George N. TRETYACHENKO, Boris S. KARPINOS,
Victor G. BARILO, and Nonna G. SOLOVYOVA

Selection of Regimes for Biaxial Thermal Fatigue Tests with the Present Thermal Stress State

Institute for Problems of Strength, National Academy of Sciences of Ukraine,
2 Timiryazevska str., Kyiv, 252014, Ukraine

Keywords: thermal fatigue, specimens, geometry and dimensions, thermal and thermal stress state.

ABSTRACT: A method is presented for choosing the shape and dimensions of specimens whose heating and cooling induce preset thermal and biaxial thermal-stress states. Based on the solutions of direct problems of nonstationary heat conduction and thermoelasticity, the plots were constructed of thermal stress state dependence on the variation of characteristic dimensions of the specimen proposed by the authors as a basic one. Those plots are used for fast solution of inverse problems of thermo-elasticity required when defining the regimes for thermal fatigue testing of materials under biaxial stress conditions.

Notation

a	thermal diffusivity coefficient
E	Young's modulus
T	temperature
α	coefficient of linear thermal expansion
α^*	heat exchange coefficient
λ	thermal conductivity
σ	thermal stress
σ_i	stress intensity, $\sigma_i = \sqrt{\left[\sigma_r^2 + \sigma_\theta^2 + (\sigma_\theta - \sigma_r)^2\right]/2}$
σ_r, σ_θ	respectively thermal radial and thermal circumferential stresses

Introduction. Basic Ideas of the Method

It was shown in Refs (1,2) that in specimens as in Fig.1 when they are heated from the inside a thermal stress state with any magnitudes of principal stresses can be induced in their thin parts. The specimens are assembled into a package of 8 to 10 pieces, the inner space of which is blown through alternately by a hot gas (products of kerosine combustion) and cold air.

Since each of the specimens consists of two massive parts (rings) which have a neck in their cross section, the mean temperature of the inner part varies with a higher rate than the outer one (ring). The neck offers essential resistance to the passage of a thermal flux. As a result of temperature difference in the specimen two parts, high thermal stresses are induced in the neck. The ratio of their components (principal stresses) are dependent on both the material thermophysical/thermomechanical properties and the ratio of the masses of the specimen two parts, the neck thickness, heat supply area, boundary and initial conditions of heat exchange.

Every package of specimens was tested under similar conditions of heat exchange. Therefore, to ensure different nonstationary thermal stress state in each specimen they should be manufactured with slightly varying dimensions. These changes should provide a different preset state in each of the specimens. The authors used the design of one of the specimens as a reference one (Fig. 1), while the rest of them differed in some dimensions. As a rule, the difference was slight and was achived by working the specimen on a lathe. The condition of the neck changes most essentially with its thickness. The change in this dimension by the tenth fraction of a millimeter results in a change in the magnitude of stresses by tens of percents. However, this does not mean that specimens should be manufactured with a very high accuracy. The thickness can be chosen such that its effect is not essential, as will be shown below.

In addition, it is important to measure specimen dimensions with a high precision rather than to fabricate it precisely. This is necessary for correct assessment of thermal stress and thermal states. The knowledge of true stresses for several specimens with different dimensions is important for the construction of thermal fatigue curves. Here, it is important to obtain experimentally the dependence of lifetime on the magnitude of those stresses with the known temperature of the metal in the specimen thin part.

