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Fatigue of smooth and notched specimens under multiaxial random loading - experimental results and prediction

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ABSTRACT: Smooth specimens of 30 CrNiMo 8 and notched specimens of 42 CrMo 4 were tested under combined bending and torsion. Tests were performed with phase shift between bending and torsion, different frequency ratios, proportional and non-correlated loading and with constant and alternating mean stresses.

Tests with noncorrelated bending and torsion show, that the lifetime is comparable to test series with proportional loads. While with smooth specimens the lifetime until crack initiation is nearly identical to the fracture-lifetime, a clear distinction has to be made in the case of notched specimens. With the proposed "Integral Multiaxial Damage Hypothesis" a lifetime prediction leads to reasonable results.

Notation

σ	- local tensile stress	τ	- local shear stress
S	- nominal tensile stress	T	- nominal shear stress
R_B	- stress ratio bending	R_T	- stress ratio torsion
N_i	- cycles to crack initiation	N_f	- cycles to final failure

Introduction

The fatigue of structures under variable multiaxial loading was subject to many research programmes in the past. As we know today the process of fatigue is influenced by many factors like the ratio of the stress components and their phase difference or correlation, the frequency of the stress components, the mean stress and the component shape.

Although there are many tests made in the automotive industry under multiaxial loading, specific studies of the influencing parameters are rare. The aim of several research programmes was to examine the above mentioned factors under combined bending and torsion (1, 2, 3). In the paper presented the main results and conclusions of multiaxial random tests under bending and torsion will be delivered.

For calculating the multiaxial fatigue life, a number of hypotheses have been developed in the past. The suggested approaches can be divided in three groups: Critical Plane Approach, Integral Approach and Empirical Approach. For ductile materials the "Integral Multiaxial Damage Hypothesis" (IMDH) was proposed (4) and will be compared with test results in the present paper.

Experimental Investigations

The tests were performed on a testing machine where bending and torsion can be applied seperately. The load sequences were stationary gaussian random processes with $I = 0{,}99$. The smooth cylindrical specimen of 30 CrNiMo 8 with a theoretical notch factor $K_t = 1$ is shown in figure 1a. The notched specimen of 42 CrMo 4 is shown in figure 1b. The notch factor for bending and torsion is $K_{tB} = 2{,}0$ and $K_{tT} = 1{,}6$, respectively.

In the case of notched specimens crack initiation is detected by direct current potential drop measurements. The detectable crack length is 1 - 1,5 mm.

Table 1. Static tensile properties

	R_m [MPa]	$R_{p0,2}$ [MPa]	A_5 [%]	Z [%]
30 CrNiMo 8	1014	912	7,6	68
42 CrMo 4	920	743	21,0	69

Figure 1a. Shape of smooth specimen

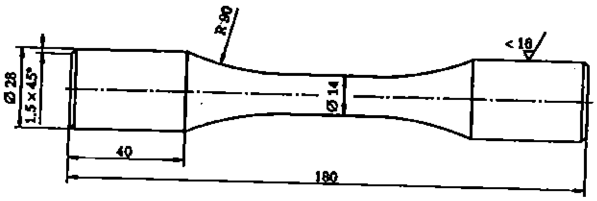
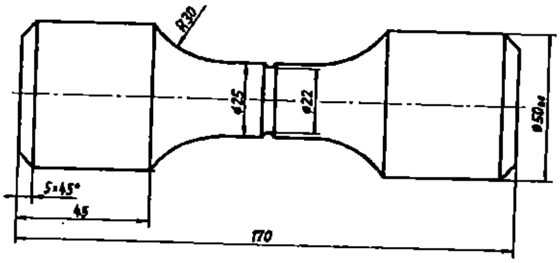


Figure 1b. Shape of notched specimen



Test results with smooth specimens

The main results and conclusions of multiaxial random tests with smooth specimens are summarised in figure 2. The results can be described as follows:

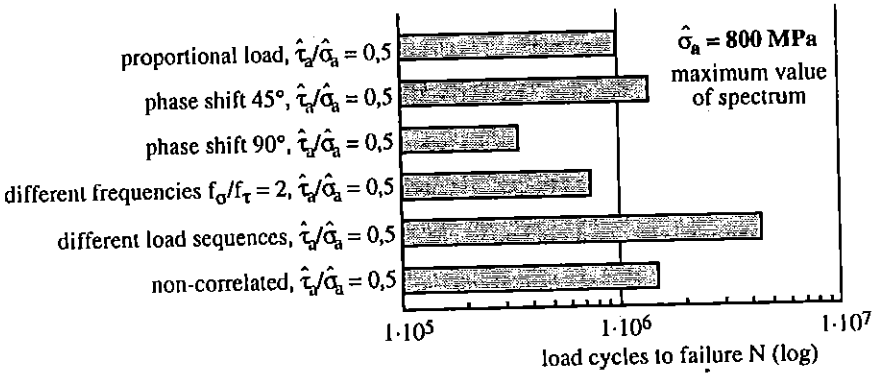
- Besides the basic test series with pure bending and pure torsion, tests with combined proportional bending and torsion have been performed. For different τ/σ - ratios the experimental results can be described on elliptical curves in a τ - σ - diagram.
- The influence of phase shift between bending and torsion has been examined in two test series with a phase shift of 45° and 90°. As a result, the random loading with a phase shift of 90° leads to a significantly decreased lifetime, whereas the significance of a 45° phase shift is negligible.
- In comparison to tests with proportional loads no evident effect was found in tests with a frequency ratio between bending and torsion of $f_\sigma/f_\tau = 2$.
- A significantly increased lifetime was found in tests, where the load sequences for bending and torsion were generated in a different way from the same gaussian type load spectrum.

- Tests with non-correlated bending and torsion, where no constant frequency- or amplitude-ratio is applied, show that the lifetime is comparable to the test series with proportional loads.

Table 2. Test results with smooth specimens under multiaxial random load

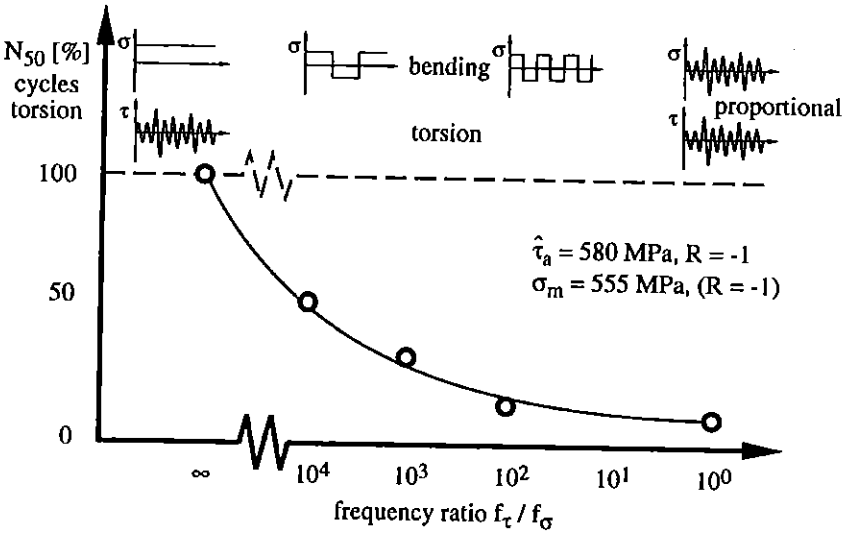
test series	$\hat{\sigma}_{a,N=10^6}$ [MPa]	slope k
proportional load, $\hat{\tau}_a/\hat{\sigma}_a = 0,5$	799	9,5
phase shift 45°, $\hat{\tau}_a/\hat{\sigma}_a = 0,5$	824	10,6
phase shift 90°, $\hat{\tau}_a/\hat{\sigma}_a = 0,5$	747	15,5
different frequencies $f_\sigma/f_\tau = 2$, $\hat{\tau}_a/\hat{\sigma}_a = 0,5$	777	10,5
different load sequences, $\hat{\tau}_a/\hat{\sigma}_a = 0,5$	901	12,5
non-correlated, $\hat{\tau}_a/\hat{\sigma}_a = 0,5$	837	8,7

Figure 2. Overview of test results with smooth specimens



- In addition to the test series with the aim to examine the influence of the load sequences, tests with constant and alternating mean stresses have been performed. A Constant mean stress shows almost no effect on the lifetime. Depending on the frequency ratio, an alternating mean stress leads to a significant lifetime reduction as shown in figure 3.

