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Proposal of an Approach in Multiaxial Fatigue

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ABSTRACT: This paper presents an experimental formula for fully reversed out-of-phase torsion and bending fatigue. Assumptions of the presented approach are based on the physical interpretation of the effect of the stress acting on the critical plane τ^* *and the stresses acting on the other planes* τ^{**}. *It was also assumed that actions of these two effective stresses can be summed and reduced stress* τ_{red} *may be then defined as:* $\tau_{red} = \tau^* + \tau^{**} = t$. *The detailed form of the above equation is presented:* $\tau_{red} = \tau^* + (e \cdot t/b \cdot \tau_{min})^t$ *The reliability of this approach is evaluated by comparing it with experimental data from various publications. It should be noted that the discussed idea has still rather cognitive than practical value.*

Notation

τ_{xy}	torsional shear stress
σ_x	bending stress
$\tau_{xy(a)}$	torsional shear stress amplitude
$\sigma_{x(a)}$	bending stress amplitude
ϕ	out-of-phase angle, σ_x leading τ_{xy}
λ	stress amplitude ratio = $\tau_{xy(a)}/ \sigma_{x(a)}$
α	angle measured from bending stress plane
α_0	angle of principal stress plane
τ_α	shear stress amplitude on a plane α
σ_α	normal stress amplitude on a plane α
σ_τ	normal stress value on a plane α, corresponded to τ_α
β	out-of-phase angle, σ_α leading τ_α

b fatigue limit under reversed bending
t fatigue limit under reversed torsion
τ_{MD} basic criterion by McDiarmid
τ^* value of τ_{MD} on critical plane
τ_{min} minimum value of τ_{MD}
τ^{**} effect of the reduced stresses acting on the other planes α
τ_{red} reduced stress according to the presented approach

Introduction

Many engineering components and structures are subjected to the multiaxial fatigue loading conditions. Engine crank shafts, propeller shafts are a few examples of shafting subjected to the combined out-of-phase bending and torsion.

Many multiaxial fatigue theories have been developed since 1900. Most of them are limited to in-phase loading cases. Although some can be used for out-of-phase loading, additional work must be done for general applicability.

The aim of this study is not to propose a "new criterion" for out-of-phase multiaxial fatigue. The purpose is to adapt existing criteria to reduce calculation error and to develop an approach based on physical observations of in-phase and out-of-phase multiaxial fatigue. Proposed approach is an attempt to connect physical sense of criteria formulated for in-phase and for out-of-phase multiaxial fatigue.

This study led to the development of one empirical relations that fit well with the experimental results. Parameters used in the formula are determined from experimental data published by Nishihara [1] and Neugebauer [2]. The experimental data used to test the validity of the criterion are from Lempp [3], Sonsino [4] and Neugebauer [5].

It must be noted that proposed empirical formula has still rather theoretical than practical value. It is because too many complex parameters are to be determined.

