{\partial}{\partial x_i}\rightarrow\frac{\partial}{\partial x_i}+\frac{1}{\alpha}\frac{\partial}{\partial\xi_i}$$

Zero approximation means that we retain terms only with zero power of parameter α in expressions for strains and stresses. The components of strain tensor in zero approximation are:

$$\varepsilon_{ij}=M_{ijpq}(\xi)\,e_{pq}(x)+\vartheta_{ij}(\xi)\,\Delta T(x) \qquad (3)$$

where e_{ij} are macrostrain components: $e_{ij}=\left\langle\varepsilon_{ij}\right\rangle=\frac{1}{2}\left(\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}\right)$; and

$$M_{ijpq}(\xi)=\frac{1}{2}\left(\frac{\partial U_{ipq}}{\partial\xi_j}+\frac{\partial U_{jpq}}{\partial\xi_i}\right); \qquad (4)$$

$$U_{ipq}(\xi)=N_{ipq}(\xi)+\xi_q\delta_{ip}\ ; \qquad (5)$$

$$\vartheta_{ij}(\xi) = \frac{1}{2}\left(\frac{\partial \Theta_i}{\partial \xi_j} + \frac{\partial \Theta_j}{\partial \xi_i}\right) ; \tag{6}$$

Applying homogenisation operator to (3), one can obtain :

$$\langle M_{ijpq}(\xi) \rangle = I_{ijpq}; \quad \langle \vartheta_{ij}(\xi) \rangle = 0; \tag{7}$$

Stress tensor components in zero approximation can be expressed through macrostrain components and temperature, using (3):

$$\sigma_{ij} = C_{ijkl}(\varepsilon_{kl} - \alpha_{kl}\,\Delta T) = C^0{}_{ijpq}(\xi)\, e_{pq}(x) - \beta^0{}_{ij}(\xi)\,\Delta T; \tag{8}$$

Elastic moduli $C^0{}_{ijpq}(\xi)$ and thermoelastic tensor components $\beta^0{}_{ij}(\xi)$ of zero approximation in (8) are:

$$C^0{}_{ijpq}(\xi) = C_{ijkl}(\xi)\, M_{klpq}(\xi) ; \tag{9}$$

$$\beta^0{}_{ij}(\xi) = \beta_{ij}(\xi) - C_{ijkl}(\xi)\,\vartheta_{kl}(\xi); \tag{10}$$

where $\beta_{ij}(\xi) = C_{ijkl}(\xi)\,\alpha_{kl}(\xi)$.

Applying homogenisation operator to (8), one can obtain the relationship between macrostresses and macrostrains together with temperature (homogenized thermoelastic equations):

$$\langle \sigma_{ij} \rangle = h_{ijpq}\, e_{pq} - b_{ij}\,\Delta T = h_{ijpq}\,(e_{pq} - a_{pq}\Delta T), \tag{11}$$

h_{ijpq} and b_{ij} being effective constants:

$$h_{ijpq} = \langle C^0{}_{ijpq} \rangle; \quad b_{ij} = \langle \beta^0{}_{ij} \rangle. \tag{12}$$

Effective coefficients of linear thermal expansion can be taken from:

$$h_{ijpq}\, a_{pq} = b_{ij} \tag{13}$$

The zero approximation of homogenisation technique include solving of homogeneous boundary-value problem:

$$h_{ijkl} \frac{\partial}{\partial x_j}\left(\frac{\partial v_k}{\partial x_l} - a_{kl}\,\Delta T\right) = 0; \tag{14a}$$

$$v_i \Big|_{\Gamma_U} = u_i^{\,o}; \quad h_{ijkl} \frac{\partial v_k}{\partial x_l} n_j \Big|_{\Gamma_S} = S_i^{\,o} , \tag{14b}$$

Effective constants, that are necessary for the solution of this boundary-value problem can be found from (12) and (13) after determining the corresponding periodical local functions. Due to the symmetry of periodicity cell shown in fig.1, one can consider only 1/8 part of it (fig.2).

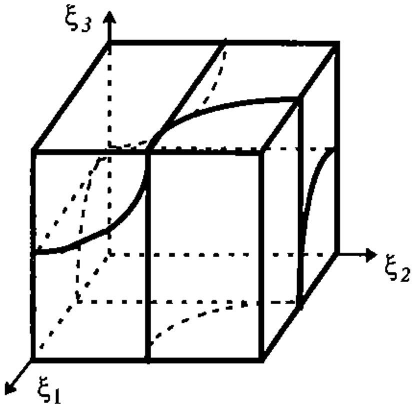

Fig.2 1/8 part of 3-D Carb periodicity cell.