Figure 3. Lifetime for combined torsion and quasistatic alternating bending



Test results with notched specimens

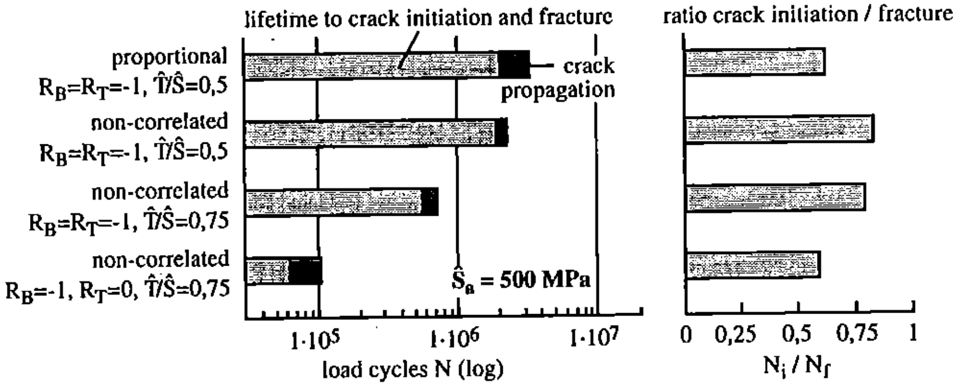
With notched specimens tests have been performed under proportional und non-correlated loads. The ratio between the nominal stresses for torsion and bending (T/S) was 0,5 with proportional loads and 0,5 or 0,75 with non-corelated loads. The stress ratio for bending and torsion was always $R_B = R_T = -1$ except for the last test series with non-corelated loads T/S = 0,75 and $R_T = 0$ (pulsating load). The test results with notched specimens are summarised in figure 4. Besides the lifetime until crack initiation N_i and fracture N_f the ratio N_i/N_f is shown.

- A comparison between test results with proportional and non-correlated loads shows a reduced N_i/N_f ratio in the case of proportional loading whereas the number of load cycles to crack initiation is rather equal.
- The non-correlated tests with T/S = 0,75 lead to a reduced lifetime compared to tests with a nominal stress ratio of 0,5 while there is no change in the crack initiation to fracture ratio.
- The test series with a torsion stress ratio $R_T = 0$ shows a significantly decreased lifetime and N_i/N_f ratio compared with the test results of stress ratio $R_T = -1$.

Table 3. Test results with notched specimens under multiaxial random load

test series	crack initiation $\hat{S}_{a,N=10^6}$ [MPa]	 slope k	fracture $\hat{S}_{a,N=10^6}$ [MPa]	 slope k
proportional, $\hat{T}/\hat{S} = 0{,}5$	544	8,2	595	6,7
non-correlated, $\hat{T}/\hat{S} = 0{,}5$	529	11,1	540	10,6
non-correlated, $\hat{T}/\hat{S} = 0{,}75$	468	8,5	480	8,2
non-correlated, $\hat{T}/\hat{S} = 0{,}75$, R_T=0	415	15,0	430	15,0

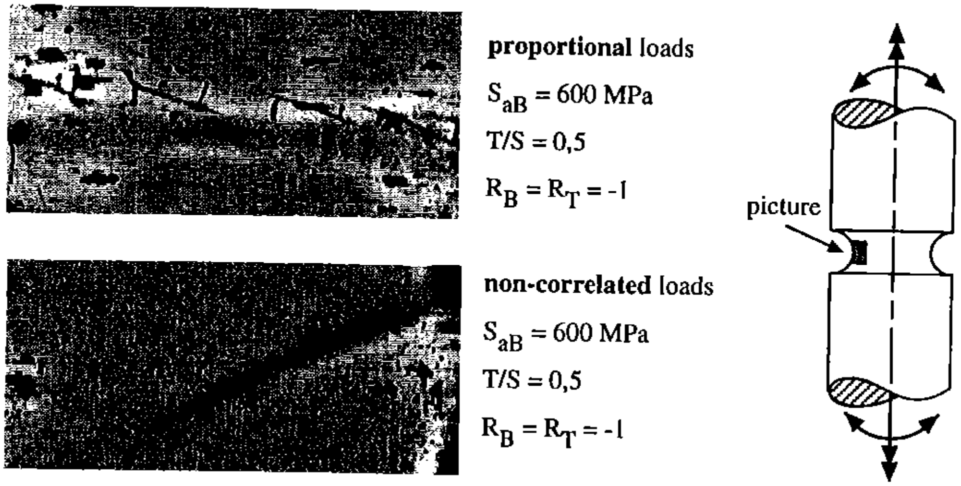
Figure 4. Overview of test results with notched specimens



Crack Initiation and Fracture

The orientation of the initial cracks in tests with proportional and non-correlated loading is shown in figure 5. In the case of proportional loading the crack propagation occurs perpendicular to the direction of the maximum principle. Whereas the non-correlated loading leads to a crack propagation in a right angle to the specimen axis.