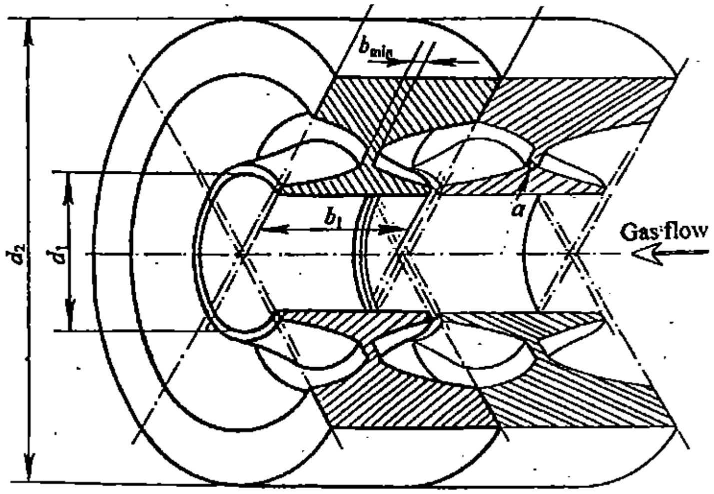

Fig. 1. Specimens for thermal fatigue tests of materials.
For the reference specimen: $d_1 = 2r_1 = 10$ mm, $d_2 = 2r_2 = 30$ mm,
$b_1 = b_2 = 10$ mm, $b_{min} = 0.8$ mm.

Since it is known that the experimental points on any curve of the material life have a wide spread, it is recommended that a few packages of similar specimens be tested under identical conditions. In this case there is no need to attain high accuracy in their manufacturing. It is important to measure them precisely, to maintain identical conditions of heat exchange and, if possible, to measure and record the temperature curves for characteristic points of several specimens using thermocouples. This is done at the initial stage of the test when the boundary conditions of heat exchange are not yet known exactly. As is known, they depend on the characteristics of the gas flow and thermal and physical properties of the material under test.

The procedure proposed for thermal fatigue testing of materials is distinguished by simplicity, the absence of the need for the application of any external mechanical loads and by the possibility of attaining the required biaxial and multiaxial thermal stress states.

In view of the fact that the specimen is in a free state, the possibility of any misalignments or abrupt irregularities in load application is excluded, although some irregularities can take place because of possible nonuniformity of the gas or air flow. Such nonuniformities cannot give rise to a high concentration of stresses. Therefore, the procedure can be used successfully for testing of both metallic and brittle refractory and cermet materials.

The main problem in the use of the proposed procedure is the choice of the shape and dimensions of specimens where the prescribed states have to be induced. The methods for solving the direct problem of heat conduction and thermoelasticity for such kind of specimens were obtained earlier **(2, 3)** In this paper, a procedure for the solution of an inverse problem is proposed. It is based on that a direct problem was solved for a set of specimens. The results of those solutions are presented in the form of plots of the components of extreme thermal stresses, the ratios of principal stresses and the corresponding temperatures as functions of the neck thickness, the specimen inner and outer diameter and the area of heat supply.

The State of the Basic Specimen.

To get a general idea of the state of the specimen shown in Fig. 1, consider a case when it is heated by a gas flow with the temperature of the medium varying from T_0=200°C to T_m=1200°C. The initial uniform temperature of the specimen is T_0=200°C. The coefficient of the convective heat transfer on the inner surface is $\alpha^* = 250$ W/m K, the Biot criterion $\mathrm{Bi} = \alpha^* r_1/\lambda = 0.05$. The specimens are fabricated of steel 1X18H10T. The inside radius $r_1 = d_1/2$ is taken as the characteristic dimension. The instants of time t=2.5 s, 10 s and 25 s are considered. They correspond to the following values of the Fourier criterion: $\mathrm{Fo} = at/r_1^2 = 0.5$, 2 and 5. In the transition regions the thickness is assumed to vary in accordance with the power law with the exponent equal to -9 for the narrowing region and to +9 for the widening region.

The results of the calculations performed by the procedure described in Refs **(2, 3)** are presented in Fig. 2, 3 where $\theta = (\mathrm{T} - \mathrm{T}_0)/(\mathrm{T}_m - \mathrm{T}_0)$, $\sigma^* = \sigma/\alpha\mathrm{E}(\mathrm{T}_m - \mathrm{T}_0)$. From this

Figure one can conclude that the maximum stress intensity is observed in the layers on the radius 7.4 mm and it is equal to $\sigma_i = 495$ MPa (in a dimensionless form $\sigma_i^* = 0.15$). In Figure 1, 3 this region is indicated by the point *a*. This state corresponds to the values of radial stress $\sigma_r = -572$ MPa $(\sigma_r{}^* = 0.173)$ and circumferential stress $\sigma_\theta = -289$ MPa$(\sigma_\theta{}^* = 0.088)$ and the temperature *T*=370°C (in a dimensionless form θ =0.17). In the layers near the inner surface the circumferential stress is equal to$\sigma_\theta = -339$ MPa which is 50 MPa higher than at the point *a*.

