Krzysztof M. Golos*, Said H. Eshtewi*

Multiaxial Fatigue and Mean Stress Effect of St5 Medium Carbon Steel

* Warsaw University of Technology, Institute of Machine Design Fundamentals,
84 Narbutta St., 02-524 Warsaw, Poland.

Keywords: multiaxial fatigue, mean stress, medium carbon steel

ABSTRACT: *In this paper a room temperature cyclic, uniaxial and multiaxial fatigue experimental data for St5 medium carbon steel have been presented. Stress controlled biaxial fatigue tests on St5 steel were performed under proportional loading, including tests with positive mean stress. Fatigue failure lives were expressed in terms of the total strain energy densities. A new multiaxial fatigue criterion including mean stress effect have been considered. Biaxial fatigue lives correlated well with considered criterion regardless of the stress/strain ratio and mean stress.*

Notation

b	fatigue strength exponent
c	fatigue ductility coefficient
C	total strain energy density at fatigue limit
E	Young's modulus of elasticity
K'	cyclic strength coefficient
N	applied cycles
N_f	number of cycles to failure
n'	cyclic strain hardening exponent
S_y	0.2 percent offset yield strength
S_u	ultimate tensile strength
S_y'	0.2 percent offset cyclic yield strength
Δ	range of strain (stress)
ΔW^t	total strain energy density
ΔW^{e+}	elastic tensile strain energy density
ΔW^p	plastic strain energy density
α	exponent in energy criterion
ε_a	axial strain amplitude
ε_e	elastic strain
ε_f	true fracture strain
ε_p	plastic strain

ε_r	radial strain
ε_t	tangential strain
ε_f'	fatigue ductility coefficient
κ	material coefficient in energy criterion
ν	Poisson's coefficient
ρ (= $\Delta\varepsilon_a / \Delta\varepsilon_t$)	strain ratio
σ	stress
$\Delta\sigma / 2$	stress amplitude
σ_f'	fatigue strength coefficient
σ_{lim}	fatigue limit
σ_{am}	alternating stress amplitude
σ_m	mean stress
σ_t	tangential stress amplitude

Introduction

It is generally believed that the fatigue damage process is controlled by the cyclic plastic deformations that depends on the loading path and the constituent properties of the materials. The steady-state stress-strain histeresis loops of materials under various cyclic loading conditions have been widely adopted for fatigue life analysis. The objective of fatigue life analysis is to develop a sound correlation between the material parameters and the life to failure to predict in a better manner the life of components in practice.

Several multiaxial fatigue life prediction theories have been presented [1-23]. These theories may be divided into three groups: stress, strain and energy based fatigue criteria.

Most of the energy based criteria utilize plastic strain energy as a correlating parameter. However, it is difficult to estimate the plastic hysteresis energy in the high-cycle fatigue regime where stress-strain behaviour is practically elastic. Therefore, models based only on plastic strain energy can not adequately predict high-cycle fatigue life. To describe fatigue in both regimes (i.e. low- and high-cycle) Golos [17-21] has proposed total strain energy density (ΔW^t) equal to the sum of the plastic strain energy density (ΔW^p) and tensile strain energy density (ΔW^{e+}) as a damage parameter. The approach has a number of desirable features such as being consistent with the crack initiation and subsequent propagation, is hydrostatic pressure sensitive. Later, this approach has been generalized for material non obeying Masing description [22] and for non-proportional cyclic loading

[23]. The purpose of this paper is to attempt to predict the multiaxial fatigue lives including mean stress effect from uniaxial fatigue test data. The question being investigated is as follows to what extent multiaxial fatigue life be calculated from uniaxial data using theoretical analysis? By comparing the theoretical and experimental fatigue lives of St5 medium carbon steel at various loading conditions, a reasonable theoretical multiaxial fatigue life prediction methodology is developed.