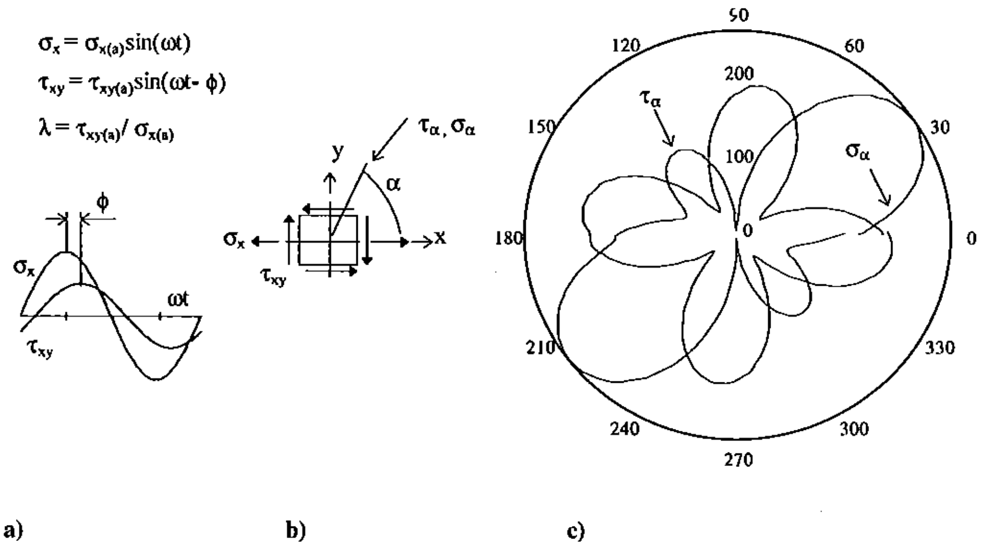
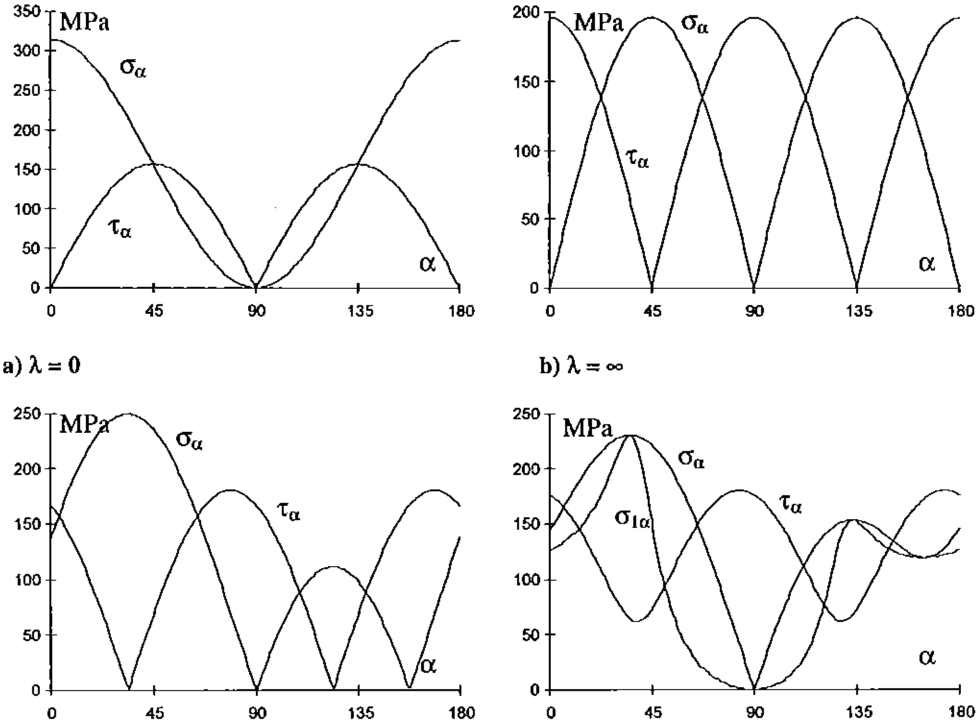


Fig. 1 Amplitude out-of-phase bending and torsion stress variation with time for $\phi \neq 0$

The effect of a phase difference in combined bending and torsion has been investigated (Fig.1a). Important parameters of such state of stress are: ϕ - out-of-phase angle, λ - stress amplitude ratio. Stresses σ_α and τ_α in polar co-ordinate system (Fig. 1b) are shown in Fig. 1c. For farther analysis it is good to show these stresses in rectangular co-ordinate system (Fig. 2). In cases a) $\lambda=0$, b) $\lambda=\infty$ and c) $\lambda=1.21$, $\phi=0$ in Fig. 2, principal stress axes are fixed, and angle α_0 has only one value: a) $\alpha_0 = 0°$, b) $\alpha_0 = 45°$ and c) $\alpha_0 \approx 40°$. For $\phi \neq 0$, when for instance d) $\lambda=1.21$, $\phi=60°$ (Fig.2), the principal stress field is rotating (Fig. 3a) and the maximum ranges of normal and shear stresses are not in phase (Fig. 3b).