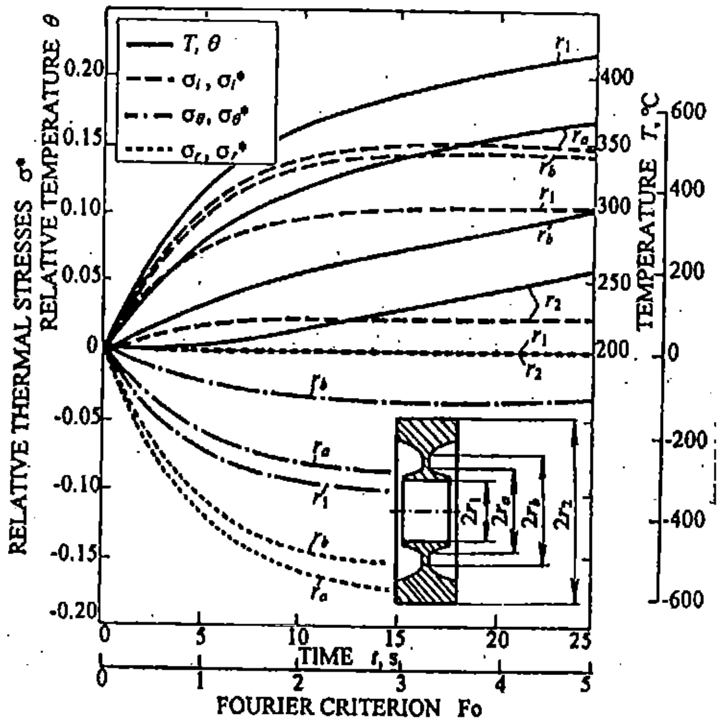

Fig. 2. Time variation in temperature and thermal stresses in different points of the specimen. Here and further r_a = 7.4 mm, r_b = 8.7 mm

The temperature on the inner surface *T*=418°C, i.e. it is 48°C higher. However, the stress intensity at the inner surface equal in the absolute value to the circumferential stress, i.e. $|\sigma_i| = |\sigma_\theta| = 339$ MPa, will be much lower than that at the point *a*, namely, by 156 MPa.

Undoubtedly, the influence of this difference in the stress levels (31%) in the temperature range 370°C - 418°C for the steel studied will be more essential than that of the variation of the temperature by 48°C. On cooling the specimen (e.g. from 1200°C down to 200°C), the sign of radial and circumferential stresses will be reversed and under symmetrical boundary conditions of heat exchange their absolute value will not be changed. Consequently, it can be concluded from the aforesaid that in testing specimens with cyclic heating under the above conditions one can anticipate the occurrence of thermal fatigue cracks in the thin part of the specimen (point *a*) where the stress intensity is maximal.

At higher test temperatures, it may appear that the difference in the extreme state temperatures near the specimen surface and in its thin part will be within the temperature range wherein a considerable reduction in the mechanical characteristics of the material

