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MODELING THE THERMOMECHANICAL COUPLING EFFECTS ON LOW-CYCLE FATIGUE LIFE OF METALLIC MATERIALS

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ABSTRACT: *The present paper presents a mechanical model to study the thermomechanical coupling effects on low cycle fatigue life of metallic materials. The ASTM standard for low cycle fatigue testing establishes that the gradient of temperature during a test must not exceed* ± 2 *K. For high inelastic amplitudes and/or high frequencies it is recommended the use of cooling devices in order to maintain the specimen temperature in the established range. Experimental curves obtained in such controlled conditions are often used to predict the lifetime of real structures, assuming the hypothesis of isothermal processes. In real problems without cooling devices, such assumption may lead to inadequate predictions if small safety factors are adopted. Simple numerical simulations of 316L stainless steel bars are presented and analysed showing that the hypothesis of isothermal processes may be inadequate when cyclic inelastic deformations are involved. The results show that part of plastic work is transformed into heat, resulting in a temperature rise that affects substantially the mechanical behaviour of the material.*

Introduction

Inelastic cyclic deformation promote heating of metallic structural elements. For high loading rates and/or high amplitudes of inelastic deformation a considerable amount of heat can be generated (1-3). The temperature rise in a mechanical component depends on loading amplitude, frequency and temperature boundary conditions. However, in traditional low-cycle fatigue models, the variation of the material temperature due to thermomechanical coupling is not considered and unreal life predictions may be obtained. Indeed, there are situations where such coupling cannot be neglected and a physically realistic model must take it into account.

Since temperature variation can interfere with the fatigue phenomena and most classical low-cycle fatigue models only take into account isothermal processes, the ASTM standard for low-cycle fatigue testing (4) establishes that the gradient of temperature during the testing program must not exceed a range of ± 2 K. For high inelastic amplitudes the standard recommends the use of cooling devices and low loading frequencies to maintain the specimen temperature on the established range. However, this is a difficult condition to achieve in real mechanical components in operation.

In this paper, a continuum damage mechanics model is proposed to study the thermomechanical coupling effects on the life prediction of metallic structures submitted to cyclic inelastic loadings (2,5). A thermodynamic approach permits a rational identification of the thermomechanical coupling in the mechanical and thermal equations.

Numerical simulations of austenitic stainless steel (AISI 316L) bars submitted to cyclic loadings are presented and analysed.

Elasto-viscoplastic Model

The model used in this paper was proposed by Pacheco (2) and is developed within the framework of the thermodynamics of the irreversible processes. Such model is a generalisation of the elasto-viscoplastic model proposed by (5) for isothermal processes. For the elasto-viscoplastic material, the thermodynamic state is completely determined by the so-called state observable variables: total deformation ($\underline{\varepsilon}$) and absolute temperature (θ) and by a set of internal variables: plastic deformation ($\underline{\varepsilon}^p$), isotropic hardening (p), kinematic hardening ($\underline{c}$) and damage (D). The macroscopic quantity D ($0 \leq D \leq 1$)

represents the material local degradation. When D = 0 the material is in a virgin state and when D = 1 the material is completely damaged.

The elasto-viscoplastic behaviour is characterised by two thermodynamic potentials: the Helmholtz free energy (Ψ) and the potential of dissipation (ϕ). The Helmholtz free energy can be written in terms of deformation energy density (W):

$$\rho\psi(\underline{\underline{\varepsilon}},\underline{\underline{\varepsilon}}^p,p,\underline{\underline{c}},D,\theta) = (1-D)\left[W_e(\underline{\underline{\varepsilon}}-\underline{\underline{\varepsilon}}^p;\theta) + W_p(p;\theta) + W_c(\underline{\underline{c}};\theta)\right] - W_\theta(\theta) \qquad (1)$$

where ρ is the density of the material. For isotropic thermoelasticity the elastic energy density is expressed as:

$$\begin{aligned} W_e(\underline{\underline{\varepsilon}}-\underline{\underline{\varepsilon}}^p,\theta) = &\frac{E}{2(1+\nu)}\left[(\underline{\underline{\varepsilon}}-\underline{\underline{\varepsilon}}^p).(\underline{\underline{\varepsilon}}-\underline{\underline{\varepsilon}}^p) + \frac{\nu}{1-2\nu}\left(tr(\underline{\underline{\varepsilon}}-\underline{\underline{\varepsilon}}^p)\right)^2\right] - \\ &- \frac{\alpha E}{1-2\nu}(\theta-\theta_0)\, tr(\underline{\underline{\varepsilon}}-\underline{\underline{\varepsilon}}^p) \end{aligned} \qquad (2)$$

The energy density associated to the hardening and to the temperature are:

$$\begin{cases} W_p(p;\theta) = b\left(p + (1/d)e^{-dp}\right) \\ W_c(\underline{\underline{c}};\theta) = (1/2)\, a\, (\underline{\underline{c}}.\underline{\underline{c}}) \\ W_\theta(\theta) = \rho \int_{\theta_0}^{\theta} C_1 \log(\xi)\, d\xi + (\rho/2)\, C_2\, \theta^2 \end{cases} \qquad (3)$$

where E, ν, α, b, d and a are temperature-sensitive material parameters, C_1 and C_2 are positive constants and θ_0 a reference temperature. To simplify the notation, a variable β is used to represent the set of internal variables ($p,\underline{\underline{c}},D$).

The thermodynamic forces ($\underline{\underline{\sigma}},B^\beta,s$), associated to the state variables ($\underline{\underline{\varepsilon}},\beta,\theta$), are defined from ψ, as follows:

$$\underline{\underline{\sigma}} = \rho\frac{\partial\psi}{\partial\underline{\underline{\varepsilon}}} \; ; \; B^\beta = -\rho\frac{\partial\psi}{\partial\beta} \; ; \; s = -\rho\frac{\partial\psi}{\partial\theta} \qquad (4)$$

The elasto-viscoplastic materials are characterised by a elastic domain in the stress space where yielding doesn't occur. There exists a yielding function: $F(\underline{\underline{\sigma}}, B^{\beta}; \underline{\underline{\varepsilon}}^{p}, \beta, \theta)$ where $\dot{\underline{\underline{\varepsilon}}}^{p} = \underline{\underline{0}}$ and $\dot{\beta} = 0$, if $F(\underline{\underline{\sigma}}, B^{\beta}; \underline{\underline{\varepsilon}}^{p}, \beta, \theta) < 0$. The yielding function has the form:

$$\begin{cases} F(\underline{\underline{\sigma}}, B^{p}, \underline{\underline{B}}^{c}; \underline{\underline{c}}, \theta) = f(\underline{\underline{\sigma}}, B^{p}, \underline{\underline{B}}^{c}; \theta) + (\frac{\varphi}{2})\left[\left(\underline{\underline{B}}^{c} . \underline{\underline{B}}^{c}\right) - a^{2}(\underline{\underline{c}} . \underline{\underline{c}})\right] \\ f(\underline{\underline{\sigma}}, B^{p}, \underline{\underline{B}}^{c}; \theta) = J(\underline{\underline{\sigma}} + \underline{\underline{B}}^{c}) + B^{p} - \sigma_{P} \\ J(\underline{\underline{\sigma}} + \underline{\underline{B}}^{c}) = \left[(3/2)(\underline{\underline{\sigma}} + \underline{\underline{B}}^{c})_{dev} . (\underline{\underline{\sigma}} + \underline{\underline{B}}^{c})_{dev}\right]^{1/2} \end{cases} \quad (5)$$

where σ_p is the limit of proportionality, φ a material parameter and $(\ )_{dev}$ represents the deviatoric part of a tensor.

