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Geometric Constraints on Fatigue Crack Paths in Tubular Welded Joints

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ABSTRACT: The complete solution of the problem of a long Stage II fatigue crack growing in a metallic material includes determination of the crack path. Macroscopic aspects of the main features of the crack path are reasonably well understood in a qualitative sense. On a macroscopic scale it is a matter of observation that cracks tend to grow in Mode I.. Geometric constraints may confine a crack to a particular path, and this sometimes leads to mixed mode fatigue crack growth. In a tubular welded joint fatigue crack growth tends to be constrained to a weld toe, and mixed mode fatigue crack growth is sometimes observed. Description of a crack path in three dimensions requires description of the surface followed by the crack (crack growth surface), and also of a family of lines on this surface which defines successive positions of the crack front. In general Mode I crack growth surfaces are curved, and there are then geometric constraints on permissible crack front families. Theoretical prediction of crack paths in three dimensions is not well understood, and in practice fatigue crack paths in structures are usually determined by large scale structural tests. General features of fatigue crack paths in tubular welded joints are discussed from a theoretical viewpoint. It is concluded that further work is needed if the prediction of crack paths in tubular welded joints is to be put on a firm theoretical basis.

Introduction

The complete solution of the problem of a long Stage II fatigue crack growing in a metallic material includes determination of the crack path. Macroscopic aspects of the main features of crack path behaviour are reasonably well understood in a qualitative sense. On a macroscopic scale fatigue crack paths, and hence crack surfaces, in metallic materials may generally be regarded as smooth, and a material as homogeneous and isotropic. It is then a matter of observation(1, 2) that, under essentially elastic conditions where crack tip stress fields may be characterised by stress intensity factors, cracks tend to grow in Mode I.

Geometric constraints may constrain a crack to a particular path, and this sometimes leads to mixed mode fatigue crack growth(2). In a tubular welded joint fatigue crack growth tends to be constrained to a weld toe(3), and if the loading axis changes this can lead to mixed mode fatigue crack growth(2). Theoretical prediction of crack paths in three dimensions is not well understood(4). In practice fatigue crack paths in tubular welded joints, and associated crack growth rate data, are determined by large scale structural tests (3).

Description of a crack path requires description of the surface followed by the crack (crack growth surface), and also of an ordered family of lines on this surface that defines successive positions of the crack front. In general Mode I crack surfaces and associated crack fronts are curved, and the methods of differential geometry(5) are required for a rigorous description. The equations governing fatigue crack path behaviour are highly nonlinear, and are only partially understood(4). This contrasts with most nonlinear dynamics problems, as discussed for example by Hillborn(6), where the governing equations are known.

Some general features of Mode I fatigue crack paths in three dimensions are discussed from a theoretical viewpoint, with emphasis on the behaviour of fatigue cracks in tubular welded joints.

Crack growth surfaces and crack front families

Consider a general, smoothly curved, crack growth surface, that is one that is differentiable at least three times. Assume that on this surface there is an ordered family of smooth curves representing successive positions of the crack front, as shown by the numbered solid lines in Figure 1. Corresponding crack growth directions are given by the family of orthogonal trajectories. These are also smooth curves, and are shown by the dashed lines on the figure. In differential geometry terms these two families are an orthogonal net(5).

The curvature of a smooth surface may be measured by constructing an arbitrary plane through a normal to the surface and the line in this plane tangent to this plane at the normal. The curvature of the surface, in the direction of the line, is the reciprocal of the radius of the osculating circle lying in the plane. In general there are two principal directions on the surface where the curvature is either a maximum or a minimum. For a given smooth surface the trajectories of these principal directions have zero torsion (twist in space), and are a unique orthogonal net(5).

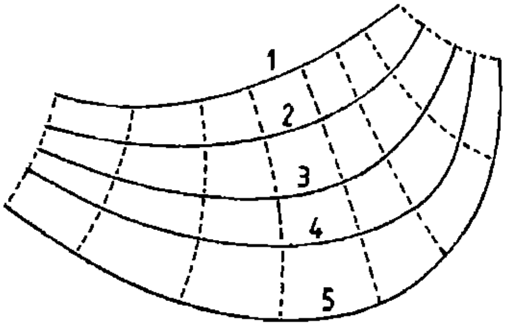

Fig. 1 Crack fronts and trajectories on a smoothly curved crack growth surface.