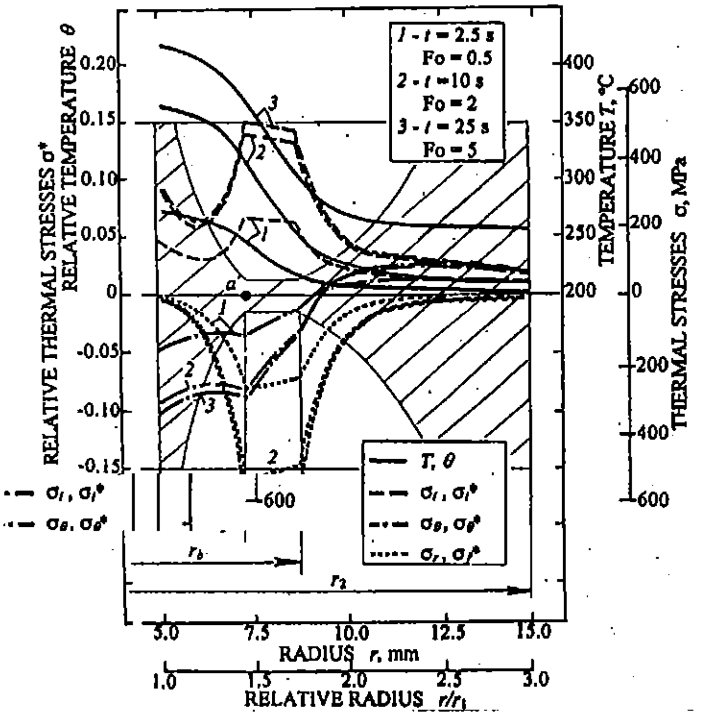

Fig. 3. Distribution of temperature and thermal stresses over the specimen section at different instants of time. Thin lines show the specimen half section.

(ultimate strength, yield stress, fatigue limit) occurs. In this case it is highly probable that the material layers in the immediate vicinity of the inner surface of the specimen may turn out to be the weakest point.

Note that maximum thermal stresses occur at the 20-th seconds after the beginning of the process (Fig. 2). With a further, actually linear rise in temperature at all the points of the body, the stresses are essentially unchanged over a relatively long period of time. Subsequently they begin to diminish and disappear completely after the leveling of the temperatures in the specimen body. Such character of thermal and thermal stress state variation which enables attaining a stable thermal stress state over a relatively long period of time is caused by the influence of high thermal resistance of the specimen thin part. Owing to this effect, one can obtain the required temperature by a simple method with practically unchanging parameters of the thermal stress state. After attaining the required temperature, the heating can be stopped abruptly by turning-off the heat source, or by changing its operating conditions so that the leveling of the temperature in the specimen body would occur at the prescribed level. In the specimen cooling half-cycle the observed effect is of no principal importance since after attaining the extreme thermal stress states, the process of their stabilization and lowering of their level proceeds at decreasing temperature

State of Specimens of Changed Dimensions

In order to induce different prescribed extreme states in each of the specimens in the package under test, the specimens are fabricated in such a way that some of their dimensions differ slightly.

1. When the thickness of the thinnest part alone ($b_{\min}$) is changed from 0.8 mm to 0.4 mm, i.e. when only 0.2 mm of metal is removed on each side, the stress intensity can increase here from $\sigma_i = 495$ MPa to $\sigma_i = 1220$MPa, i.e. by a factor of 2.46 (Fig. 4).

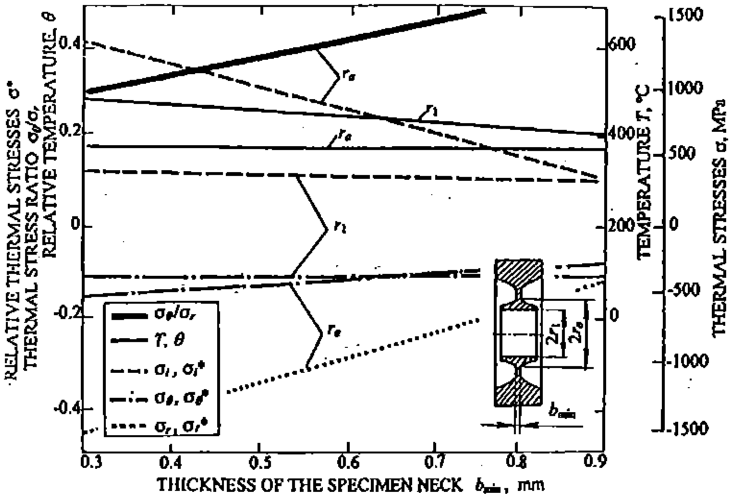

Fig. 4. The dependence of thermal stresses which are maximum in time and their corresponding temperatures at points $r = r_1$ and $r = r_a$ upon the neck thickness b_{min}

This is indicative of the possibility of generating the required stresses over a wide variation range of their magnitude, on the one hand, and of the necessity of very high requirements to the accuracy of specimen fabrication in the neck region and the measurement its dimensions, on the other hand.