Note, that local functions U_{ipq} comply with differential equations that formally are the equations of elasticity problem for displacements $U_{i(pq)}$, if we fix indexes p and q:

$$\frac{\partial}{\partial \xi_j}\left(C_{ijkl}(\xi) \frac{\partial U_{k(pq)}}{\partial \xi_l} \right) = 0 \qquad (15)$$

The local functions Θ_i comply with differential equations of thermoelasticity problem for displacements Θ_i (temperature difference $\Delta T = 1$) :

$$\frac{\partial}{\partial \xi_j}\left(C_{ijkl}(\xi) \left(\frac{\partial \Theta_k}{\partial \xi_l} - \alpha_{kl}(\xi) \right)\right) = 0 \qquad (16)$$

Functions $M_{kl(pq)}(\xi)$ from (4) and $\vartheta_{kl}(\xi)$ from (6) act as strains while $C^0_{ijpq}(\xi)$ from (9) and $-\beta^0_{ij}(\xi)$ from (10) act as stresses in these problems.

If there is an ideal contact in fiber- matrix interfaces in initial problem, then we have the analogous conditions in fiber-matrix interfaces for cell problems:

$$\begin{cases} \left[\left[C^0_{ij(pq)}\right]\right] n_j \Big|_{\Gamma_{fm}} = 0; \\ \left[\left[U_{i(pq)}\right]\right]\Big|_{\Gamma_{fm}} = 0. \end{cases} \quad \text{for (15) and} \quad \begin{cases} \left[\left[\beta^0_{ij}\right]\right] n_j \Big|_{\Gamma_{fm}} = 0; \\ \left[\left[\Theta_i\right]\right]\Big|_{\Gamma_{fm}} = 0. \end{cases} \quad \text{for (16).}$$

Note, that general conditions of non- ideal contact, that keep the linearity of the problem were written in [3]. The cell problems with linear conditions of "elastic sliding" were solved, for example in [6]. One should take into account that non-linear contact conditions, including fiber-matrix debonding lead to nonlinear problems, in which we can't solve the periodicity cell problems and homogenised boundary problem independently.

Because of the symmetry of the cell for considered structure (fig.1), it is sufficient to take into account only two combinations of indexes p and q instead of six. These for instance are 1) p,q = 1,1 and 2) p,q = 1,2. Boundary conditions for cell sides in these problems are:

1) p,q = 1,1.

Tangential components of quasi-stresses are equal to zero :

$$\xi_i = 0,1: \ C^0_{ij(11)} = 0; \ (i = 1,2,3; \ j \neq i). \tag{17a}$$

Normal components of quasi-displacements are equal to zero in all sides of the cell except $\xi_1 = 1$:

$$\xi_i = 0, i = 1,2,3 \text{ and } \xi_i = 1, i = 2,3 \ : \ U_{i(11)} = 0. \ ; \quad \xi_1 = 1: \ U_{1(11)} = 1. \tag{17b}$$

2) p,q = 1,2.

$$\xi_1 = 0,1: \ C^0_{1j(12)} = 0, \ (j = 1,3); \ U_{2(12)} = 0. \tag{18a}$$

$$\xi_2 = 0,1: \ C^0_{2j(12)} = 0, \ (j = 2,3); \ \xi_2 = 0: U_{1(12)} = 0; \ \ \xi_2 = 1: U_{1(12)} = 0. \tag{18b}$$

$$\xi_3 = 0,1: \ C^0_{3j(12)} = 0, \ (j = 1,2); \ U_{3(12)} = 0. \tag{18 c}$$

In thermoelastic problem (16) normal components of quasi-displacements and tangential components of quasi-stresses are equal to zero in all sides of the cell:

$$\xi_i = 0,1: \ \Theta_i = 0, \ \beta^0_{ij(11)} = 0; \ (i = 1,2,3; \ j \neq i). \tag{19}$$

The named boundary conditions provide the fulfilment of the relations (7), concerning homogenised quasi-strains.

Periodicity Cell Problems for the Carbon-Carbon Composite.

Effective constants.

