Artur CICHAŃSKI and Janusz SEMPRUCH

Modelling of Biaxial Fatigue Loading State

Department of Mechanical Engineering,
University of Technology and Agriculture in Bydgoszcz, Poland

Keywords: biaxial fatigue , plastic behaviour .

ABSTRACT: The method discussed in this paper aims at determining fatigue tests in load conditions including specific features of biaxial state of fatigue loading. The possibility of modelling , the known from Troost's work , „stress anisotropy" effect by means of plastic strain controlled in respect of its turn and direction has been mentioned here. Using the suggested method of modelling leads to formulating the methodology for fatigue tests which may be performed in a uniaxial test machine. The results of tests have been presented, which in consequence lead to identification of plastic properties, and experimental verification the experimental assumptions of the suggested methodology.

Notation:

σ_x	normal stress
σ_{xa}	normal stress amplitude
σ_{xm}	static (mean) normal stress
τ_{xy}	shear stress
τ_{xya}	shear stress amplitude
τ_{xym}	static (mean) shear stress
α	angle for considerated plane
ϕ_y	out-of-phase angle, σ_x leading σ_y
ϕ_{xy}	out-of-phase angle, σ_x leading τ_{xy}
σ_A	uniaxial fatigue strength
$\sigma_{a(All)}$	allowable normal stress amplitude
ψ	material dependent parameter
σ_{pl}	initiation yield stress
R_e	yield stress

Introduction

One of significant elements which affect fatigue life is the stress distribution in the object under discussion. Analysis of the stress courses on particular planes defined by the observation angle lets us draw the conclusion that despite isotropic properties of the loaded material state of stress occurs in it, and it changes with changing of the observed direction. This phenomenon in relation to fatigue loading showing mean value of cycle has been described by Troost as „anisotropic fatigue behaviour of isotropic material"[1,2].
One way to achieve the anisotropy of mechanic material properties is causing in them plastic changes through subjecting it to loading above the yield point. Exactly determined direction of the object or sample may be a possibility to model the above described fatigue effects by means of plastic properties changes.

Literature shows attempts to adjust the descriptions of yield theories, which are being introduced to determining fatigue point in biaxial state of fatigue loading [2]. For the case of loading which reveal mean values of cycle it is „quadratische Versagungshypotese" based on the Hill's yield criterion.

In this paper it is attempted to describe the fatigue phenomena by means of values which are easy to gain, and which describe plastic properties whereas fatigue life is the parameter of such a description.

Identification of fatigue and plastic properties made let us formulate a thesis that there exists a model course (Fig. 1) which binds both phenomena. Experimental verification of taken assumptions has been presented in this paper, as well.

