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Equivalent Stress Calculation for Biaxial / Multiaxial Fatigue and Fracture (Experiment and Theory)

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Keywords: stress, multiaxial, fatigue, fracture, strength

ABSTRACT: *In the paper, a brief critical analysis of the fatigue criteria proposed previously is given and the possible use of the static fatigue criteria for describing biaxial/multiaxial fatigue is considered wherein the stress state parameters are the stress intensity and maximum normal stress. Cases of oscillating loading are considered and physical unreliability of some widely used analytical approximation of relationships between mean and amplitude stress values is shown. A new, more validated approximation is proposed, the ways of its use to determine limit stress under conditions of a multiaxial stress state are considered making use of the proposed formulae for equivalent stresses. The validity of the formulae proposed to determine equivalent stresses is verified by the test results for a large group of materials (cast irons, steels, etc.) which are available in the literature and have been obtained by the authors.*

Notation

$\sigma_1, \sigma_2, \sigma_3$	Principal normal stresses
σ_{max}, τ_{max}	Maximum normal and shear stresses
σ_{lim}, τ_{lim}	Limit normal and shear stresses
$\sigma_u = \frac{\sqrt{2}}{3}\tau_{oct}$	Equivalent stress
σ_{-1}, τ_{-1}	Endurance limits for fully-reversed cycles in tension-compression and pure shear
σ_{eq}	Equivalent stresses
σ_a, σ_{m}	Amplitude and mean stress values under loading with an oscilating cycle
t^*, N^*	Test duration in terms of time and number of cycles
σ_s, σ_t	Ultimate strength and long-term ultimate strength
A, χ, χ_a, χ_m	Parameters of the material properties

Introduction

The problem of enhancing the reliability of methods for assessing fatigue strength of structural elements under multiaxial stress state has been gaming greater urgency. The complexity of this problem lies in the fact that in the general case of loading, the principal axes can rotate, whereas the frequency and amplitude of the stress tensor components pulsation can vary according to different laws. As a result, the problem of equivalence criteria as to metal fatigue becomes practically unsolvable. Therefore, most of the proposed criteria have been established as applied to the simplest particular cases of loading.

Loading with a fully-reversed cycle

The analysis of the results of the experimental investigation into fatigue strength under conditions of multiaxial stress state (mainly, in torsion and torsion with bending) reveals that the ratio of endurance limits in a fully-reversed cycle of shear τ_{-1} and tension-compression, is 0.5...0.7 for steels and 0.75...0.9 for cast irons which corresponds to the ratio of limit stresses in shear and in tension under conditions of a single static load application. Similar data were obtained for thermosetting plastics of different brittleness. The observed correlation between the characteristics of static cyclic strength of materials of different classes points to the possibility of extending the use of the criteria verified experimentally under static loading to the case of fatigue.

Thus, to describe fatigue strength of plastic materials, Soderberg (1933) used the theory of maximum shear stress whereas Marin (1937) used the theory of shape changing energy. Using the principle of reduction of cyclic stresses to static ones, Serensen (1938) proposed the condition of constancy of the shape changing energy with the correction factor $\sigma_{\lim}/\tau_{\lim}$ which at $\tau_{\lim} = 0.5\sigma_{\lim}$ coincides with the condition of $\tau_{\max} = const$ (Soderberg's condition) and at $\tau_{\lim} = \sigma_{\lim}/\sqrt{3}$ transforms into $\tau_{oct} = const$ condition (Marin's condition). To assess the fatigue strength of brittle materials, Marin (1942) proposed to use the theory of maximum normal stresses with the equivalent stress taken for that acting on the plane where the stress normal from the standpoint of fatigue strength varies in the worst way.

Another group of criteria (the relationships of Kinososhvili, Birger, Auding, Heiwood, Science, etc.) reflects to a greater extent specific behaviour of the material at variable stresses. However, when using those criteria additional data are required which can be obtained only by way of carrying out additional complex experiments. The use of energy-based fatigue strength criteria requires special care. The specially performed experiments [6] revealed that those approaches are not always justified. This is evidenced in particular, by the known fact that in pulsating tension the endurance limit is half and over as much as that in pulsating compression.

In this respect an interesting experiment was carried out by V.N. Findley (1961). The experiments were performed on special specimens wherein the zone under study was subjected to loading by variable stresses with the strain energy in this zone being constant. Since energy is a scalar quantity invariant to the direction of principal axes, fatigue cracks should not develop in the region with a constant strain energy. However, there is no experimental evidence in favour of this assumption. The occurrence of fatigue cracks in the zone where the strain energy was kept constant testifies that the energy-based concepts do not fit the fatigue mechanism adequately. Fatigue fracture occurs obviously due to a cyclic variation of some stress or strain component on individual planes, shear stress for instance.