In the thin part, with its twofold thinning, the radial stresses increase by about the same factor. The ratio of principal stresses, σ_θ/σ_r, is reduced by approximately 40%. In this case, the extreme temperature of the thin part is practically unchangeable. The extreme state of layers at the inner surfaces of specimens (curves σ_i and σ_θ for $r = 5$ mm) are scarcely affected by the variation in the b_{min} - value.

2. When only the thickness of the outer massive part b_2 is changed in the specimen, i.e., for example, with the dimension b_2=10 mm being in place of b_2=5 mm, we get the dependence shown in Fig. 5.

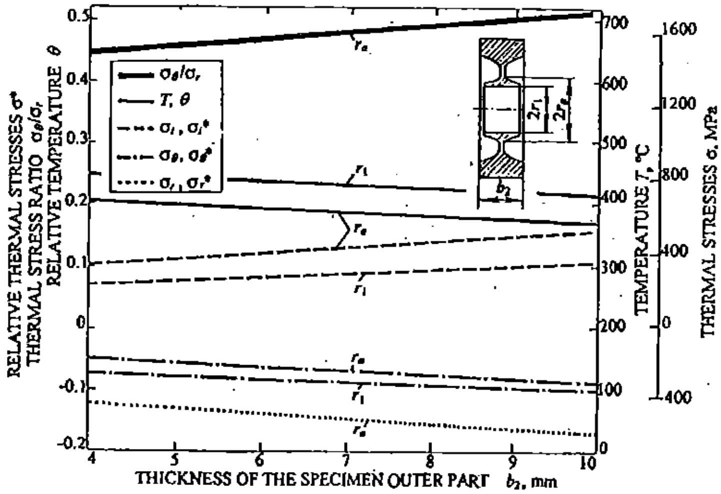

Fig. 5. The dependence of thermal stresses which are maximum in time and their corresponding temperatures at points $r = r_1$ and $r = r_a$ upon the thickness b_2 of the specimen outer part

As the thickness is reduced by half (5 mm instead of 10 mm), the stress intensity in the thin part will vary from $\sigma_i = 495$ MPa to $\sigma_i = 366$ MPa, i.e. will be reduced by 26%. As this takes place, compressive stresses (both circumferential, σ_θ, and radial, σ_r, ones) decrease accordingly. The temperature in this point will increase by 4.8% (from T =370°C to T=402°C). The ratio σ_θ/σ_r will decrease by approximately 9.5%.

3. The material temperature in the specimen thin part is affected to the greatest extent by a decrease in the thickness of the inner massive ring (dimension b_1) (Fig. 6). As the thickness b_1 changes from 10 mm to 5 mm, the temperature changes by 71°C (from T=370°C to T=299°C).

This is an important factor when it is considered that the variation in the thickness $b_{\min}$ from 0.8 mm to 0.4 mm results in the increase in the stress intensity by more than twice, but in this case the temperature remains actually unchanged. The change of the thickness b_1 results a decrease in both the inner ring mass and in the heat supply area. As a consequence, in addition to a considerable change in temperature corresponding to the extreme thermal stress state, the level of the induced stress can change almost two-fold.

As b_1 changes from 10 to 5 mm, the σ_θ/σ_r-value increases by almost 30%. The circumferential stresses near the inner surface change slightly. Note that since at $b_1 < b_2$ a slot is formed between the specimen inner rings through which the gas may leak, it is filled by special rings of heat-insulating material.

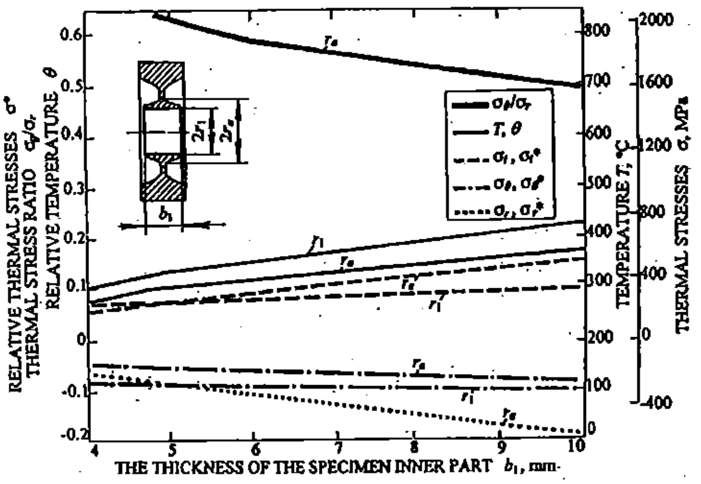

Fig. 6. The dependence of thermal stresses which are maximum in the time and their corresponding temperatures at points $r = r_1$ and $r = r_a$ upon the thickness of the specimen inner part b_1