The potential of dissipation can be separated in two parts: $\phi^{*}(\underline{\underline{\sigma}}, B^{\beta}, \underline{g}) = \phi_{1}^{*}(\underline{\underline{\sigma}}, B^{\beta}) + \phi_{2}^{*}(\underline{g})$, with

$$\begin{cases} \phi_{1}^{*} = \dfrac{k}{n+1} \left\langle \dfrac{F(\underline{\underline{\sigma}}, B^{\beta}; \underline{\underline{\varepsilon}}^{p}, \beta, \theta)}{k} \right\rangle^{n+1} \\ \phi_{2}^{*} = (\theta / 2)\Lambda\left[\underline{\underline{I}}\, \underline{g}\right] \underline{g} \end{cases} \quad (6)$$

where $\underline{q}$ are the heat flux, $\underline{g} = \nabla\theta/\theta$ and k, n and Λ are material parameters that depends on the temperature.

A set of constitutive equations, called the evolution laws, which characterises the evolution of the dissipative processes are obtained from ϕ^{*}:

$$\dot{\underline{\underline{\varepsilon}}}^{p} = \frac{\partial\phi^{*}}{\partial\underline{\underline{\sigma}}} \; ; \; \dot{\beta} = \frac{\partial\phi^{*}}{\partial B^{\beta}} \; ; \; \underline{q} = -\frac{\partial\phi^{*}}{\partial\underline{g}} \quad (7)$$

Using the set of constitutive equations (4,7), the heat equation can be written as (2):

$$\operatorname{div}\left(\Lambda \underline{\underline{I}} \nabla\theta\right) - \rho c\dot{\theta} = -\underline{\underline{\sigma}}.\dot{\underline{\underline{\varepsilon}}}^{p} - B^{\beta}\dot{\beta} + \theta\left(-\frac{\partial\underline{\underline{\sigma}}}{\partial\theta}.(\dot{\underline{\underline{\varepsilon}}} - \dot{\underline{\underline{\varepsilon}}}^{p}) + \frac{\partial B^{\beta}}{\partial\theta}\dot{\beta}\right) \quad (8)$$

The following local version of the second law of thermodynamics can be obtained:

$$\begin{cases} d_1 = \underline{\underline{\sigma}}.\underline{\underline{\dot{\varepsilon}}}^p + B^\beta\dot{\beta} \geq 0 \\ d_2 = -(\underline{q}.\underline{g}) \geq 0 \end{cases} \tag{9}$$

where d_1 represents the mechanical dissipation and d_2 the thermal dissipation. It can be shown that the set of constitutive equations formed by (4) and (7) will always verify the inequalities (9).

The term d_1 appears in the right hand side of the heat equation (8) and will be called internal coupling. It is always positive and has a role in (8) similar to a heat source in the classical heat equation for rigid bodies. The last term in the right hand side of the heat equation, can be positive or negative and will be called the thermal coupling:

$$acpT = \theta\left(\frac{\partial \underline{\underline{\sigma}}}{\partial \theta}.\underline{\underline{\dot{\varepsilon}}} - \frac{\partial B^\beta}{\partial \theta}\dot{\beta}\right) \tag{10}$$

In metal forming the thermomechanical coupling is usually taken into account by an empirical constant called the heat conversion factor **(1,6)**. It represents the part of plastic power transformed into heat:

$$\chi = \frac{d_1 + acpT}{\underline{\underline{\sigma}}.\underline{\underline{\dot{\varepsilon}}}^p} \tag{11}$$

Elasto-viscoplastic Bars

Uniaxial bars immersed in a medium with constant temperature and subjected to prescribed axial displacement loadings are considered in this work. This simple geometry offer a good enlightenment in the analysis of the thermocoupling effects in fatigue life of metallic components and permits a direct comparison with experimental results obtained from specimens of traditional low-cycle fatigue testing. For the uniaxial case, the set of equations (4,7) can be reduced to **(5)**:

$$
\begin{cases}
\dot{\varepsilon}^p = (3/2)\left\langle \dfrac{|\sigma - X| - R - \sigma_P}{k} \right\rangle^n \dfrac{\sigma - X}{|\sigma - X|} \\
\dot{p} = \left|\dot{\varepsilon}^p\right| \\
\dot{c}_1 = \dot{\varepsilon}^p - (2/3)(\varphi_1 / a_1)(1-D)X_1\dot{p} \;\; ; \;\; \dot{c}_2 = \dot{\varepsilon}^p - (2/3)(\varphi_2 / a_2)(1-D)X_2\dot{p} \\
\dot{D} = \dfrac{B^D}{S_0}\dot{p} \;\; ; \;\; B^D = \rho\left(W_e + W_p + W_c\right) \\
\sigma = (1-D)E\left[(\varepsilon - \varepsilon^p) - \alpha(\theta - \theta_0)\right] \\
R = (1-D)\, b\left[1 - e^{-dp}\right] \\
X = (3/2)(1-D)(a\,c) \;\; ; \;\; X = X_1 + X_2 \;\; ; \;\; c = c_1 + c_2
\end{cases}
\tag{12}
$$

When there is no internal source present and the traditional hypothesis adopted in the study of fins are considered, the heat equation (8) can be written as (x denotes the axial direction):

$$
\frac{\partial}{\partial x}(\Lambda\frac{\partial \theta}{\partial x}) - \frac{hP}{A}(\theta - \theta_\infty) - \rho c\dot{\theta} = -\sigma\dot{\varepsilon}^p + R\dot{p} + X\dot{c} - B^D\dot{D} -
- \theta\left(\frac{\partial \sigma}{\partial \theta}(\dot{\varepsilon} - \dot{\varepsilon}^p) + \frac{\partial R}{\partial \theta}\dot{p} + \frac{2}{3}\frac{\partial X}{\partial \theta}\dot{c} - \frac{\partial B^D}{\partial \theta}\dot{D}\right) \tag{13}
$$

where X and R are auxiliary variables, B^D a variable associated with the damage evolution and θ_0 the initial temperature of the bar which is the same as the medium temperature θ_∞. Two kinematic hardening variables (c_1 and c_2) are used to model the material behaviour (5). In the following analysis a linear dependency on temperature is considered for the material parameters k, n, σp, φ_1, φ_2, a_1, a_2, S_0, E, α, b, d, Λ, ρ and c. The constants h, P e A are, respectively, the convection coefficient, the perimeter and the cross section of the bar. The right side of equation (13) contains the thermomechanical coupling terms.

In the analysis of problems with prescribed displacement (u) loadings, the equilibrium equation and the strain-displacement relation must also be considered:

$$
\frac{\partial \sigma}{\partial x} = 0 \;\; ; \;\; \frac{\partial u}{\partial x} = \varepsilon = \frac{\sigma}{(1-D)E} + \varepsilon^p + \alpha(\theta - \theta_0) \tag{14}
$$

with the following boundary conditions: u(x=0) = 0 ; u(x=L) = u_L (t), where L is the bar length.

An operator split technique is used to treat the set of coupled non-linear differential equations (12-14). This approach results in three uncoupled problems (elastic, thermal conduction and evolution problems) that can be solved with traditional numerical techniques (predictor-corrector integrator schemes, finite differences, etc.). An iterative approach guarantees good convergence. Details about the numeric technique employed can be found in the reference (2).

Numerical Simulations

To study the thermocoupling effects in fatigue life of metallic components, the following analysis consider bars of AISI 316L stainless steel with diameter (d) of 5 mm and length (L) of 50 mm submitted to prescribed cyclic displacement loadings (triangular shape). The bar have an initial temperature of 293 K and is immersed in a medium with a constant temperature of 293 K. Table 1 shows the material parameters for two distinct temperatures (5,7).

Table 1 - Material parameters (AISI 316L).