Background

Is assumed, that damage due to the cyclic multiaxial loading can be modelled as a function of the absorbed plastic strain energy per cycle and that part of the elastic energy which facilitates the crack growth [17-23]. Thus, both tensile elastic and plastic parts of the strain energy per cycle, have to be determined (Fig. 1), i.e.

$$\Delta W^{t} = \Delta W^{p} + \Delta W^{e+} \tag{1}$$

For an isotropic elastic material the cyclic elastic strain energy density for the positive stress parts of the cycle can be calculated from

$$\Delta W^{e+} = \frac{1}{2E}\left[\left(I_1^{max}\right)^2 - 2(1+\nu)I_2^{max}\right] \tag{2}$$

where I_1^{max}, I_2^{max} are the first and second invariant of the stress tensor and H is the Heaviside function.

The cyclic plastic strain energy density, ΔW^{p}, can be calculated from

$$\Delta W^{p} = \int_{cycle} \Delta\sigma_{ij} d\left(\Delta\varepsilon_{ij}^{p}\right) \tag{3}$$

where the integration is carried over the closed cyclic loop. The plastic strain components for a proportional or nearly proportional loading are given by [19, 22]:

$$\Delta\varepsilon_{ij}^{p} = 3(2K')^{-1/n'}(\overline{\Delta\sigma})^{(1-n')/n'}\Delta S_{ij} \tag{4}$$

here $\overline{\Delta\sigma} = (3/2\ \Delta S_{ij}\ \Delta S_{ij})^{1/2}$ and K' and n' are material parameters, in general functions of multiaxial stress state. According to the J_2 theory the equivalent plastic strain can be defined as follows

$$\overline{\Delta\varepsilon}^{p} = \left(2/3\Delta\varepsilon_{ij}^{p}\Delta\varepsilon_{ij}^{p}\right)^{1/2} \tag{5}$$

Combining equation (4) and (5) we obtain the effective generalized cyclic stress-strain curve for multiaxial proportional loading

$$\overline{\Delta\varepsilon^{p}} = 2\left(\overline{\Delta\sigma}/2K'\right)^{1/n'} \tag{6}$$

Therefore for ideal Masing behaviour the plastic strain energy density can be expressed as

$$\Delta W^{p} = \frac{1-n'}{1+n'}\overline{\Delta\sigma}\,\overline{\Delta\varepsilon^{p}} \tag{7}$$

Thus the total damaging cyclic strain energy density, ΔW^{t}, is obtained by combining eqn. (2) and (7), i.e.

$$\begin{aligned}\Delta W^{t} = \Delta W^{p} + \Delta W^{e+} = \frac{1}{2E}\left[\left(I_1^{max}\right)^2 - 2(1+\nu)I_2^{max}\right] + \\ + \frac{2(1-n')(2K')^{-1/n'}}{(1+n')}\left(\overline{\Delta\sigma}\right)^{(1+n')/n'}\end{aligned} \tag{8}$$

Failure criterion with mean stress

The failure criterion in the case of proportional multiaxial cyclic loading can be expressed as

$$\Delta W^{t} = \kappa(\rho)N_f^{\alpha} + C(\rho) \tag{9}$$

where $\kappa(\rho)$ and $C(\rho)$ are the functions of ρ, defined as triaxiality constraint factor

$$\rho = \left(1+\nu^{*}\right)\frac{\varepsilon_1}{\gamma_{max}} \tag{10}$$

where $\varepsilon_1 = \max[\varepsilon_a \text{ or } \varepsilon_t]$ and $\gamma_{max} = \max[(\varepsilon_a - \varepsilon_r) \text{ or } (\varepsilon_t - \varepsilon_r)]$.

The effective Poisson's ratio, ν^* is calculated from

$$\nu^* = \frac{\nu_p(1-\nu_e)(\varepsilon_a + \varepsilon_t) + (\nu_e - \nu_p)(\varepsilon_a^e + \varepsilon_t^e)}{(1-\nu_e)(\varepsilon_a + \varepsilon_t) + (\nu_e - \nu_p)(\varepsilon_a^e + \varepsilon_t^e)} \tag{11}$$

As a first approximation, the functions $\kappa(\rho)$ and $C(\rho)$ can be expressed in a linear form

$$\begin{aligned} \kappa(\rho) &= a_1\rho + a_2 \\ C(\rho) &= a_3\rho + a_4 \end{aligned} \tag{12}$$

where a_1, a_2, a_3, a_4 are material coefficients. Therefore, the explicit form of the failure criterion is obtained by combining eqn. (9) and (12), i.e.