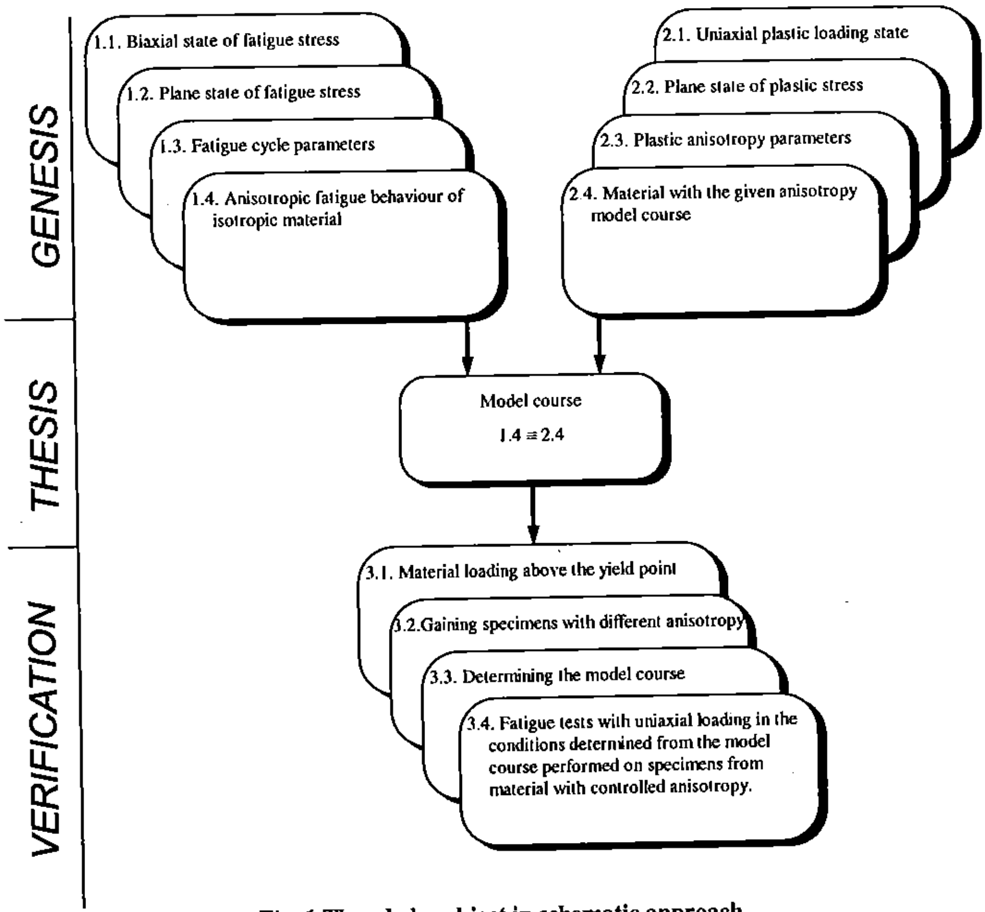


Fig. 1 The whole subject in schematic approach.

Characteristics of complex fatigue loading and fatigue stress states

In Fig. 2 a co-ordinate system X*0Y* connected with the existing nominal loading are presented The values σ^*_x, σ^*_y, τ^*_{xy} are nominal stresses resulting from the loading applied. With the changing of indicating the searched critical plane α angle, a rotation of the XOY system and stress σ_x, σ_y, τ_{xy} connected with it follows.

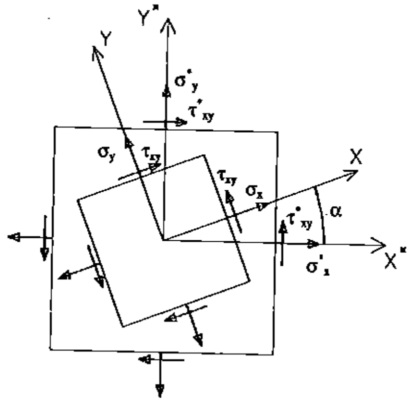


Fig.2 Elementary area subjected to fatigue loading (after Ref. [3]).

After Ref. [3] nominal stresses σ^*_x, σ^*_y, τ^*_{xy}, dependent on phase angle of the loading change ω, express the dependency (1a)÷(1c).

$$\sigma^*_x(\omega) = \sigma^*_{xm} + \sigma^*_{xa}\sin(\omega) \tag{1a}$$

$$\sigma^*_y(\omega) = \sigma^*_{ym} + \sigma^*_{ya}\sin(\omega - \phi_y) \tag{1b}$$

$$\tau^*_{xy}(\omega) = \tau^*_{xym} + \tau^*_{xya}\sin(\omega - \phi_{xy}) \tag{1c}$$

In the elementary area which together with its own system of XOY co-ordinates is turned an α angle towards X*OY* system there will occur σ_x, σ_y, τ_{xy}, stresses dependent both on the loading change phase angle ω and on the α angle. The mean values σ_x, σ_y, τ_{xy}, show dependencies (2a)÷(2c).