	293 K (20°C)	873 K (600°C)
E (GPa)	196	150
σ_p (MPa)	82	6
k (MPa)	151	150
n (-)	24	12
b (MPa)	60	80
d (-)	8	10
a_1 (GPa)	108.3	17.5
φ_1 (-)	2800	350
a_2 (GPa)	4.5	1.0
φ_2 (-)	25	15
α (1×10^{-6}/K)	15.4	18.0
c (J/Kg K)	454	584
Λ (W/m K)	13	21

The coefficient S_0 presents a dependency with plastic deformation amplitude, and can be adequately represented by the following equation (in MPa) (2):

$$S_p = (1.524 \times 10^3)\, e^{-64.4\, \Delta\varepsilon^p} \qquad (15)$$

Figure 1 shows a comparison between experimental data from a low-cycle fatigue test (8) and model prediction for the 316L stainless steel at room temperature. In the simulations, a combination of slow loadings rates and high convection coefficients is used (T=100s and h = 1 kW/m^2 K, where T is the loading period) to guarantee the temperature variation restriction of ± 2K of the ASTM standard. This figure shows a good agreement between the experimental and the predictions.

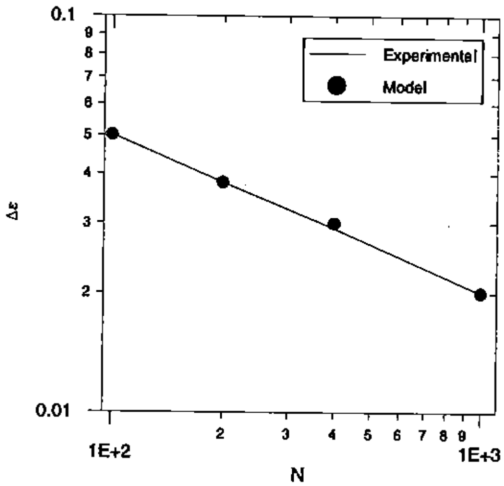

Figure 1 - Curve ε x N for the stainless steel 316L (293 K) obtained with the ASTM recommendations for low-cycle fatigue test.

For ordinary mechanical components in operation this is a difficult condition to achieve and wrong predictions can be obtained if the thermomechanical coupling is not considered. As an example, Figure 2 shows the temperature and damage evolution in the middle position of a bar submitted to prescribed displacement loading amplitude of 0.75 mm with constant temperature boundary conditions. Three conditions are presented for the same loading displacement amplitude:

①- T=100s and h=1 kW/m^2 K

②- T=100s and h=10 W/m^2 K

③- T=10s and h=10 W/m^2 K

These conditions permits to study the influence of loading frequency and heat removal characteristics in the low-cycle fatigue life predictions. Situation ① can be seen as a testing of a specimen in compliance with the ASTM standard for low-cycle fatigue, where the variation of temperature does not exceed the ± 2 K range. In the other situations this requirement was not obeyed and the temperature rise was enough to decrease the fatigue resistance of the material. A comparison between condition ① and conditions ② and ③ reveals a decrease in the life of the bar of about 15% and 26% respectively.