a) λ = 0

b) λ = ∞

c) λ = 1.21 and ϕ = 0°

d) λ = 1.21 and ϕ = 60°

Fig. 2 Variation of maximum values of σα and τα with α for different ϕ and λ

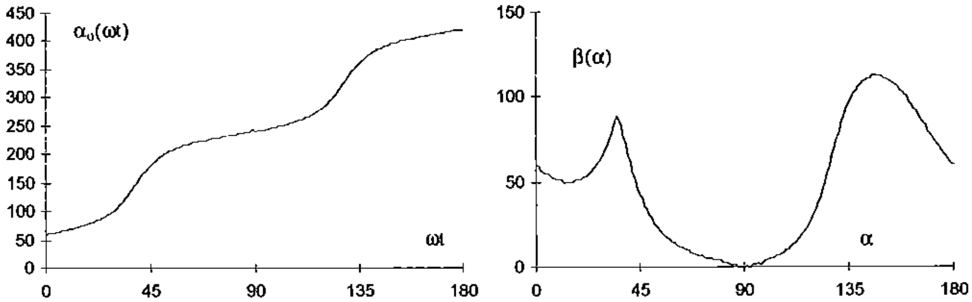


Figure 3. Variation of α and β for ϕ=1.21 and λ=60°

Fatigue behaviour under out-of-phase bending and torsion

Rotation of the principal stress field has the significant impact on fatigue limits. Nishihara and Kawamoto [1] have shown that the fatigue limits, based on the normal value of the maximum shear stress, are higher for combined bending and torsion with phase difference than for in-phase stressing case. Little [6] has shown that this increase in fatigue strength can be misleading, as when the fatigue limit data are expressed in terms of the true maximum shear stress amplitudes in the out-of-phase cases it becomes clear that the fatigue limit actually decreases as the phase difference increases. McDiarmid [7] shows that the decrease is of the order of 25% for λ=0.5 and ϕ=90°.

Microscopic evidence of the out-of-phase impact on the fatigue behaviour was shown by Rios [8]. For uniaxial and in-phase biaxial loading dislocations are clustered together into diffuse cell walls. For non-proportional loading the character of the cell boundary changes from loose tangle to a tight, dense network, increasing in perfection and misorientation as the phase angle ϕ is raised from 0 to 90°. According to Rios, this is because of the multiple slip brought about by constantly changing the preferential slip plane. In the low-cycle fatigue regime, the extra hardening can be observed as a result of multiplication and interaction of dislocations.

Physical sense of proposed approach

Physical sense of criteria formulated for in-phase multiaxial fatigue

Macha [9] has defined an assumption for creating stress and strain fatigue criteria: fatigue fracture of materials is defined only by those components of the stress or strain states which were acting on an existing fatigue fracture plane. In case of the energy based criteria it is assumed that fatigue fracture is determined by the amount of energy equal to the specific work related to the strain in one direction, namely that connected with the fatigue fracture plane. This direction determines the position of the critical plane. This is a view based on the microscopic observations of fatigue crack initiation and growth.

Physical sense of criteria formulated for out-of-phase multiaxial fatigue

The various authors like Rios [8], Lee [10] or Sonsino [11], explain the decrease in fatigue strength in the same way. The detailed analysis was made by Bentachfine [12]. He distinguishes two modes of the deformation process: stable, when the phase angle ϕ is zero, and the strain components stay always at the same proportion, and unstable, when ϕ deviates from zero and the deformations are not proportional. In the stable mode, the strain direction can change into opposite and the same slip systems can be active. In the unstable mode, the grains are subjected to a loading which constantly rotates. The consequence is that different slip and twinning systems combinations become active. The same slip system can not be active and new dislocation sources are necessary to form. The number of defects increases. According to Bentachfine this is the main reason why fatigue life must decrease.