The finite element model is used to investigate the above mentioned periodicity cell problems. Tetrahedral finite elements with linear approximation of displacements are utilised. 1/8 part of a periodicity cell is divided into parallelepipeds. Each one of them in its turn is divided into six tetrahedrons by three cross-sections, parallel to coordinate axes and containing side diagonals. The example of cell's discretisation is shown in Fig.3.
Technical thermoelastic constants for fibers and matrix under three different temperatures were taken from [1] and are given in Table 1. Note that carbon fibers possess severe anisotropy.

After solving two elastic and one thermoelastic cell problem one can define effective thermoelastic constants, using (12) and (13). For 3-D Carb composite we have three independent elastic constants, h_{1111}, h_{1122}, h_{1212} for instance. Effective technical elastic constants can be calculated as:

$$E = h_{1111} - \frac{2 \cdot h_{1122}^2}{h_{1111} + h_{1122}}; \quad \nu = \frac{h_{1122}}{h_{1111} + h_{1122}}; \quad G = h_{1212}. \tag{20}$$

Effective CLTE is defined from (13) as:

$$a = b_{11} / (h_{1111} + 2 \cdot h_{1122}). \tag{21}$$

Two types of fibers cross- sections were taken into consideration: 1) square one and 2) regular eight-sided polygon.

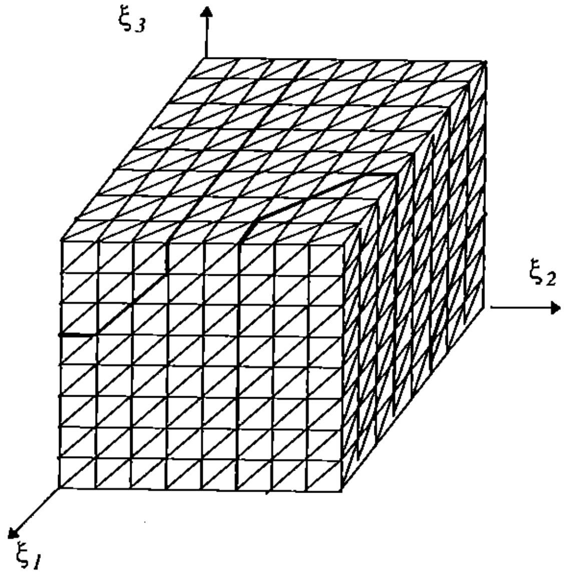

Fig. 3 Discretisation of 1/8 part of 3-D Carb periodic cell.

Table 1. Thermoelastic constants of fibers and matrix under different temperatures.

	20^0 C	1000^0 C	2300^0 C
Elastic moduli, GPa			
E_L^f	200	205	120
E_T^f	2.2	2.4	3.0
G_{LT}^f	3.4	4.0	6.5
E^m	9.0	4.6	2.4
Poisson ratios			
ν_{LT}^f	0.23	0.23	0.19
ν_{TT}^f	0.16	0.16	0.16
ν^m	0.12	0.14	0.17
CLTE $\cdot 10^6$, 1/K			
α_L^f	-0.5	1.9	3.9
α_T^f	6.0	9.7	12.8
α^m	4.3	6.6	8.6

The effective thermoelastic constants for different fiber volume ratios: 0.1875; 0.4219 and 0.5 are posed in Table 2. The ratios 0.1875 and 0.4219 correspond to half-width of square fibers 0.25 and 0.375 respectively. The analysis of the effective thermoelastic constants for two types of cross-sections shows their practical coincidence for the same fiber volume ratios.

Table 2. The effective thermoelastic constants for 3-D Carb composite.

	20^0 C	1000^0 C	2300^0 C
$v_f = 0.1875$			
E, GPa	19.8	16.9	9.94
v	0.052	0.041	0.053
G, GPa	3.58	2.09	1.25
$a \cdot 10^6$, 1/K	1.59	3.39	5.57
$v_f = 0.4219$			
E, GPa	33.3	32.2	19.2
v	0.27	0.021	0.029
G, GPa	3.03	2.13	1.57
$a \cdot 10^6$, 1/K	0.49	2.63	4.90
$v_f = 0.5$			
E, GPa	37.9	37.3	22.3
v	0.022	0.018	0.025
G, GPa	2.87	2.14	1.71
$a \cdot 10^6$, 1/K	0.29	2.51	4.79

The local functions analysis.