4. Figure 7 shows how the state of the specimen is affected by a change in its outer radius r_2. The decrease in the outer diameter from 30 mm down to 24 mm results in a considerable change in the principal stress ratio, σ_θ/σ_r. As this takes place, even the sign of this ratio changes. The magnitude of the stress intensity both in the specimen thin part and near its inner surface decreases. The temperature of the thin part rises by almost 40%.

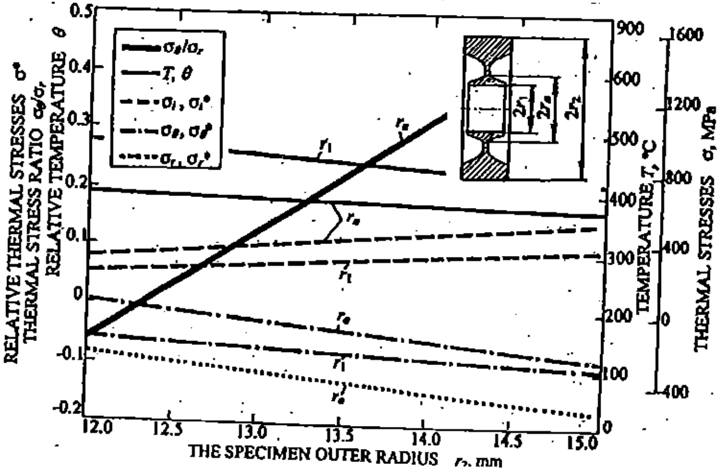

Fig. 7. The dependence of thermal stresses which are maximum in time and their corresponding temperatures at points $r = r_1$ and $r = r_a$ upon the specimen outer radius r_2.

A Simplified Method for Choosing Some Dimensions of Specimens.

The specimen dimensions required for inducing preset thermal and thermal stress states can be defined using the dependences given in Figs 4 through 7. However, to do this in practice is rather difficult since a change in each dimension of the specimen influences simultaneously all the parameters of the thermal and thermal stress states. This can be done approximately by choosing an appropriate minimal thickness of the specimen to preset the required magnitude of the stress intensity with the use Fig. 4 and an appropriate magnitude of the outer diameter to preset the principal stress ratio with the use of Fig. 7. The required temperature can be given to a first approximation by choosing the thickness of the specimen part being heated (Fig. 6) or the thickness of the specimen massive part (Fig. 5). The material state can be defined more accurately by the successive approximation method or by solving a set of equations, namely, dependences of the material state parameters upon the specimen dimensions with certain dimensions of the specimen being chosen as unknown ones. However, the choice between the varied dimensions of the specimen will not necessarily be optimal since there are a lot of combinations of the specimen dimensions wherein the thermal and stressed states will be rather close. In this case, most of those wherein the thermal and stressed states will be rather close. In this case, most of those combinations include the dimensions which present certain difficulties in specimen fabrication.

A Method for Choosing the Combinations of Specimen Dimensions which Provide Producing of the Preset Thermal Stress States

The choice between specimen dimensions can be made more conveniently and clearly using the interrelations between the state parameters with the specimen dimensions being changed, which are plotted in Fig. 8. The change in the state of the specimen material with the change in each dimension is given in Fig. 8 as the lines in the space of the material state parameters. The line length corresponds to the change in the specimen dimension, whereas its direction is determined by the type of specimen dimension.