$$\sigma_{xm}(\alpha) = (1+\cos 2\alpha)\frac{\sigma^*_{xm}}{2} + (1-\cos 2\alpha)\frac{\sigma^*_{ym}}{2} - \sin 2\alpha \cdot \tau^*_{sxm} \tag{2a}$$

$$\sigma_{ym}(\alpha) = (1-\cos 2\alpha)\frac{\sigma^*_{xm}}{2} + (1+\cos 2\alpha)\frac{\sigma^*_{ym}}{2} + \sin 2\alpha \cdot \tau^*_{sxm} \tag{2b}$$

$$\tau_{xym}(\alpha) = \sin 2\alpha \cdot \frac{\sigma^*_{xm} - \sigma^*_{ym}}{2} + \cos 2\alpha \cdot \tau^*_{xym} \tag{2c}$$

The stress amplitude values σ_{xa}, σ_{ya}, τ_{xya} in the elementary areas turned at the α angle show dependencies (3a)÷(3c).

$$\sigma_{xa}(\omega,\alpha) = (1+\cos 2\alpha)\frac{\sigma^*_{xa}\sin\omega}{2} + (1-\cos 2\alpha)\frac{\sigma^*_{ya}\sin(\omega-\phi_y)}{2} - \sin 2\alpha \cdot \tau^*_{sxa} \cdot \sin(\omega - \phi_{xy}) \tag{3a}$$

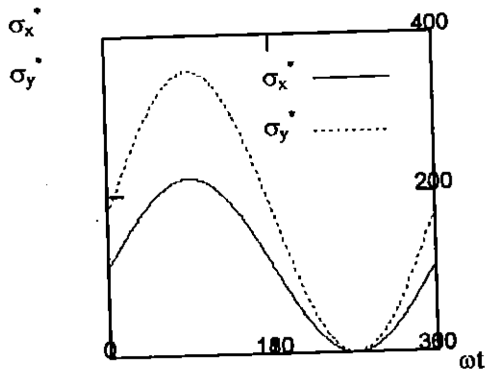
$$\sigma_{ya}(\omega,\alpha) = (1-\cos 2\alpha)\frac{\sigma^{*}_{xa}\sin\omega}{2} + (1+\cos 2\alpha)\frac{\sigma^{*}_{ya}\sin(\omega-\phi_{y})}{2} \tag{3b}$$

$$+\sin 2\alpha \cdot \tau^{*}_{xxa} \cdot \sin(\omega - \phi_{xy})$$

$$\tau_{xya}(\omega,\alpha) = \sin 2\alpha \cdot \frac{\sigma^{*}_{xa}\sin\omega - \sigma^{*}_{ya}\sin(\omega-\phi_{xy})}{2} \tag{3c}$$

$$+\cos 2\alpha \cdot \tau^{*}_{xya} \cdot \sin(\omega - \phi_{xy})$$

Dependencies (3 a)÷(3c) transformed in relation to phase difference angles may be found in Ref. [3]. Exemplary stress courses on particular, described by α angle, planes are shown in Fig. 4. Parameters of these courses are described in Fig. 3.



σ_{xm}^{*} = 110 MPa

σ_{xa}^{*} = 110 MPa

σ_{ym}^{*} = 177.5 MPa

σ_{ya}^{*} = 177.5 MPa

Fig.3 Parameters of fatigue cycle taken for analysis.