We have tried to analyse the spatial distribution of "quasi-stresses" $C^0_{ij(pq)}(\xi)$, (p,q)=1,1 and (p,q)=1,2 from (9) and $-\beta^0_{ij}(\xi)$ from (10), calculating some integral characteristics. These characteristics are the mean values and measures of deviation from them in that or other component of the composite, namely fibers of three mutually orthogonal directions and matrix. The mean value of a variable in a certain component with number k (k=1,2,3 corresponding to fiber of k-direction and k=4 corresponding to the matrix) is obtained by volume averaging through this component of volume Y_k:

$$\left\langle C^0_{ij(pq)} \text{ or } \beta^0_{ij} \right\rangle_k = \frac{1}{Y_k} \cdot \int_{Y_k} (C^0_{ij(pq)} \text{ or } \beta^0_{ij}) \, d\xi_1 \, d\xi_2 \, d\xi_3 \tag{22}$$

The measure of deviation is taken analogically to standard deviation in probability theory by replacing ensemble averaging by volume averaging:

$$D^0_{ij(pq)} = \sqrt{\left\langle \left(C^0_{ij(pq)} - \left\langle C^0_{ij(pq)} \right\rangle_k \right)^2 \right\rangle_k}\ , \tag{23a}$$

$$\delta^0_{ij} = \sqrt{\left\langle \left(\beta^0_{ij} - \left\langle \beta^0_{ij} \right\rangle_k \right)^2 \right\rangle_k}\ , \tag{23b}$$

The relative characteristic of the deviation, the variation factor of a variable, can be calculated by dividing the "standard deviation" from (23) by corresponding mean value within a component from (22).

1. The cell problem with (p,q) = (1,1) - uniaxial deformation in ξ_1 direction.

The tensor component $C^0_{11(11)} \cong C^f_{1111}$ in the fiber 1 (of ξ_1 direction) has a scatter of about 1% at 20° C ($v_f = 0.42$ and $v_f = 0.5$) and a scatter < 1% in all other considered cases. The tensor components in the transversal directions in fibers have maximum values in ξ_1 direction in fibers 2 and 3. The results for $C^0_{11(11)}$ are summarized in table3 for two types of fiber cross-section, with variation factor placed in parentheses.

Table 3. The local functions characteristics $\langle C^0_{11(11)} \rangle_k / D^0_{11(11)}$ in fibers 2 and 3.

	20° C	1000° C	2300° C
$v_f = 0.1875$			
square	3.61/0.44(12%)	3.23/0.21(7%)	2.82/0.06(2%)
polygon	3.55/0.23(6%)	3.03/0.11(4%)	2.82/0.03(1%)
$v_f = 0.4219$			
square	3.16/0.5/(16%)	3.0/0.24(8%)	2.9/0.07(2%)
polygon	3.15/0.39(12%)	3.0/0.2(6%)	2.89/0.06(2%)
$v_f = 0.5$			
square	3.04/0.52(17%)	2.91/0.25(9%)	2.92/0.08(3%)
polygon	3.01/0.42(14%)	2.91/0.21(7%)	2.91/0.07(2%)

Maximum normal components of elastic moduli tensor in matrix are shown in Table 4. The mean values of shear components of this tensor in matrix are more than 10 times less than mean values of normal components in all cases.

Table 4. The local functions characteristics $\left\langle C^{0}{}_{11(11)} \right\rangle_4$ / $D^{0}{}_{11(11)}$ in matrix (Gpa).

	20^0 C	1000^0 C	2300^0 C
$v_f = 0.1875$			
square	8.44/1.61(19%)	4.56/0.43(9%)	2.62/0.07(3%)
polygon	8.47/1.57(18%)	4.56/0.48(10%)	2.62/0.07(3%)
$v_f = 0.4219$			
square	7.47/2.11/(28%)	4.32/0.6(14%)	2.66/0.11(4%)
polygon	7.47/2.0(27%)	4.31/0.75(13%)	2.66/0.10(4%)
$v_f = 0.5$			
square	7.13/2.15(30%)	4.24/0.58(14%)	2.68/0.11(4%)
polygon	7.21/2.0(28%)	4.24/0.58(14%)	2.68/0.11(4%)

2. The cell problem with (p,q) = (1,2) - shear deformation in (ξ_1, ξ_2) plane.

The characteristics of shear "quasi-stresses" with maximum mean values are placed in Table 5 for fibers of 1 and 2 directions and in Table 6 for matrix. The mean values of this "quasi-stress" in fiber of 3 direction is lower than the mean values from Table 5 for corresponding cases with variation factor not exceeding 7%.