$$\Delta W^t = (a_1\rho + a_2)N_f^\alpha + (a_3\rho + a_4) \tag{13}$$

To obtain an extended multiaxial fatigue criterion the relationship (9) can be modified as

$$f(\sigma_m)\Delta W^t = \kappa(\rho)N_f^\alpha + C(\rho) \tag{14}$$

where $f(\sigma_m)$ is the function of the mean stress, i.e.

$$\sigma_m = \left(2/3\sigma_{ij(m)}\sigma_{ij(m)}\right)^{1/2} \tag{15}$$

The following form of the function $f(\sigma_m)$ is proposed:

$$f(\sigma_m) = \left(1 - \left(\frac{\gamma\sigma_m}{\sigma_f'}\right)^\delta\right)^{(1+n')/n'} \tag{16}$$

where σ_f' is the fatigue strength coefficient, n' is the cyclic strain hardening exponent and γ, δ are materials parameters to fit uniaxial data.

Therefore, the extended multiaxial criterion including mean stress effect can be expressed as

$$\Delta W^{t} = \kappa^{m}(\rho,\ \sigma_{m})N_{f}^{\alpha} + C^{m}(\rho,\ \sigma_{m}) \qquad (17)$$

where $\kappa^{m}(\rho,\sigma_{m}) = \kappa(\rho)/f(\sigma_{m})$ and $C^{m}(\rho,\sigma_{m}) = C(\rho)/f(\sigma_{m})$.

Experimental results and discussion

In tests a medium carbon steel St5 has been selected. The chemical properties of this material are listed in Table 1.

Table 1. Chemical compositions in %wt for St5 medium carbon steel

St5	C	Mn	P	S	Si	Cr
	0.40	0.51	0.02	0.05	0.27	0.05

The material used in this research was sand cast from one heat into approximately 200*200*30 mm blocks from which all test specimens were taken. Monotonic tensile, cyclic and uniaxial fatigue specimens had a cylindrical cross section diameter of 6.4 mm and cylindrical grip ends of 12.7 mm. Thirty such specimens were cut, machined to final dimensions, and then polished with 400 and then 600 grit emery paper. Monotonic tensile tests were performed on six specimens at room temperature, following ASTM Standard Methods B557 and E8. The cyclic stress-strain curve was obtained by the companion specimen method that connects the maximum stress of the stabilized half-life hysteresis loops from fully reversed axial fatigue tests.

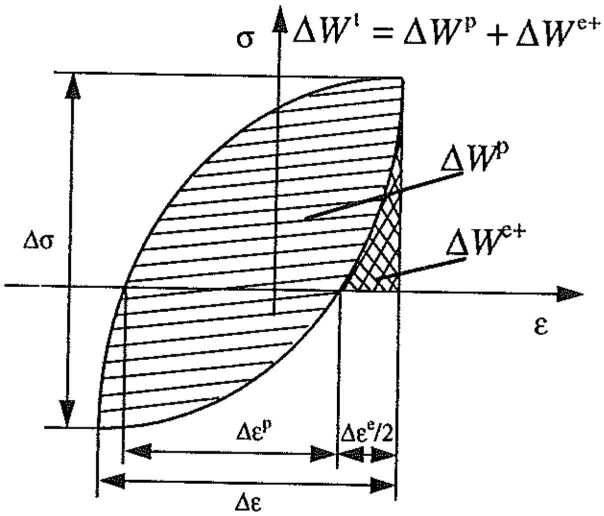

Fig. 1 Definition of damage parameter.

All axial fatigue tests were performed at room temperature on an 100kN closed-loop servohydraulic material testing system. System alignment prior to all testing was checked and corrected according to ASTM Standard E1012. A triangular waveform with a frequency ranging from 0.5 to 20 Hz was used in all axial fatigue tests. Fatigue failure was defined as specimen fracture or 20 percent drop in maximum tensile load. Monotonic, cyclic and fatigue parameters for examined steel are given in Table 2.