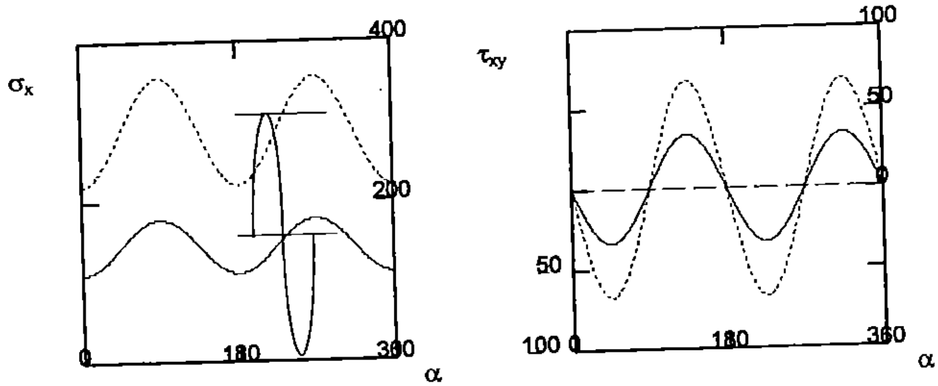


Fig.4 Stress courses on particular α planes.

Mean stress of fatigue cycle occurring in the presented case do not cause any damages of the elementary area because their values are smaller than the yield point. However, they cause that the connected with them, time steady, action directions are characteristic directions. In these directions the material is exposed to stronger static loading than in other directions.

Influence of mean value on fatigue life.

The analysis of dependencies (1)÷(3) show that the course of loading on particular planes α are influenced by: mean value of cycle, phase difference between loading components or by frequency. In this paper the mean value influence will be discussed more thoroughly. Numerous descriptions of the mean value of cycle have been already made and are known. Using, for example, linear dependency between σ_a i σ_m, on the basis of Haigh's diagram, as it was in the Ref. [4], for normal stresses dependent on α angle we will receive equation (4).

$$\sigma_{a(All)}(\alpha) = \sigma_A - \psi \cdot \sigma_m(\alpha) \tag{4}$$

The value ψ is material constant. This constant for normal stresses occurring in steel is in the range 0.1÷0.3. In Fig. 5 permissible amplitude distribution determined on the basis of parameters show in Fig. 3, and σ_A=210 MPa & ψ=0.2 are presented.

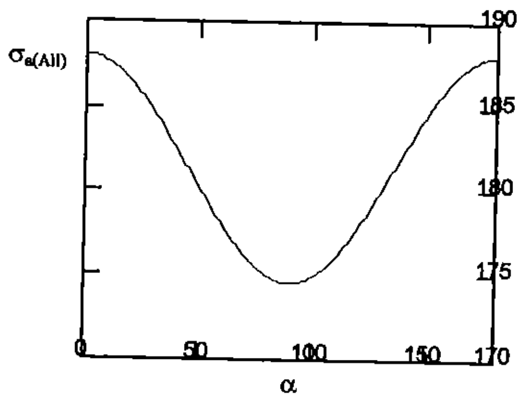


Fig.5 Permissible amplitude distribution on particular planes α.

Analysis of course changes of permissible amplitude distribution on particular planes described by α angle indicates that in spite of isotropic properties of the material exposed to loading its fatigue properties change with changing of the observed direction revealing in this way anisotropy.

Plastic properties identification

Rectangular plate made of isotropic material was subjected to uniaxial loading which brought about plastic changes in it (Fig. 6). Let us now discuss initiation yield stress distribution σ_{pl} in elementary area (Fig. 7) cut out in the crossing point of the plate's symmetry axis.

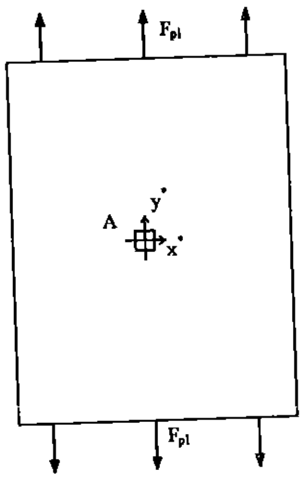


Fig.6 Scheme of the point of loading application causing plastic changes.