Now consider a smooth initial crack. If this initial crack is in Mode I, then by symmetry an element of Mode I branch crack growth at any point along the crack front is in the initial direction, and the crack growth surface remains smooth. However, if the initial crack is in mixed Modes I and II there will be a kink at a Mode I branch crack (Figure 2), for mixed Modes I and III a twist (Figure 3), and for all three modes a combination of the two (Figure 4). In no case can the crack growth surface remain smooth. Hence, a necessary condition for crack growth to be in Mode I is that the crack growth surface must be smooth. From the formal definition of torsion(5) it follows that the crack fronts and crack growth trajectories of a Mode I crack must have zero torsion(7). Therefore they must coincide with the orthogonal net of principal curvature trajectories. On a crack growth surface either family of principal curvature trajectories can define a family of crack front positions. Further, since a family of crack front positions is ordered, crack growth can be in either direction along the trajectories. Hence, for a given crack growth surface there are four possible crack front families.

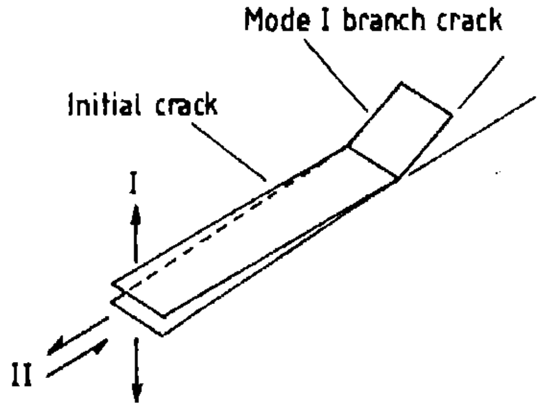

Fig. 2 Quasi two dimensional mixed Modes I and II crack with Mode I branch crack.

There is one important exception to this geometric constraint on permissible crack front families. At an umbilic point the curvature is equal in all directions, so principal directions cannot be defined, and the orthogonal net has a point missing(5). Umbilic points are usually isolated on a smooth surface. However, in the special cases of a plane and a sphere the surface consists entirely of umbilic points, and there is no orthogonal net of principal curvature directions. Therefore, for a flat Mode I crack there are no geometric constraint on permissible crack front families, and a wide range of crack front families is possible and, indeed, observed. See, for example References 8 and 9.

For example, consider a crack growth surface which is part of a right circular cylinder. To visualise an orthogonal net imagine wrapping a piece of graph paper around the cylinder. If the graph paper is aligned so that one family of lines is parallel to the axis of the cylinder, then this family of lines are generators of the cylinder, and the other family are circular cross sections. Both these families are principal curvature trajectories, and hence are permissible crack front families. At any other alignment the families become circular helices. These have non-zero torsion, so are not permissible crack front families.

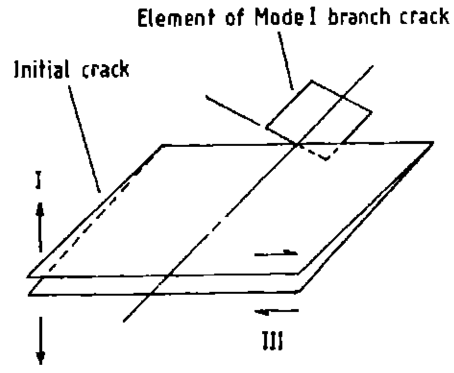

Fig. 3 Mixed Modes I and III crack with element of Mode I branch crack.

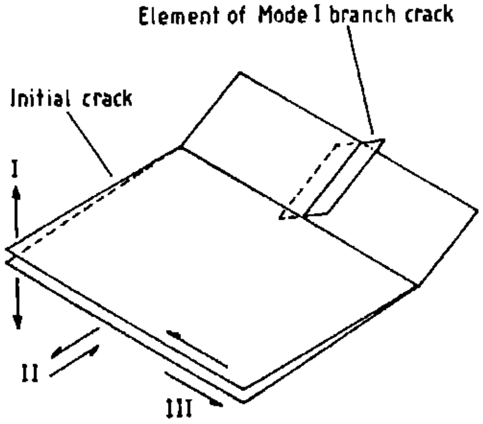

Fig. 4 Mixed Modes I, II and III crack with element of Mode I branch crack.