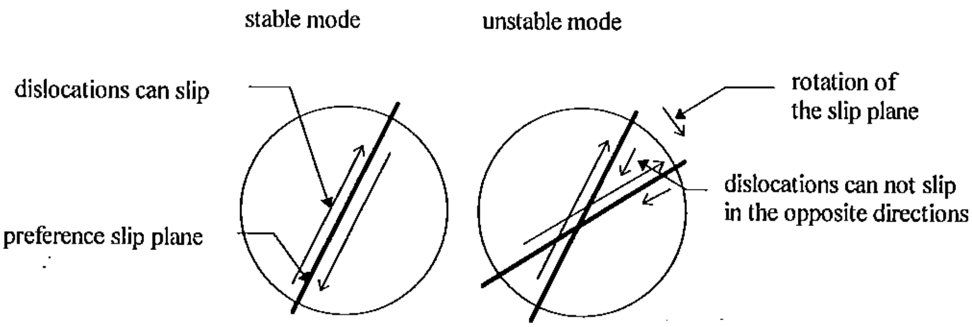


Fig. 4 Slip systems in stable and unstable mode

Definition of the approach

Assumptions

According to the Macha's suggestions [9], proposed approach is based on the critical plane theory. It assumes that the effective shear stress τ^{*} governing the fatigue failure is a linear function of the shear and normal tensile stresses on the critical plane. The plane is determined by maximum rage of shear stress.

The decrease of the fatigue limit as the phase difference ϕ increases testifies to the participation in fatigue process stresses τ^{**} acting on another planes α (Fig. 4). Therefore, previous assumption must be supplemented by Bentachfine's considerations.

A more general assumption can be introduced now: actions of these two effective stresses can be summed and reduced stress τ_{red} may be then defined as:

$$\tau_{red} = \tau^{*} + \tau^{**} = t \tag{1}$$

This value should be constant and equal to the fatigue limit in a full range of angle ϕ (Fig. 5).

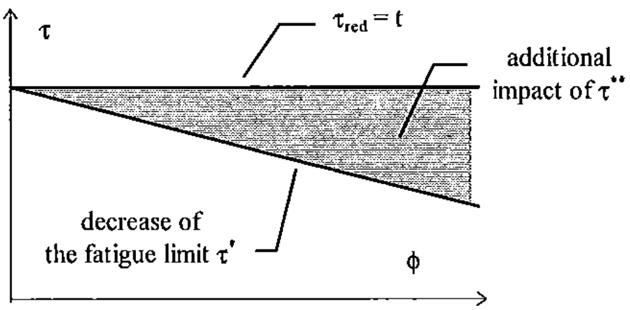


Fig. 5 Participation in fatigue process stresses τ**

Basic criterion τ^*

As a sample of the basic criterion τ^* critical value of McDiarmid's formula τ_{MD} [7, 13, 14] was accepted. This is stress based critical plane criterion of failure, modified for the effect of normal stress on the plane of maximum range shear stresses. According to the author this damage parameter is conservative for the long life out-of-phase bending and torsion.
For the later analysis the following modified form of McDiarmid formula was accepted:

$$\tau_{MD} = \tau_\alpha + (t - b/2) \cdot (2\sigma_\tau / b)^{1.5} \tag{2}$$

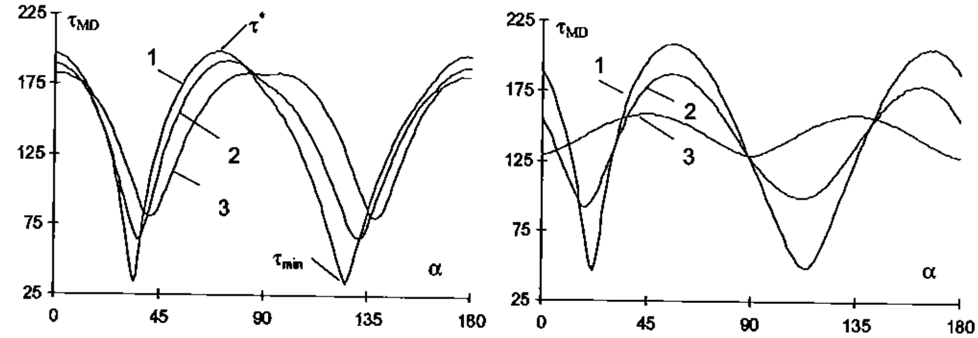
It should be noted that σ_τ is a normal stress value on a plane α corresponding to τ_α, when at McDiarmid σ_τ means normal stress amplitude.

Form of τ^{**}

Impact of the stresses acting on the other planes τ^{**}, can be defined on the ground of analysis of $\tau_{MD}(\alpha)$ distribution. Necessary test data shown in Table 1, come from Nisihara [1] and Neugebauer [2].
It is evident from Fig. 6 that the maximum value decreases and the minimum value increases as the phase difference ϕ increases. It confirms the assumption that the fatigue limit decrease can be caused by the greater impact of the stresses acting on the other planes τ^{**}. Minimal values of $\tau_{MD}(\alpha)$ distribution τ_{min} was selected as a parameter τ^{**}. Therefore, quantity τ^{**} can be expressed as a following function:

$$\tau^{**} = \tau^{**}(\tau_{min}) \tag{3}$$

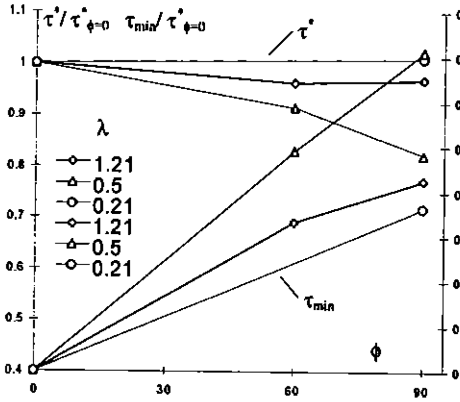
The more detailed analysis was made in Fig. 7. Values of τ^* and τ_{min} are referenced to the values τ^* at ϕ=0°. One can notice that for the different types of materials (different t/b) when the maximum value decreases, the minimum value increases as the phase difference ϕ increases.



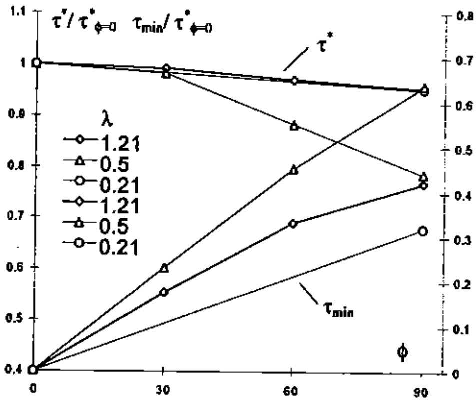
a) λ =1.21

b) λ =0.5

Fig. 6 Variation of τMD for t/b=0.625 and ϕ= 0° (1), ϕ=60° (2), ϕ=90° (3)

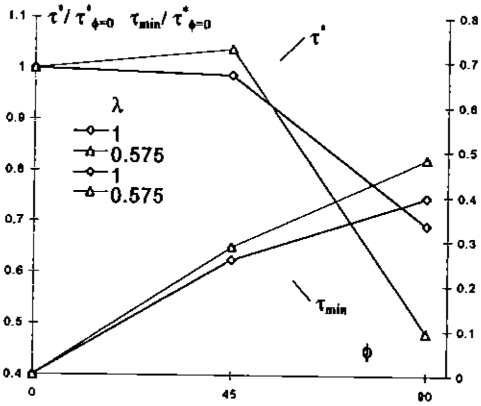


) t/b=0.583



a

b) t/b=0.625



c) t/b=0.938

Fig. 7 Decrease of maximum and increases of minimum τ_{MD} for different materials