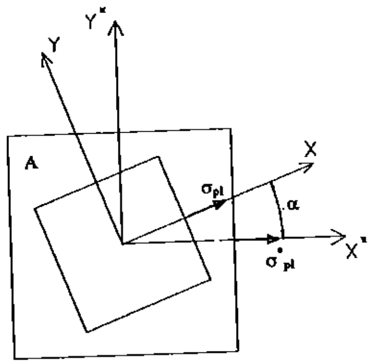


Fig.7 Initiation yield stress in the elementary area.

Stresses σ_{pl}^* correspond with the R_e yield point for $\alpha = 0°$ angle in the reference system X^*0Y^* for the considered construction element. Stresses σ_{pl} correspond with yield point determined in the direction α which is being considered. The material which is not loaded will have the same value R_e in all directions α because of the isotropy of its properties. After loading the material above the yield point, as has been shown in Fig. 6 permanent plastic strains will occur, and anisotropic properties together with them. It may be assumed that the R_e yield point changes with the α angle changes (Fig. 7) are described by dependency (5).

$$R_e^2(\alpha) = (R_{e0} \cdot \cos\alpha)^2 + (R_{e90} \cdot \sin\alpha)^2 \tag{5}$$

Values R_{e0} and R_{e90} are yield points determined respectively in 0° and 90° direction. The above assumption has been verified positively with the data contained in Fig. 11. Because of double symmetry of the geometrical shape of the plate under discussion it may be assumed that the R_e courses of our interest will be changing within the range $0° < \alpha < 90°$, whereas the other parts of the runs may be gained through mirror reflection towards appropriate symmetry axis. As it is easy to prove the dependency (5) meets the above requirements. Exemplary course R_e for the data included in Fig. 11 are shown in Fig. 8. The dash line corresponds with the material with no plastic changes (isotopic), while the continuous line describes the material with plastic loading (anisotropic). Undoubtedly the length and width of the stretched plate affects the consistence R_e described in the direction $0° < \alpha < 90°$.

Model course

Juxtaposition of stress run σ_{xm}, described with the dependency (2a) and parameters included in Fig. 3, with yield points R_e described by dependency (5) and parameters included in Fig. 11, shows significant similarities (Fig. 9).

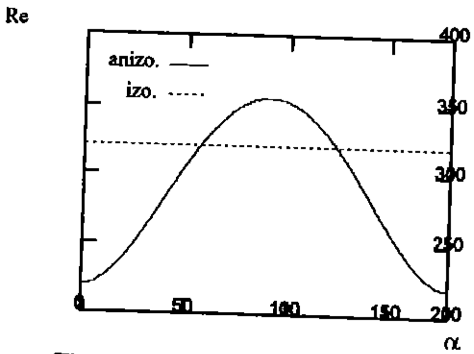


Fig. 8 Courses R_e for isotropic and anisotropic materials.

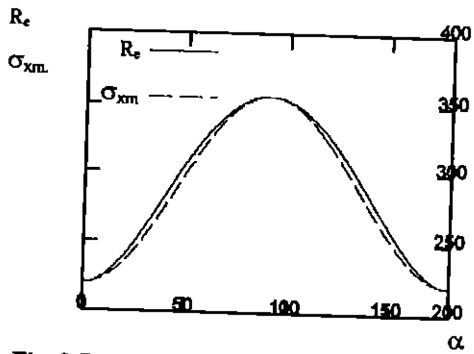


Fig. 9 Juxtaposition of R_e and σ_{xm} courses.

Analysing courses σ_x which correspond with biaxial state of loading (Fig. 2), we notice that stress changes in one fatigue cycle occur in relation to mean value σ_m with amplitude σ_a, whereas both values depend on α angle. If we assume that plastic changes have similar fatigue effect as the material strain under static loading, then we can say that $\sigma_{pl\alpha}$ run corresponds with the change run σ_m (Fig. 9) in relation to the fatigue behaviour of the material. Basing on the above assumption we can say that if the elementary area with anisotropic properties is subjected to uniaxial full-reversal loading with σ_{xa} amplitude, determined for the discussed angle α from the dependency (3a), it may be expected that the attained fatigue life will be close to fatigue life determined in the conditions of biaxial loading applied to elementary area from isotropic material.