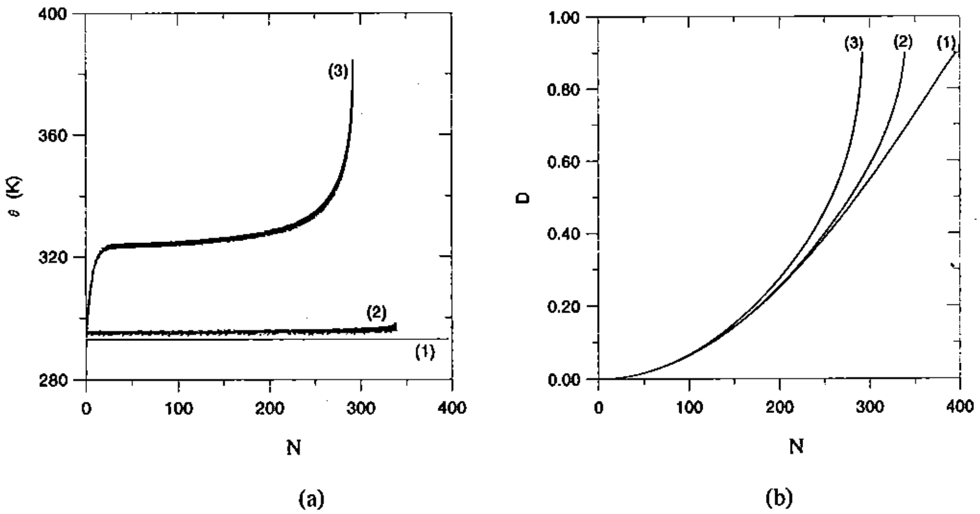

Figure 2 - Temperature (a) and damage (b) evolution in the middle bar position versus number of loading cycles (N), for the three conditions considered.

A comparison between conditions ② and ③ shows a strong influence of the loading frequency in the amount of heat generated and indeed in the life of the bar.

The thermomechanical coupling can be seen as a feedback phenomenon. The heat generated by the mechanical process causes an increase of temperature which promotes a decrease in the mechanical strength. As a consequence, the plastic strain amplitude tends to increase causing a greater temperature rise and so on. Also it is important to observe that the temperature boundary conditions can lead to a localisation process which can accelerate the feedback phenomenon. Figure 3 shows for condition ③ the localisation of the damage processes in the middle position, promoted by the temperature distribution.

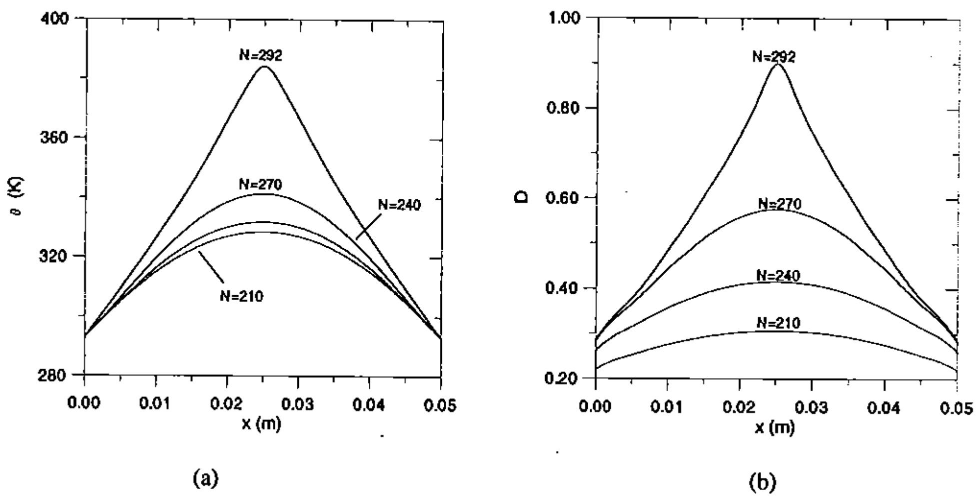

Figure 3 - Temperature (a) and damage (b) distribution for the condition ③.

The mechanic behaviour strongly depends on the temperature distribution. Other mechanical and thermal conditions lead to different predictions. For example, adiabatic temperature boundary conditions leads to a homogeneous temperature distribution and different life predictions are observed for the three defined conditions. In this case a reduction of 3% and 6% is predicted for conditions ② and ③, respectively. In real life structures and mechanical components, thermal boundary conditions between adiabatic and constant temperature can be expected. In tensile tests, the perturbation in the temperature field caused by the end connections geometry is sufficient to strongly localise deformation in the middle of the bar, even if adiabatic conditions are considered

Conclusion

In this paper an internal variable theory was proposed to study the thermomechanical coupling effects in elasto-viscoplastic bars subjected to mechanical loadings. This formulation provides a rational methodology to study complex phenomena like the amount of heat generated during plastic deformation of metals.

The numerical simulations show that is important to consider the thermomechanical coupling effects in low-cycle fatigue projects of mechanical components, particularly when high loading rates are involved. In this situations, if the thermomechanical effect is not included in the model wrong predictions can be obtained and unexpected failures may occur.

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