Moreover, the impact of material type is also visible. For the same λ and $\phi=90°$ fatigue limits are changing variously for the different values of ratio t/b. This effect is much better visible in Fig. 8 where $\tau^*_{\phi=90}/\tau^*_{\phi=0}$ are collected for two values λ and three ratios t/b. This analysis indicates that τ^{**} should be sensitive also to t/b, so relation (3) must be changed to:

$$\tau^{**} = \tau^{**}(\tau_{min}, t/b) \qquad (4)$$

The above analysis indicates that interaction of dislocations during out-of-phase loading discussed by Bentachfine can be expressed as a function of the stress amplitude ratio λ (with parameter τ_{min}) and as a function of material type (with parameter t/b).

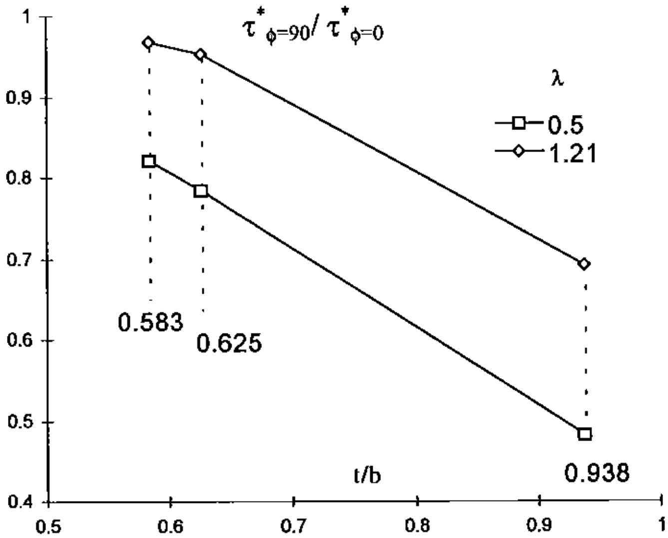


Fig. 8 Impact of material type

Form of τ^{**} considering the λ influence

Equation (4) depends on two variables: τ_{min} and t/b. To find detailed form of the function, variable t/b was preliminary accepted as constants, so $\tau^{**}(\tau_{min}, c)$ and the general form was assumed as $\tau^{**} = c(\tau_{min})^n$. For each material type the influence of λ was analysed separately. It is necessary to find such 'n' that for stress values suited fatigue limit, for any ratio λ and angle ϕ, τ_{red} is equal t:

$$\tau_{red} = \tau^* + c(\tau_{min})^n = t \quad (5)$$

It is required, for each material type, different ratios λ (1.21, 0.5, 0.21) and for full range of ϕ (from 0 to 90°) to statistically examine behaviour of the ratio:

$$(\tau^* + c(\tau_{min})^n) / t \quad (6)$$

However to avoid an error which can be introduced by the basic criterion τ^*, relative quantity was examined:

$$\tau^* / \tau^*_{\phi=0} + c(\tau_{min} / \tau^*_{\phi=0})^n \quad (7)$$

For different 'n' values, obtained results was compared in Table 1. Each row of the table contains the statistical characteristics for assumed material t/b. It is clear that one can get improvement (see the standard deviation) by increase the exponent till n=3. So the character of the stress influence (and their effects i.e. dislocation interactions) at the different planes than the critical plane, is an exponential relationship with the exponent n=3.