Experimental verification

In the experimental part, methodology for determining plastic properties changes of metals [5] suggested by Szczepiñski was used.

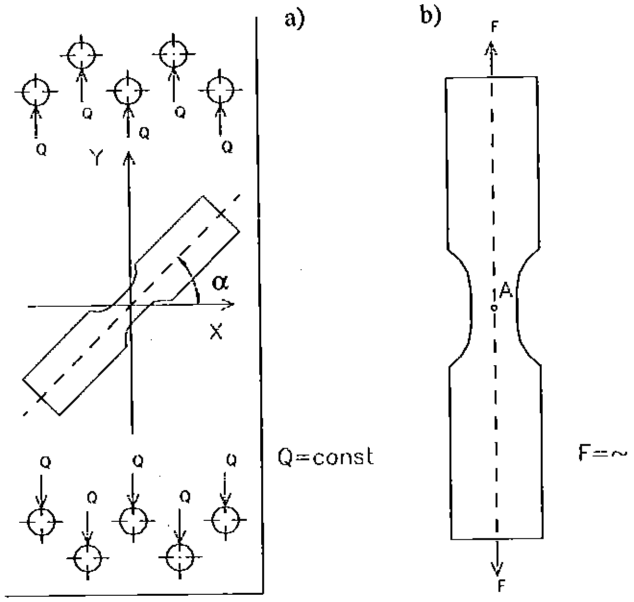


Fig.10 Fatigue test methodology.

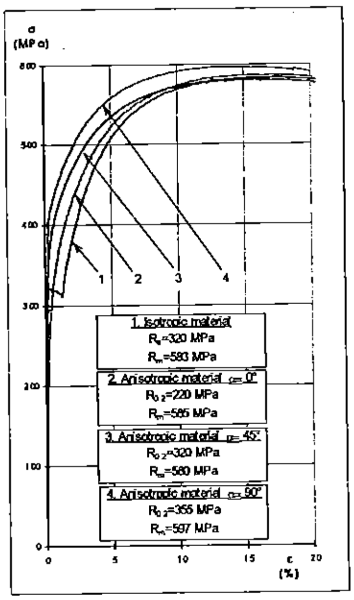


Fig.11 Plastic properties identification.

Steel 45 in normalisation state was used for tests. Material in the shape of steel sheet was subjected to stretching in a special mount constructed for this purpose (Fig. 10a). Next, specimens of different α angles towards the loading directions are being cut out from initially deformed material (Fig. 10b). These specimens undergo fatigue tests in a uniaxial test machine INSTRON 8501.

Tests have been carried out on the samples cut out of angles a equal 0°, 45°, 90° (Fig. 11). Determined yield points for the material in delivery state were R_e^*=320 MPa. These points showed no dependency on the α angle indicating isotropy of the material properties. The material's yield points after plastic loading in dependence on a angle were juxtaposed in Fig. 11. We can notice here clear anisotropy of plastic properties.

Fatigue tests were made on flat samples (Fig. 10b) to witch full-reversal loading was applied with amplitudes 200 MPa, 225 MPa, 250 MPa. The results in forms of Wöhler's diagrams are shown in Fig. 12 - for isotropic and Fig. 13 - for anisotropic materials .