Table 2. Mechanical and cyclic properties of St5 medium carbon steel

Property	E [MPa]	$S_{y(0.2\%)}$ [MPa]	$S'_{y(0.2\%)}$ [MPa]	σ_{lim} [MPa]	K' [MPa]	σ_f' [MPa]	
Value	207000	306	325	220	1104	971	
Property	n'	ε_f'	K_u [MJ/m^3]	C_u [MJ/m^3]	a_1 [MJ/m^3]	a_2 [MJ/m^3]	a_3 [MJ/m^3]
Value	0.193	0.63	1069	0.15	1000	69	0.15

To examine the applicability of the investigated criterion to multiaxial cyclic loading and determine the material constants, biaxial fatigue tests for $\rho = \Delta\varepsilon_a / \Delta\varepsilon_t = 1$, 0.5, and 0 have been performed. The axial strain ranges are chosen between 0.075% to 0.18%. Additionaly stress controled tests with $\rho_\sigma = \sigma_{am}/\tau_a = 1.05$ have been done. The axial mean stress, σ_m is choisen between 0 and 50 [MPa]. A series of tests were carried out on thin-walled tubular specimens. The thin-walled specimen was mounted in a pressure test cell device, this assembly then being used in a standard fatigue test machine.

The tests were conducted in oil and the definition of failure was based on the detection of small crack, normally indicated by a distortion in the hysteresis loop. The predictions of the presented criterion are compared with the experimental data for different strain and stress ratio in Fig. 2. For examined material two sets of experimental strain controlled data, e.g. uniaxial case ($\rho = -\nu$) and biaxial for example at $\rho = 0$ were sufficient to specify $\kappa(\rho)$ and $C(\rho)$. For St5 steels we obtained that $a_1 = 1000$ MJ/m^3, $a_2 = 69$MJ/m^3, $a_4 = C_{uniaxial} = 0.15$ MJ/m^3, E=207000 MPa. The value of $C_{uniaxial}$ is that part of total strain energy density which does not cause the damage and can be calculated as

$$C_{uniaxial} = 1/(2E)\sigma_{lim}^2 \tag{18}$$

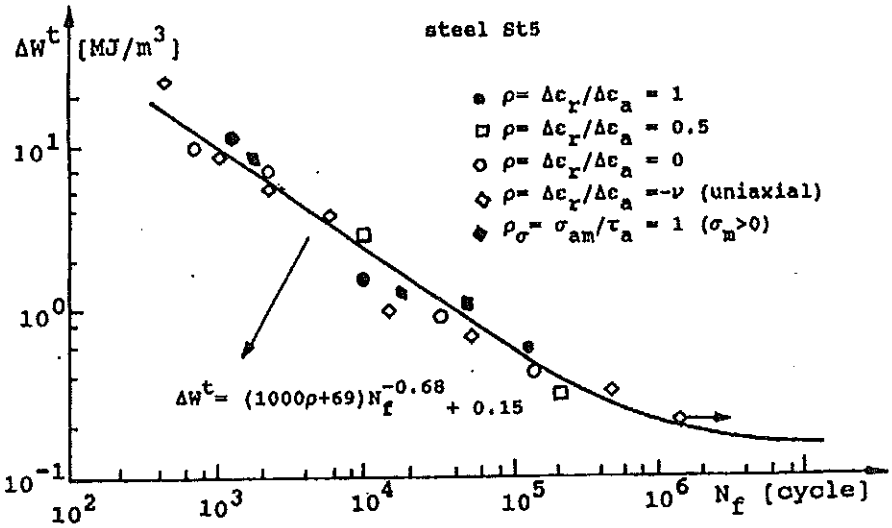

Fig. 2 Total strain energy density versus number of cycles to failure.

One can observes that the correlation between criterion and experimental data for examined materials is fairly good.

Conclusions

A form of the cyclic strain energy density equal to the sum of plastic and tensile elastic strain energy densities is used as a damage parameter for axial and multiaxial fatigue failure. It is show that the used criterion applies to uniaxial and proportional multiaxial loading in both low and high cycle regimes, including mean stress effect. The predictions of the total strain energy density criterion are compared with the experimental results for analyzed St5 medium carbon steel. The agreement for examined material is found to be fairly good.