Figure 5. Initial crack orientation observed in tests



proportional loads

$S_{aB} = 600$ MPa

$T/S = 0,5$

$R_B = R_T = -1$

non-correlated loads

$S_{aB} = 600$ MPa

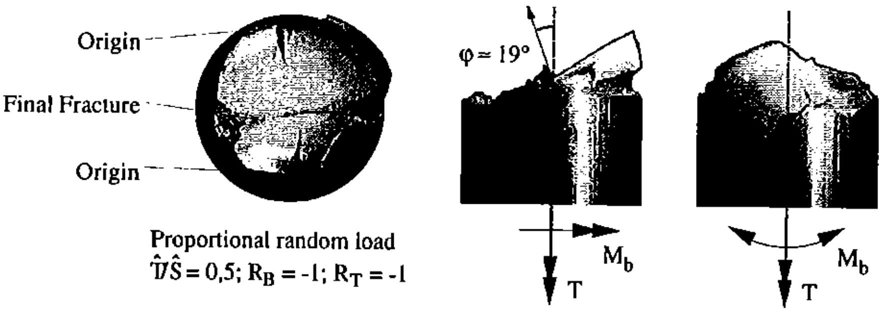
$T/S = 0,5$

$R_B = R_T = -1$

It was pointed out in figure 4, that a clear distinction has to be made between crack initiation and fracture in the case of notched specimens.

Proportional bending and torsion causes a fracture surface in the direction of the first principle. The crack propagates with constant direction out of the notch region until it separates the specimen. The observed striations on the fracture surface indicate a convex crack front, figure 6.

Figure 6. Fracture surface of specimen subjected to proportional loads



Proportional random load

$\hat{T}/\hat{S} = 0,5$; $R_B = -1$; $R_T = -1$

For tests with the same maximum loads but non-correlated bending and torsion the crack propagation shows a different behaviour. The crack propagates perpendicular to the specimen axis. The surfaces show evidence of rubbing against each other but the striations give rise to the assumption, that the crack propagates in a concave way, figure 7.

In the case of vibrating bending $R_B = -1$ and pulsating torsion $R_T = 0$ the crack propagates in different directions on the upper- and under-side (related to the neutral bending plane). Both cracks join near the neutral plane, figure 8.

Figure 7. Fracture surface of specimen subjected to non-correlated loads

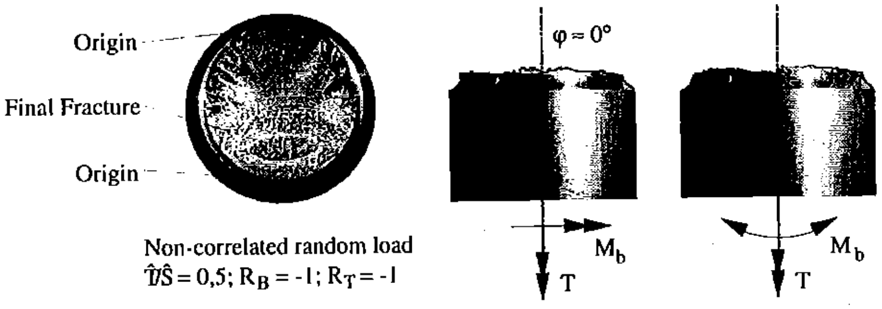
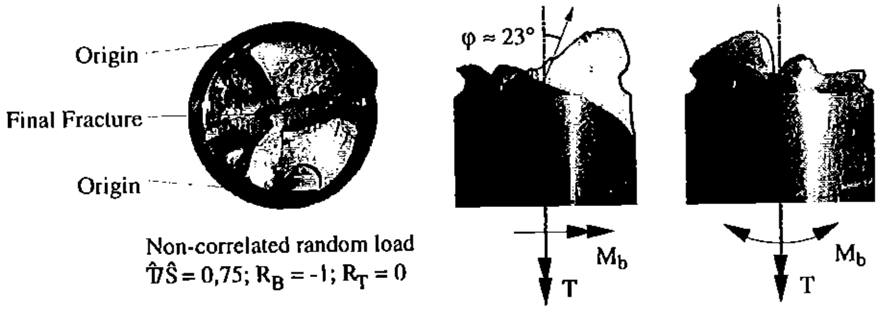