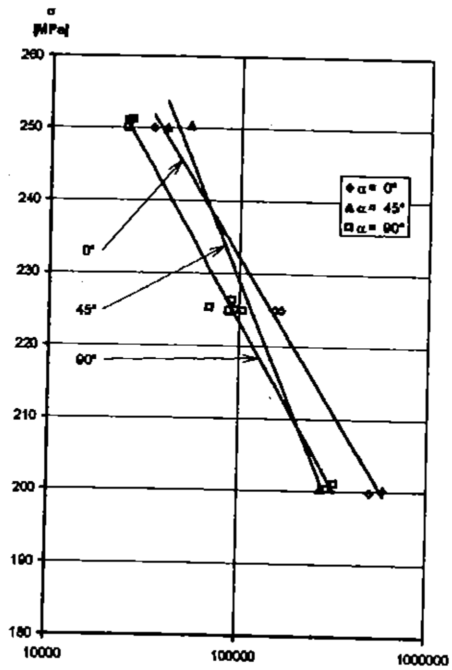


Fig. 12 Wöhler's diagrams for isotropic material.

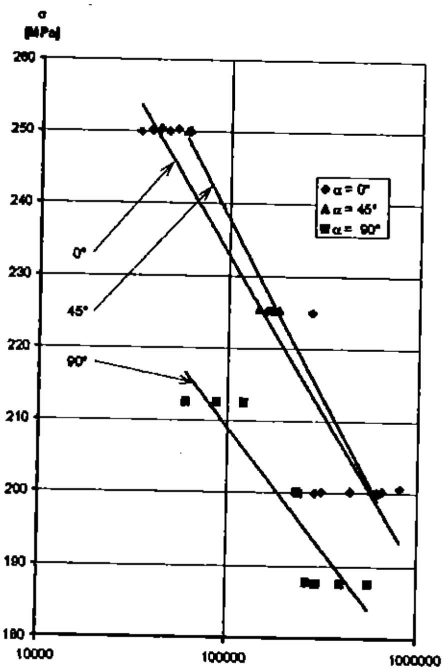


Fig. 13 Wöhler's diagrams for anisotropic material.

Results shown in Fig. 12 do not reveal significant differences in the gained fatigue life. In Fig. 13 appears a clear difference between the position of fatigue characteristics for the angle 90°, and the characteristics concerning the angles 0° and 45°.

Conclusions

The method described in the paper aims at determining fatigue properties in loading conditions including specific features of the complex state of fatigue loading. Possibility of modelling of, known from Troost's papers, „stress anisotropy" effect by means of plastic strain controlled in respect of its turn and direction has been pointed out. Using of the suggested modelling method leads to formulating fatigue test methodology possible to be carried out in a uniaxial test machine.

References

1. El. -´ Magd E., Mielke S. Dauerfestigkeit bei überlagerter zweiachsiger statischer Beanspruchung, Konstruktion 29, 1977.
2. Troost A.,El. - Magd E., Schwingfestigkeit bei mehrachsiger Beanspruchung ohne und mit Phasenverschiebung, Konstruktion 33, 1981.
3. Troost A.,El. - Magd E., Anisotropes Ermüdungsverhalten isotroper metallischer Werkstoffe, Metall - Internationale Zeitschrift für Technik und Wirtschaft, Heft 1, 1974.
4. Sempruch J., Kryterium wytrzymałości zmęczeniowej stali konstrukcyjnej 25CrMo4 w warunkach złożonego stanu naprężenia, Praca habilitacyjna, Zeszyt ATR, Nr 51/1992.
5. Szczepiński W., On the effect of plastic deformation on yield condition , Arch. Mech. 15, 1963.
6. Miastkowski J., Własności mechaniczne materiału odkształconego plastycznie przy złożonych wielokrotnych obciążeniach, Prace IPPT, 53/1976
7. Miastkowski J., Szczepiński W., Doświadczalna analiza powierzchni wpływu plastycznego odkształcenia na wytrzymałość zmęczeniową metali, Prace IPPT, 6/1980.
8. Dietrych L., Doświadczalne określenie anizotropii metali, Prace IPPT 33/1994
9. Dietrych L., Kirk R., Socha G., Śliwowski M., Identyfikacja plastycznej anizotropii aluminium, Prace IPPT 26/1994.
10. Malinin N., Rżysko J., Mechanika materiałów, PWN, Warszawa 19981.