This is valid within the linear dependence of the state parameters upon the specimen dimensions. For an appreciable nonlinearity, it is necessary to use curvilinear segments of the dependences plotted separately for each specific region of the state parameters variation rather than the straight line segments. Figure 8 shows two projections of the lines segments corresponding to the variation of the specimen minimum thickness from 0.8 to 0.4 mm (segment AA_1), of the thickness of the specimen part from 10 to 6 mm being heated(AA_2), of the thickness of the massive parts of the specimen from 10 to 4 mm (AA_3) and of the specimen outer diameter from 30 to 24 mm (AA_4) in the following planes: stress intensity versus principal stress ratio (*a*) and temperature versus principal stress ratio (*b*). Considered here is the state of the specimen thin part. All the lines in Fig. 8 emerge from the point A corresponding to the state of the reference specimen whose dimensions are given in Fig. 1, whereas the distribution of temperatures and stresses over the specimen cross-section is presented in Fig. 3. The direction of each of the line segments and its magnitude proportional to the variation of the corresponding dimension of the specimen are seen in Fig. 8. The possibility exists of choosing the required segments of those lines in order to fall on the given point of the material state parametric space.

Consider an example of choosing the dimensions such that the given thermal and thermal stress states in the specimen are set up with the following three parameters: the stress intensity 800 MPa, the ratio of the circumferential to radial stresses 0.2, the temperature 400⊕C. This state is shown as point B is Fig. 8.

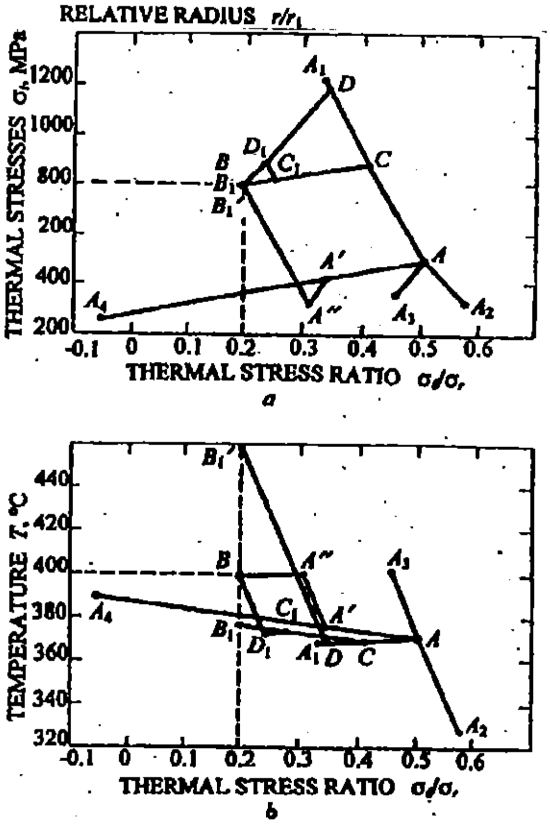

Fig. 8. Scheme of choosing specimen dimensions to induce preset thermal and thermal stressed states of the specimen.

By varying the specimen dimensions, the material state can be moved from the point A to point B by different ways. One of them is shown in Fig. 8 as segments of the lines AA', $A'A''$ and $A''B$ which are parallel to segments AA_4, AA_3 and AA_1, respectively. The segment lengths reflect the changes in the specimen dimensions that are required to set up the prescribed state. For example, to get from point A into point A_4 it is necessary to reduce the outer diameter by 6 mm (from 30 down to 24 mm) and into the point A' by 1.66 mm (from 30 down to 28.34 mm), i.e. by the value which is less by as much as the segment AA' is less than the segment AA_4.

To find the points A' and A'' is not always easy. It is much easier to prescribe the state with two parameters. For example, in Fig. 8a, the segment BC is drawn from point B parallel to the segment AA_4 until it intersects the segment AA_1 at point C. The changes in the minimal thickness and the specimen outer diameter which are required to set the stress intensity equal to 800 MPa and the principal stress ratio to 0.2 can be determined from the lengths of the resulting segments AC and CB. With the resulting segments AC and CB_1 having been constructed, as in Fig. 8b, we shall see that the state obtained (point B_1) differs from the preset one (point B) in temperature.