References

[1] Brown M.W., Miller J., (1985), Multiaxial fatigue, ASTM STP 853

[2] Third International Conference on Biaxial/Mulitiaxial Fatigue, (1989), Staatliche Materialprufungsanstalt (MPA), Universitat Stuttgart.

[3] Fourth International Conference on Biaxial/Multiaxial Fatigue, (1994), St.Germain en Laye (SF2M), ESIS.

[4] Kocanda S., (1978), Fatigue Failure of Metals, Alphen ad den Rijn: Sijthof a. Noordhoff Int. Pub.

[5] Gough H.J., Pollard, H.V., (1935), Strength of metals under combined alternating stress, Proc. Inst. Mech. Eng., 131, pp. 3-54.

[6] Findley W.N., (1957), Fatigue of metals under combinations of stress, Transaction ASME, pp.1337-1348.

[7] McDiarmid, D.L., (1974), A new analysis of fatigue under combined bending and twisting, Aeronautical Journal, pp. 325-329.

[8] Sines G., (1981), Fatigue criterion under combined stresses or strains, ASME Journal of Engineering Materials and Technology, Vol.103, pp. 82-90.

[9] Dowling N., (1978), Stress-strain analysis of cyclic plastic bending and torsion, ASME Journal of Engineering Materials and Technology, Vol.100, pp.157-163.

[10] Brown M.W., Miller K.J., (1973), A theory of fatigue failure under multiaxial stress-strain condition, Proc. Inst.Mech. Engrs., 187, pp. 745-755.

[11] Brown M.W., Miller K.J., (1979), High temperature low-cycle biaxial fatigue of two steels, Fatigue of Engineering Materials and Structures, Vol.1, No1, pp. 217, 229.

[12] Lohr R.D., Ellison E.G., (1980), A simple theory for low cycle multiaxial fatigue,

Fatigue of Engineering Materials ans Structures, Vol.3, pp.1-17.

[13] Fatemi A., Socie D., (1988), A critical plane approach to multiaxial fatigue including out-of-phase loading, Fatigue Fracture of Engineering Materials and Structures, Vol.11, pp.145-165.

[14] Morrow J.D. , (1965), Cyclic plastic strain energy and fatigue of metals, Internal Friction, Damping and Cyclic Plasticity, ASTM STP 378, pp. 48-84.

[15] Halford G.K., (1966), The energy required for fatigue, Journal of Materials, Vol. 1, No 1, pp.3-18.

[16] Garud Y.S., (1979), A new approach to the evaluation of fatigue under multiaxial loadings, in:Methods for Predicting Material Life in Fatigue, ASME, pp.247-264.

[17] Golos K. , (1988), Energetistic formulation of fatigue strength criterion , Archiwum Budowy Maszyn, Tom 35, Nr 1/2, pp. 5-16.

[18] Golos K., (1988), Multiaxial energy based fatigue criterion, Rozprawy Inzynierskie (Engineering Transactions), Vol. 36, nr 1, pp. 55-63.

[19] Golos K., (1988), Plastic strain energy density under multiaxial proportional loading, Mechanika Teoretyczna i Stosowana, Vol.25, nr 1, pp. 170-177.

[20] Golos K., Ellyin F., (1988), Generalization of Cumulative Damage Criterion to Multilevel Cyclic Loading, Theoretical and Applied Fracture Mechanics, Vol.7, pp.169-176.

[21] Golos K., Ellyin F., (1988), A Total Strain Energy Density for Cumulative Fatigue Damage, ASME Journal of Pressure Vessel Technology, Vol.110 , pp.36-41.

[22] Ellyin F., Golos K., (1988), Multiaxial fatigue damage criterion, ASME Journal of Engineering Materials and Technology, Vol.110, pp. 63-68.

[23] Golos K., (1995), Multiaxial fatigue criterion under proportional and non-proportional cyclic loading, 7th Int. Con. on Mech. Behaviour of Materials, Book of abstracts, The Hague, ESIS.