Fatigue crack path data have been collected during a number of tests on tubular welded joints(3). Some data for a tubular welded Y-joint (Figure 5), test number YOPBC1, are shown in Figures 6 and 7. The joint was made from BS4360:50D steel, and it was tested as welded. During the test the joint was immersed in artificial sea water, with cathodic protection applied. The brace was loaded perpendicularly at its end, in the out-of-plane direction, using a variable amplitude fatigue loading. One side was in tension only, and the other in compression only. The chord was built in at its ends. Crack depths were recorded automatically using alternating current potential drop (acpd) equipment. Full details of the test method are given in Reference 10. Crack growth was in the chord at the toe of the weld.

In a tubular welded joint, such as this test, individual fatigue cracks originate along the toe of a weld, and these link up to form a single crack(3). It is in this sense that a fatigue crack in a tubular welded joint is constrained to the toe of a weld.

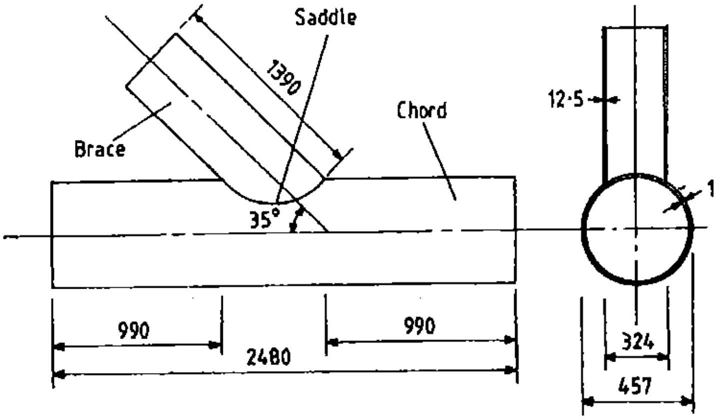

Fig. 5 Tubular welded Y-joint. Dimensions in mm.

Figure 6 shows the crack front family obtained. Crack surface lengths are measured along the toe of the weld. Crack depth measurements, from the nature of the acpd technique, are based on lines on the crack growth surface. Sectioning showed that the crack growth surface is approximately perpendicular to the chord surface (Figure 7). The two sections shown are near the saddle position. The average deviation from the perpendicular, on a number of similar sections, was about 10°. The different angles in the two sections shown in Figure 7 probably reflect the crack linking up process. Sometimes crack growth surfaces in tubular welded joints are tortuous or kinked. Figures 8 and 9 show examples(11).

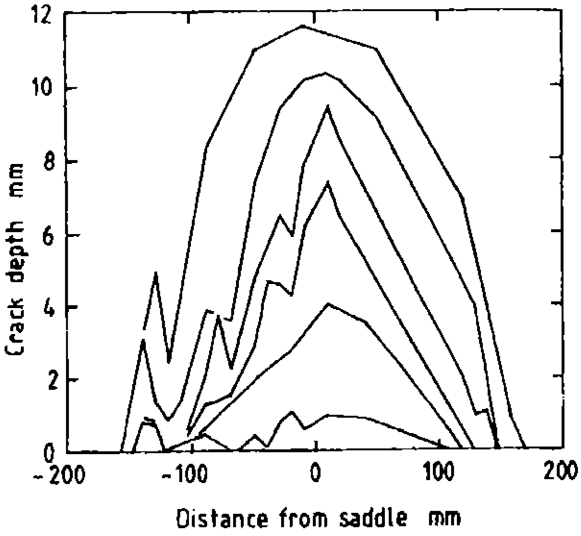

Fig. 6 Crack front family for YOPBC1.

Crack growth surface and crack front intersection angles

It is well known that the elastic stress and displacement fields in the vicinity of a crack front may be characterised by a stress intensity factor, which is a singularity. In a Mode I crack these fields are symmetrical about the crack plane(1, 12). It follows that a crack growth surface must be perpendicular to the surface of a body. That is the crack growth surface intersection angle must be 90°. In this section it is taken as 90°.