Figure 8. Fracture surface of specimen subjected to non-correlated loads with pulsating torsion



Lifetime Prediction and Verification

For calculating the fatigue life in the case of multiaxial stress, a number of hypotheses have been developed. However the classical strength hypotheses are only applicable, if the loads are known to be in phase. As shown in figure 2 for example, a significantly decreased lifetime might occur if the loads are out of phase.

Most real design situations involve a multiaxial stress state of non-proportional and non-correlated cyclic stress. For this case three groups of different approaches to predict the fatigue life have been proposed. They can be subdivided into hypotheses of the critical intersection plane, hypotheses of integral strain, as well as empirical strength hypotheses.

For the group of ductile materials, like those mentioned here, the Critical Plane Approach, where failure is the result of the damage sum in the "most damaging plane", proved to be not suitable.

The Integral Approach, in comparison, gathers all damaged planes of the specific critical volume. Proceeding on this assumption the "Integral Multiaxial Damage Hypothesis" (IMDH) was proposed (4). With this new proposal all accumulated plane damage sums are summarised to an integral damage sum which is assumed to be determinative for the component lifetime.

Based upon the real-time load sequences a transformation into all plane's φ is executed. After that, the equivalent stress $\sigma_{eq,\varphi a}$ has to be determined by superimposing the plane stresses $\tau_{\varphi a}$ and $\sigma_{\varphi a}$.

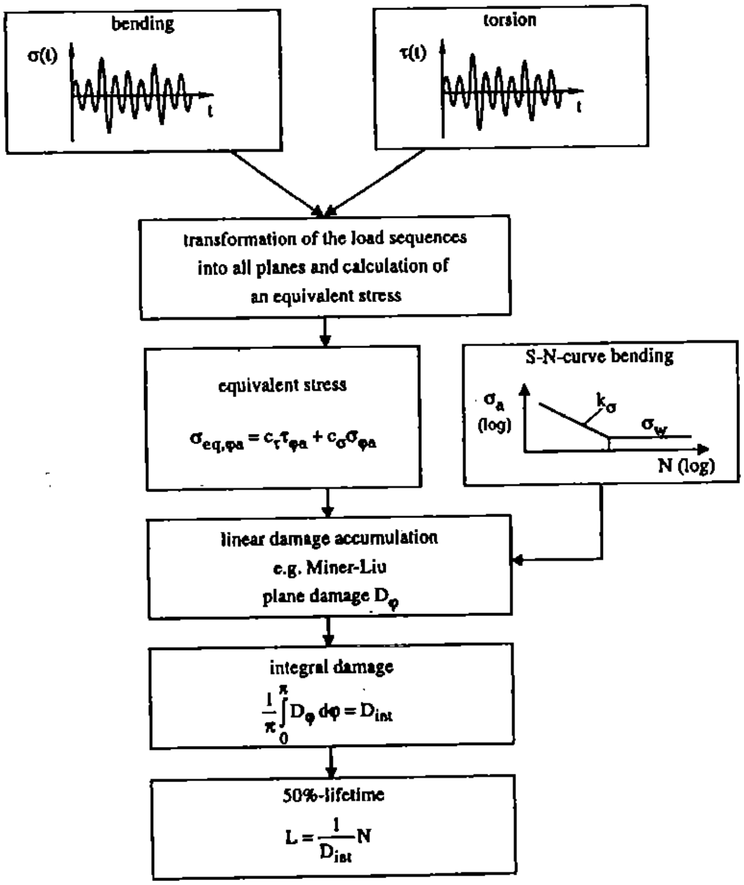
$$\sigma_{eq,\varphi a} = c_{\tau} \cdot \tau_{\varphi a} + c_{\sigma} \cdot \sigma_{\varphi a} \qquad (1)$$

To determine the plane damage sum D_{φ} a linear damage accumulation with the modification by Miner-Liu (5) follows. The integral evaluation of all accumulated plane damages is lifetime decisive.

$$\frac{1}{\pi}\int_{0}^{\pi} D_{\varphi}\, d\varphi = D_{int} \qquad (2)$$

Figure 9 shows the flowchart of the proposed prediction algorithm for combined bending and torsion.