It is seen from Fig.8b, that the material temperature is most strongly affected by changes in the inner thickness (the line segment AA_2). A reduction of the inner thickness results in a decrease in temperature. The temperature at point B_1 was found to be lower than the required one. To increase the temperature we reduce the outer thickness (the line segment AA_3). The decrease in the outer thickness will cause not only a rise in temperature, but it will also change the stress intensity and principal stress ratio. Find the sum of changes in the specimen dimensions such that the stress intensity and the principal stress ratio remain unchanged. For this purpose, in Fig. 8a, we go from point B e.g. to point C using the segment BC which has already been constructed. Then we return again to point B using the changes in the outer thickness and in the specimen minimum thickness, i.e. the segment BD is drawn from point B parallel to the line segment AA_3 until they intersect at point D with the segment AA_1 passing through point C (Fig. 8a).

With the sum of the same changes plotted on the "principal stress ratio-temperature" coordinates (segments B_1C, CD and DB_1O in Fig. 8b), we shall see that the temperature at point B_1O has increased as compared to point B_1 by the value somewhat higher than that required to attain the prescribed temperature of 400#C with the unchanged principal stress ratio and stress intensity. With the segments B_1C, CD, DB_1O having been reduced in equal proportion to B_1C_1, C_1D_1, D_1B_1, i.e. as much as the temperature diference at points B and B_1 is less than at points B and B_1**O** , we get to point B in all the prescribed parameters. On summation of the lengths of the line segments AC, C_1D_1, we obtain the change in the specimen minimal thickness from 0.8 to 0.52 mm which is proportional to the lines total length. Similarly, a change in the outer diameter from 30 to 28.34 mm corresponds to the total length of the line segments CB_1, B_1C_1 with the account taken of the changes direction, i.e. actually to the length of the segment CC_1. The length of the segment D_1B corresponds to a change of the specimen outer thickness from 10 to 6.42 mm. By reducing those three dimensions of the reference specimen we get the material thermal and thermal stress state prescribed in three parameters under the specimen test conditions identical to those of the reference specimen. When it is required to specify the thermal and thermal stress states in terms of a great number of parameters, e.g., the stress gradient over the specimen cross-section, the temperature variation rate, etc., apart from the above mentioned parameters, the process of more accurate determination of the given state may be continued.

The plotted sum of changes does not influence the magnitudes of the parameters already specified but changes the additional parameter by some value. Comparing this value with the required change of the additional parameter, we increase or reduce, in a similar proportion, all the members of the plotted sum in such a way as to obtain the given value of the additional parameter.

Thus using the proposed presentation of the dependences of the thermal and thermal stress state parameters of the specimen material on its dimensions, one can choose the most optimal proportions of the specimen dimensions to produce the material state specified by a certain number of parameters.

When it is required to vary the specimen dimensions over a wide range, the length and directions of the line segments in Fig 8 can be distorted due to non-linearity of the dependences. For better accuracy, it is necessary to use more plots with a narrower range of the specimen dimensions variation.

Conclusion

A method is proposed for choosing specimen shape and dimensions for thermal fatigue tests of materials which allows inducing preset combinations of the extremes of nonstationary biaxial thermal stress state components and corresponding temperature.

References

(1) TRET'YACHENKO G.N., BARILO V.G., and SOLOVJOVA N.G., (1993), Modeling of inhomogeneous biaxial thermally stressed states of materials in the range of principal-stress ratios from -◎ to +◎. Strength of Materials. Vol. 25, *N* 3, March, November, pp. 188-195.

(2) TRET'YACHENKO G.N., KARPINOS B.S., BARILO V.G., and SOLOVJOVA N.G. (1994) Method and testing results of biaxial thermal fatigue with nonuniform strain distribution. Fourth International Conference on Biaxial/Multiaxial Fatigue, May 31 - June 3, 1994, St Germaine en Laye (France) European Structural Integrity Society (ESIS) vol. 11, pp. 379-390.

(3) TRET'YACHENKO G.N., KARPINOS B.S., BARILO V.G., and SOLOVJOVA N.G., (1996). Biaxial thermal fatigue testing with non-uniform strain distributions. Fatigue & Fracture of Engineering Materials & Structures, Vol. 19, *N* 10, pp. 1239-1253.