The analyses(12) on which the concept of stress intensity factor is based are two dimensional in nature, and the crack front is a point. When analysis is extended to three dimensions the crack front becomes a line. Derivations then include the (usually unstated) assumption that a crack front is continuous. This is not the case at a corner point where a crack front intersects a free surface. The nature of the crack tip singularity changes in the vicinity of a corner point, but understanding of such corner point singularities is incomplete(13). There is only limited information on the size of the region in which the

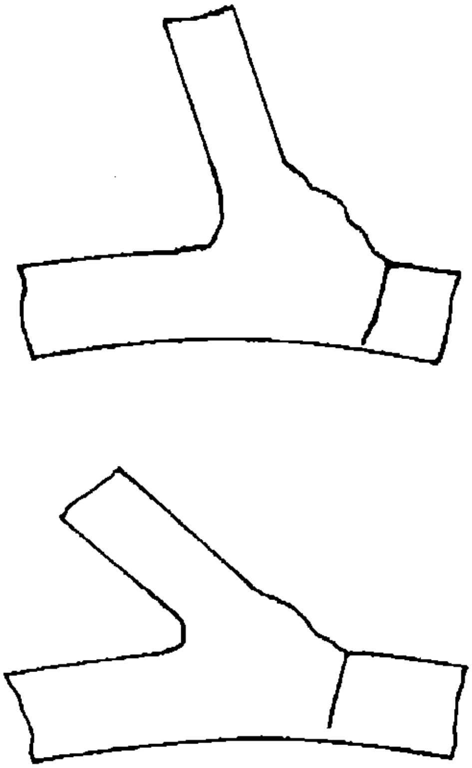

Fig. 7 Sections through crack growth surface for YOPBC1.

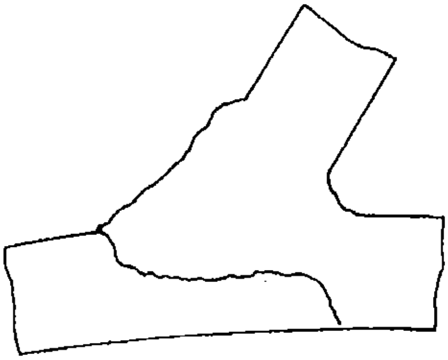

Fig. 8 Example of tortuous crack growth surface.

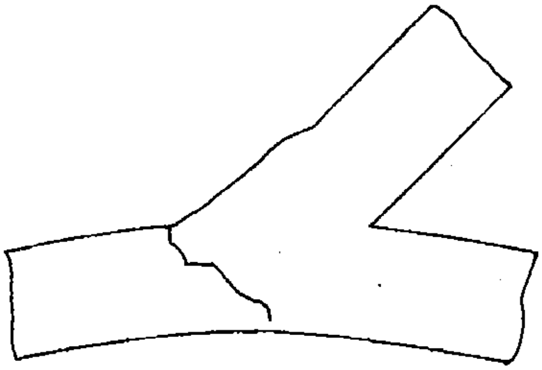

Fig. 9 Example of kinked crack growth surface.

corner point singularity dominates the crack tip stress field. Within the corner region the crack tip singularity is in general of a different order and it is only possible to define stress intensity factors in an asymptotic sense(13). Precise behaviour is a function of Poisson's ratio and the inclination of the crack front to the free surface. If this crack front intersection angle, β (Figure 10), is small then stress intensity factors tend to zero at the corner point. If β is large stress intensity factors tend to tend to infinity. At a critical angle, which is a function of Poisson's ratio, ν, and type of loading, stress intensity factors have a finite value(13).

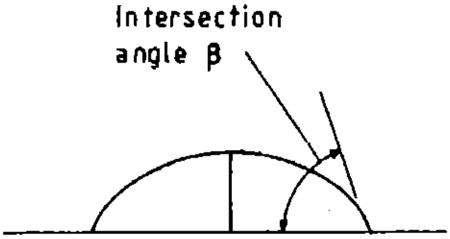

Fig. 10 Symmetrical part-elliptical surface crack.

Mode II or Mode III crack tip surface displacements cannot exist in isolation in the vicinity of a corner point. The presence of one of these modes always induces the other. It is therefore only possible to make a distinction between a symmetric mode (Mode I), and an antisymmetric mode (a combination of Modes II and III). As a free surface is approached the ratio of the Mode II to the Mode III stress intensity factors tends to a finite limiting value, which is a function of Poisson's ratio, ν, and the crack front inclination angle. For $\nu = 0.3$, and crack front perpendicular to the surface ($\beta = 90°$) the limiting ratio is 0.5. At the critical intersection angle, β_c, the limiting ratio appears to be equal to Poisson's ratio(13).