Table 1

n	t/b	c	standard deviation	max.	min.	range
1	*2*	*3*	*4*	*5*	*6*	*7*
1	0.583	0.1956	**0.03404**	1.08	0.96	0.12
	0.625	0.2735	**0.03238**	1.07	0.96	0.11
	0.938	0.86668	**0.13282**	1.29	0.9	0.39
2	0.583	0.34923	**0.01959**	1.05	1	0.05
	0.625	0.5178	**0.01565**	1.05	0.99	0.06
	0.938	2.26625	**0.08379**	1.23	1	0.23
3	0.583	0.50376	**0.01626**	1.03	0.97	0.06
	0.625	0.82956	**0.01419**	1.02	0.96	0.06
	0.938	4.96386	**0.05609**	1.16	1	0.16
4	0.583	0.68931	**0.01883**	1.02	0.96	0.06
	0.625	1.27238	**0.01912**	1	0.94	0.06
	0.938	10.3221	**0.04937**	1.11	0.95	0.16

Form of τ^{**} considering the t/b influence

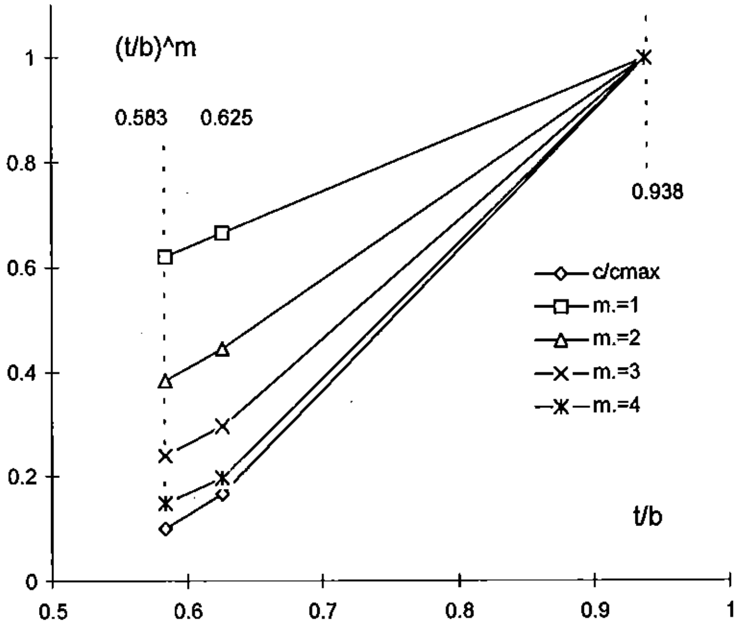


Fig. 9 Fitting exponent 'm'

In the above analysis variable t/b was preliminary accepted as constants 'c'. It can be seen in Table 1 column 3 that 'c' is a function of material type. General form of this function was assumed as:

$$c(t/b) = d(t/b)^m \tag{8}$$

Fig. 9 shows that the correlation between $(t/b)^m$ function and real changes of c(t/b), are better for the bigger exponent 'm'. Taking into account smaller changes for the bigger exponent and aspire to simplicity, exponent 'm' was accepted as equal 'n'. For construction steel Eq. 1 is then given by:

$$\tau_{red} = \tau^* + (\, e \cdot t/b \cdot \tau_{min})^3 \tag{9}$$

where constant 'e' for such basic criterion is 0.041.

Results and summary

In each table, column 11 contains fatigue strength predictions. Because τ_{MD} is a basic criterion, column 6 in Table 2 contains the results published by McDiarmid [7], and in Table 3, the results obtained using his formula.

The formula was determined from experimental data shown in Table 2. It is obvious that predictions in this table correlate the experimental data with good precision.