3. Thermoelastic cell problem.

When evaluating structural stresses according to (8) we are to multiply the thermoelastic tensor by temperature difference. Such products are given in Table 7, where normal component of thermoelastic tensor are presented. The shear components are much less than normal ones, even in matrix.

The data obtained in thermoelastic cell problem show that practically the spatial distribution of normal components of "quasi-stresses" in fibers is uniform and their mean values are almost constant for different fiber volume ratios and forms of cross-sections.

Table 5. The local functions characteristics $\left\langle C^0_{12(12)} \right\rangle_k$ / $D^0_{12(12)}$ in fibers 1 and 2 (Gpa).

	20^0 C	1000^0 C	2300^0 C
$v_f = 0.1875$			
square	3.29/0.3(9%)	2.63/0.22(8%)	2.0/0.3(15%)
polygon	3.32/0.26(8%)	2.61/0.18(7%)	1.93/0.09(5%)
$v_f = 0.4219$			
square	2.82/0.53/(19%)	2.49/0.38(15%)	2.19/0.42(19%)
polygon	2.88/0.51(18%)	2.49/0.35(14%)	2.11/0.25(12%)
$v_f = 0.5$			
square	2.67/0.58(22%)	2.45/0.43(18%)	2.29/0.48(21%)
polygon	2.75/0.56(20%)	2.46/0.41(17%)	2.23/0.34(15%)

Table 6. The local functions characteristics $\left\langle C^0_{12(12)} \right\rangle_4$ / $D^0_{12(12)}$ in matrix.

	20^0 C	1000^0 C	2300^0 C
$v_f = 0.1875$			
square	3.68/0.41(11%)	2.03/0.22(11%)	1.13/0.24(21%)
polygon	3.74/0.42(11%)	2.03/0.23(11%)	1.14/0.22(19%)
$v_f = 0.4219$			
square	3.28/0.53/(16%)	2.03/0.31(15%)	1.29/0.38(30%)
polygon	3.39/0.57(17%)	2.05/0.31(15%)	1.3/0.34(26%)
$v_f = 0.5$			
square	3.15/0.53(17%)	2.03/0.31(16%)	1.35/0.42(31%)
polygon	3.32/0.6(18%)	2.06/0.33(16%)	1.36/0.37(27%)

Table 7. Characteristics of $b_{ij} \cdot \Delta T$ at 2300^0 in fibers and matrix (Mpa).

	$\langle b^0_{kk} \rangle \cdot \Delta T / \delta^0_{kk}$ in fiber k.	$\langle b^0_{jj} \rangle \cdot \Delta T / \delta^0_{jj} \cdot \Delta T$ in fiber k $(j \neq k)$	$\langle b^0_{ii} \rangle \cdot \Delta T / \delta^0_{ii} \cdot \Delta T$ in matrix
$v_f = 0.1875$			
square	1083/1(<1%)	106/3(3%)	75/4(6%)
polygon	1083/1(<1%)	106/1(1%)	75/4(5%)
$v_f = 0.4219$			
square	1086/3(<1%)	111/3(3%)	79/6(8%)
polygon	1085/2(<1%)	111/3(2%)	79/6(7%)
$v_f = 0.5$			
square	1086/3(<1%)	113/4(3%)	80/7(8%)
polygon	1086/3(<1%)	113/3(3%)	80/6(8%)

Conclusions

The zero approximation of homogenisation technique for micro-heterogeneous solids with periodic structure has been applied to evaluate the structural stresses in spatially reinforced at three mutually orthogonal directions carbon-carbon composite material, working in structures, subjected to multiaxial loading and high temperatures. The finite element procedure has been used to solve a set of periodicity cell problems and the local functions, namely the components of elastic moduli tensor and thermoelastic tensor of zero approximation, have been defined. The spatial distribution of these functions has been investigated and the their mean values within fibers of different directions and matrix together with measures of deviation from these values have been analysed for various fiber volume ratios, forms of fiber cross-sections at three different temperatures. This form of data representation is convenient for modelling of damage accumulation in fibers and matrix. The effective thermoelastic constants have been calculated for cases considered. We need these constants to solve homogenised thermoelastic boundary-value problem to evaluate macrostrains and at least obtain the structural stresses at multiaxial loading of heterogeneous structure, using the local functions obtained.

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