Many of the fatigue cracks found in components and structures are part-through cracks, as in Figure 10, and these are often modelled as a semi-ellipse with the major axis along the surface of the body. They are then referred to as semi-elliptical surface cracks. It is often assumed that during fatigue crack growth successive positions of the crack front are of semi-elliptical form. However, it has been argued(14), from energy and other considerations, that a crack front must intersect the surface at β_c. For the symmetric mode (Mode I), and $\nu = 0.3$ β_c is 100.4°. For the antisymmetric mode it is 67.0°. A physical consequence(14) of the existence of two distinct critical angles is that at the corner point crack growth must be either in the symmetric mode (Mode I), or in the antisymmetric mode. A combination of the two is not possible.

Crack front intersection angles of about the theoretical values have been observed for both the symmetric(14) and, in a tubular welded joint, for the antisymmetric(2) modes. Hence the assumption that a part-through crack is of semi-elliptical form may not be adequately realistic. Modelling a part-through crack as a symmetrical part-elliptical surface crack (Figure 10) has been suggested(15) for the antisymmetric mode. In tubular welded joints surface irregularities due to the weld usually make it difficult to judge whether the crack front intersection angle is close to the theoretical value.

Crack front line tension

A crack front has some analogies with a crystal dislocation(13). In particular the elastic stress field associated with both is a singularity. The associated energy means that a dislocation has a line tension, which controls its shape under an applied stress field. Similarly, a crack front may be regarded as having a line tension, which controls its shape, but with the important difference that the motion of a crack front is irreversible since a crack cannot contract. At a corner point the corner point singularity provides a point source which balances the line tension. The direction of this point force determines the crack front intersection angle (Figure 10). The line tension concept accounts qualitatively for two well known aspects of fatigue crack front behaviour. Firstly, on a macroscopic scale a crack front is smooth, and any initial sharp corners rapidly disappear. This is demomstrated by some finite element calculations(16). Secondly, in a particular set of circumstances a crack front tends to a particular stable shape. See, for example, Reference 8.

Directional stability of a Mode I crack

The directional stability of an initial Mode I crack has been investigated theoretically for the two dimensional case using a linear elastic analysis(17). When only Mode I displacements are present on the initial (main) crack the crack tip stress field is symmetrical about the crack tip(1, 12). Intuitively, crack growth would be expected to continue in the initial direction. However, a fatigue crack growing in Mode I is not necessarily directionally stable, even for a completely symmetrical initial configuration such as a centre cracked panel loaded in uniaxial tension, or a double cantilever beam (Figure 11).

The elastic stress field in a cracked body may be expanded as a series(12). The first term is the stress intensity factor, a singularity, which dominates the crack tip stress field. Other terms are non singular. For a Mode I crack the second term is a stress parallel to the crack, sometimes called the T-stress. No values of the T-stress appear to be available for tubular welded joints. The third and higher terms can usually be neglected. The directional stability of a growing Mode I crack is governed by the T-stress(17). If the T-stress is compressive, and there is a small random crack deviation, perhaps due to microstructural irregularity, then the direction of Mode I crack growth is towards the initial crack line. A Mode I crack in a centre cracked panel loaded in uniaxial tension is directionally stable in this sense. Repeated random deviations mean that the crack follows a zig-zag path about the initial crack line.

When the T-stress is tensile then a crack is directionally unstable, and following a small random deviation, it does not return to its initial line. A fatigue crack in a double cantilever beam specimen is directionally unstable in this sense(18). Typical crack path behaviour is shown in Figure 11. An initial random deviation can be either above or below the centre line, so there are two possible crack paths. These are shown as solid and dashed lines in the figure. The stability of a crack may change as it grows(17), and a stable crack may follow a curved path. Cracks tend to be attracted by boundaries, as in a double cantilever beam specimen, and are increasingly stable as a boundary is approached.