Table 2

experimental data Nisihara [1]					McDiarmid [7]	**calculation results**	proposed formula			
material	λ	ϕ	$\sigma_{x(a)}$	$\tau_{xy(a)}$	pred./act.	τ^{*}	τ_{min}	τ^{**}	τ_{red}	τ_{red}/t
		°	MPa	MPa		MPa	MPa	MPa	MPa	
1	2	3	4	5	6	7	8	9	10	11
t 137,3	1,21	0	99,9	120,9	1,02	136,06		0,00	136,06	0,99
b 235,4	1,21	60	103,6	125,4	1,03	131,06	45,32	1,29	132,35	0,96
t/b 0,583	1,21	90	108,9	131,8	1,03	131,80	57,95	2,71	134,51	0,98
	0,5	0	180,3	90,2	0,97	141,14		0,00	141,14	1,03
	0,5	60	191,4	95,7	1,05	129,13	69,43	4,65	133,78	0,97
	0,5	90	201,1	100,6	1,08	116,03	100,60	14,15	130,18	0,95
	0,21	0	213,2	44,8	0,99	132,90		0,00	132,90	0,97
	0,21	90	230,2	48,3	1,03	134,05	48,30	1,57	135,62	0,99
t 196,2	1,21	0	138,1	167,1	1,03	191,90		0,00	191,90	0,98
b 313,9	1,21	30	140,4	169,9	1,03	190,54	33,73	0,66	191,20	0,97
t/b 0,625	1,21	60	145,7	176,3	1,02	186,29	64,02	4,49	190,78	0,97
	1,21	90	150,2	181,7	1,06	183,05	80,65	8,97	192,02	0,98
	0,5	0	245,3	122,7	0,97	201,60		0,00	201,60	1,03
	0,5	30	249,7	124,9	0,99	197,99	46,70	1,74	199,73	1,02
	0,5	60	252,4	126,2	1,08	178,05	91,73	13,20	191,25	0,97
	0,5	90	258	129,0	1,10	158,25	129,00	36,71	194,96	0,99
	0,21	0	299,1	62,8	0,99	199,50		0,00	199,50	1,02
	0,21	90	304,5	63,9	1,04	189,75	63,90	4,46	194,21	0,99
Neugebauer [2]					McDiarmid [7]					
1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*
t 300	1	0	190	190,0	1,11	274,36		0,00	274,36	0,91
b 320	1	45	205	205,0	1,05	270,38	70,71	20,51	290,89	0,97
t/b 0,938	1	90	190	190,0	1,50	190	108,74	74,58	264,58	0,88
	0,575	0	255	146,6	1,04	297,79		0,00	297,79	0,99
	0,575	45	290	166,8	0,88	309,33	85,4	36,12	345,45	1,15
	0,575	90	250	143,8	0,98	143,8	143,8	172,47	316,27	1,05

The experimental data used to test the validity of the criterion are collected in Table 3. It should be noted that calculation results in this table are much better then for basic criterion. It is especially important for the difficult prediction case when ϕ=90°.

Table 3

experimental data Nisihara [1]						**calculation results**				
material	λ	ϕ	$\sigma_{x(a)}$	$\tau_{xy(a)}$	McDiarmid pred./ act.	τ^{*}	proposed τ_{min}	formula τ^{**}	τ_{red}	τ_{red}/t
		°	MPa	MPa		MPa	MPa	MPa	MPa	
1	2	3	4	5	6	7	8	9	10	11
t 100	0,48	0	140	67,2	0,94	105,50		0,00	105,50	*1,06*
b 176	0,48	90	168	80,64	1,06	95,20	80,64	6,82	102,02	***1,02***
t/b 0,568										
t 146	0,48	0	201	96,48	0,96	152,61		0,00	152,61	*1,05*
b 254	0,48	90	234	112,32	1,10	133,8	112,32	18,42	152,22	***1,04***
t/b 0,575										
Sonsino [4]					McDiarmid					
1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*
t 120	0,58	0	135	78,3	1,05	114,12		0,00	114,12	*0,95*
b 200	0,58	90	152	88,16	1,40	88,16	86,66	9,76	97,92	***0,82***
t/b 0,600										
Neugebauer [5]					McDiarmid					
1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*
t 175	0,57	0	183	104,31	1,03	170,99		0,00	170,99	*0,98*
b 250	0,57	90	195	111,15	1,44	111,15	111,15	32,96	144,11	***0,82***
t/b 0,7										

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