Figure 9. Integral Multiaxial Damage Hypothesis (IMDH)
Prediction algorithm for combined bending and torsion



A comparison between experimental and calculated results shows a good agreement for proportional loading with different τ/σ - ratios, figure 10. The damage sum D_{eff}, which is the ratio of the experimentally determined to the calculated lifetime, takes into consideration the uncertainties of the linear damage calculation.

In the case of non-correlated cyclic loads, the integral evaluation of all accumulated plane damages leads to a decisive improvement in lifetime calculation compared to the Critical Plane Approach. Figure 11 shows a comparison of experimentally determined and calculated results for tests with proportional and non-correlated loads.

Figure 10. Experiment and Calculation, comparison for proportional loading

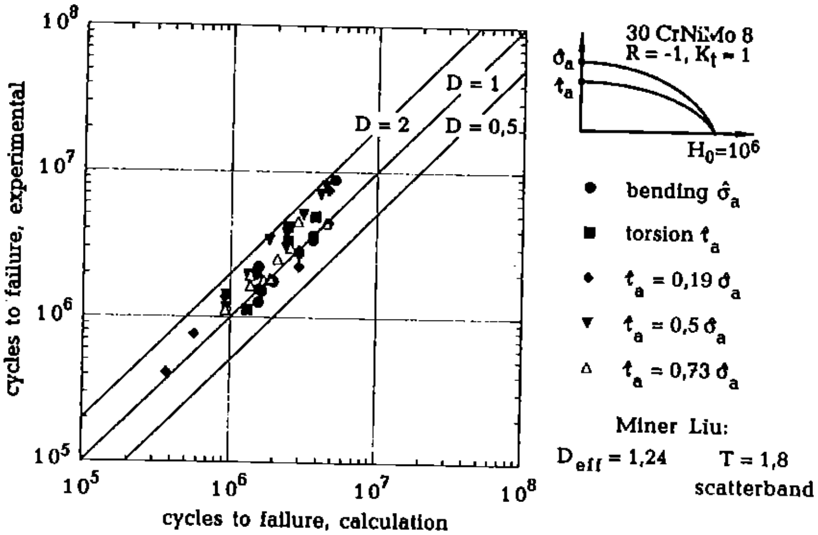
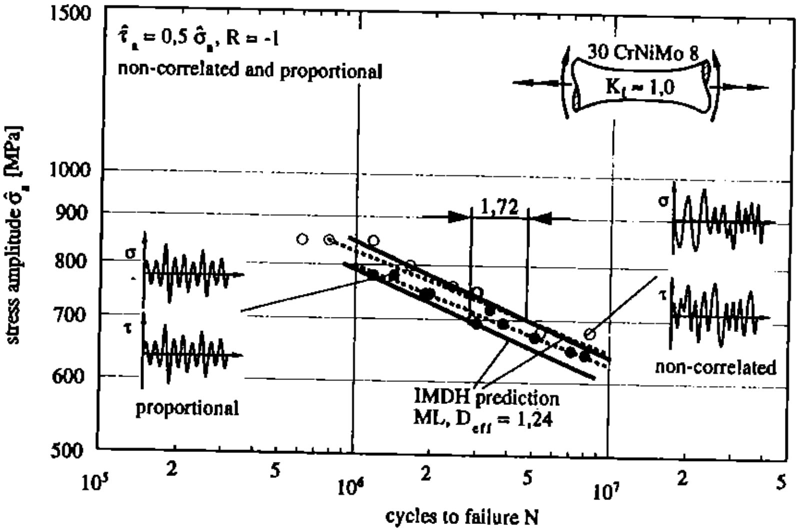


Figure 11. Lifetime prediction for proportional and non-correlated loads



Conclusion

The process of fatigue under multiaxial cyclic stress is influenced by factors like the ratio of the stress components and their phase difference or correlation, the frequency of the stress components, the mean stress and the component shape. Comprehensive investigations with smooth and notched specimens under combined bending and torsion gives the opportunity to study the influencing parameters.

As a result, an integral approach was proposed for lifetime calculation of ductile materials. With this proposal an improved prediction under multiaxial loading is possible.

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