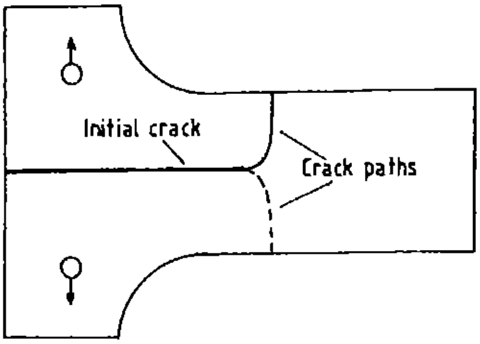

Fig. 11 Double cantilever beam specimen.

Theoretical determination of crack paths

Theoretical determination of the path taken by a Mode I crack is usually based on a two dimensional linear elastic analysis. Related experimental work on fatigue crack paths is carried out on thin sheets, which are regarded as quasi-two dimensional. Experimentally observed crack paths are either straight lines or smooth curves, and are often remarkably reproducible(19). Two dimensional Mode I crack path predictions have been carried out by a number of authors, for example Reference (20), using the same general scheme. Calculations are carried out numerically using small increments of straight crack growth. The direction taken by each increment is selected such that it is Mode I. Care is needed to ensure that the crack path approaches the correct limit as the increment size is reduced. Predicted paths are in general curved. Agreement between theoretical predictions and experimental data obtained using thin sheets is variable(2).

In three dimensions the numerous possible crack configurations(21) make a systematic approach difficult. In particular, when Mode III displacementsare present on an initial crack a Mode I branch crack can only intersect the initial crack at one point, as shown in Figures 3 and 4. Theoretical prediction of crack paths is much less well understood than in two dimensions. It appears to be largely restricted to determination of the initial crack direction and plane of a Mode I branch crack, using methods analogous to those for two dimensions. See, for example, Reference 22.

Discussion

The data shown in Figures 6 and 7 are for a joint which was tested in the as welded condition. Therefore, there are residual stresses in the vicinity of the weld, and these stresses are superimposed on stresses due to the uniaxial loading applied. As a result, the stress tensor rotates during a fatigue cycle, and the loading is effectively a multiaxial out of phase loading. it follows that it is not possible to define a fatigue path which is Mode I throughout the loading cycles. Nevertheless, it is possible to define a loading path which is Mode I in some average sense(23), and it is reasonable to assume that its behaviour will be the same as a crack which is Mode I throughout loading cycles.

The only generalisation that it is possible to make about crack paths in tubular welded joints is that they tend to be constrained to the weld toe. Some crack trajectories are close to straight lines (Figure 7) so that the crack growth surface could be approximated as a ruled surface. The tortuous crack path in Figure 8 is probably the result of crack directional instability. Towards the end of crack growth the crack growth surface is being attracted to a free surface, but it clearly does not intersect this surface at the theoretical angle of 90°. Intersection at about the theoretical angle is sometimes observed, as in Figure 9, and in some double cantilever beam specimens(18). The kinked crack path shown in Figure 9 is probably due to a very large grain size at the weld, with the crack growth surface following crytallographic planes. In this case the crack does finally intersect the surface at about 90°.

If the crack growth surface in a tubular welded joint could be modelled as part of a general cone, then the orthogonal net would consist of the generators of the cone, and of a family of lines each of which is equidistant from the vertex. This family of lines would then predict successive crack front positions. Unfortunately, at present insufficient information on crack paths in tubular welded joints is available for this approach to be utilised.

Conclusions

(1) Crack growth in tubular welded joints is normally in Mode I. There is a wide range of crack paths. These are normally constrained to a weld toe, and the crack growth surface is curved. When the load axis changes during loading mixed mode crack growth may occur.

(2) When a Mode I crack growth surface is curved there are theoretical constraints on permissible crack front familes. The two families of crack fronts and crack trajectories form an orthogonal net. Modelling a crack growth surface as a general cone may permit prediction of the corresponding crack front family, but there are insufficient experimental data available to ascertain whether this is possible.

(3) There are theoretical constraints on the angles at which both a Mode I crack growth surface and a Mode I crack front may intersect a free surface. Comparison with experimental data shows that these theoretical constraints do not necessarily apply.

(4) Even under uniaxial loading residual stresses may mean that the loading is effectively multiaxial and out of phase, but it appears to be possible to generalise the theoretical results for uniaxial loading.

(5) Further work is needed if the prediction of crack paths in tubular welded joints is to be put on a firm theoretical basis.

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