The findings of recent investigations have shown that promising approaches to assess fatigue resistance of materials are those based on the Prandtl conception of two types of fracture by tearing and by shear. As is known, this conception was developed in the works of Russian scientists N.N. Davidenkov [1] and B. Friedman [2], and is used as the basis for the generalized criterion of static strength [6]. By extending this criterion to the case of cyclic loading when stresses vary according to a fully-reversed cycle and coincide in phase, we get:

$$\sigma_{eq} = \chi\sigma_u + (1-\chi)\sigma_1 \,. \qquad (1)$$

Here σ_i and σ_u are the amplitude values of the maximum normal stress and stress intensity. If the base experiments are tension-compression and pure shear (torsion of thin-walled pipes), then

$$\chi = 1.37\left(\frac{\sigma_{-1}}{\tau_{-1}} - 1\right) . \quad (2)$$

The validity of Eqn (1) is illustrated by Fig.1 where the results of testing three grades of cast irons in bending with torsion [5] are presented, as well as the curves plotted using Eqn (1) and, for comparison, those plotted according to Soderberg and Marin (for plastic and brittle materials).

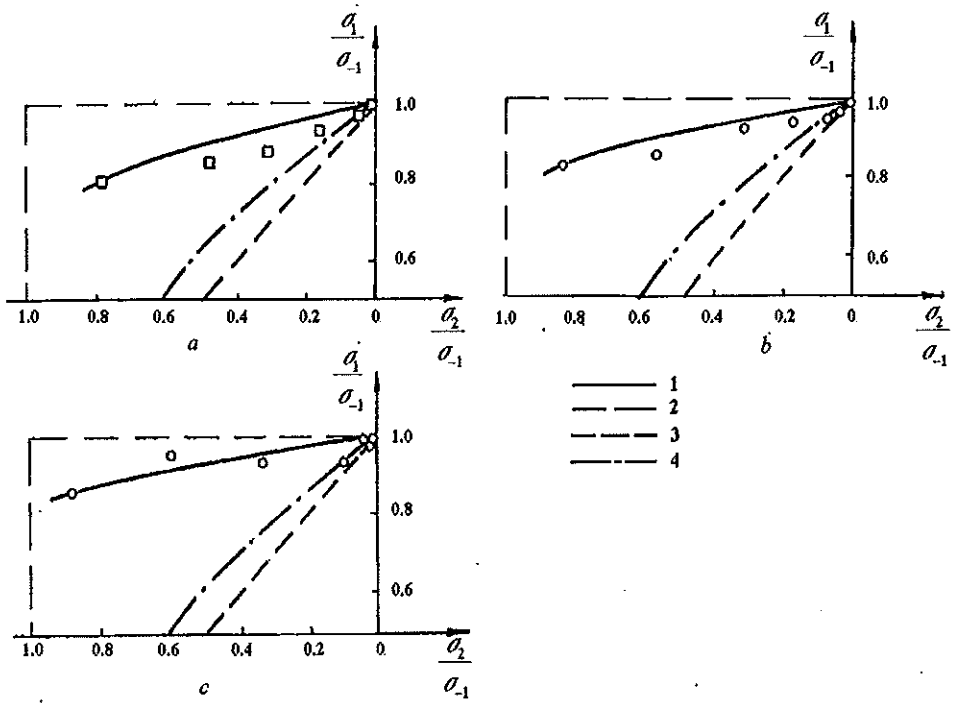

Fig. 1. The results of testing cast irons in synchronous tension with torsion: a. Cu-Cr cast iron; b. Mo-cast iron; c. Si-Al cast iron. 1 - according to Eqn (1); 2 - according to the τ_{max} = const condition; 3 - according to the σ_{max} = const condition; 4 - according to the von Mises theory

Loading with an oscillating cycle

For the majority of real structures typical is the oscillating cycle loading. In this case, the validity of calculations depends not only on the criterion chosen, but also on the validity of the ratios which approximate the relationship between the amplitude, σ_a and mean, σ_m, values of stresses. As possible variations, there were considered a straight line, ellipse, parabola, cosine curve and more complex-shaped curves [5, 9-11].

The parameters of the limit stress curve depend on the properties of the material studied and the test duration adopted (lifetime) Fig. 2 shows a general view of a limiting surface on the coordinates $\sigma_a - \sigma_m - N$. Under single load application, the region of safe stresses is bounded by the line of static loading in the plane $\sigma_a - \sigma_m$. With increasing number of cycles the endurance limit σ_{-1} under fully-reversed cycle changes in accordance with the fatigue curve $\sigma(N)$ (plane $\sigma_m = 0$), whereas stress-to-rupture σ_t, changes in accordance with the long-term strength curve $\sigma_t(t)$ (plane $\sigma_a = 0$). The experiments show that the rate of the endurance limit drop with an increase in the test duration (lifetime). exceeds considerably that of the long-term strength. Therefore for the majority of materials at room temperature it can be taken that $\sigma_t = \sigma_s$. However, at elevated and high temperatures the above assumption can result in essential errors. For this reason, the σ_t value for the test duration $t*$ corresponding to the base number, of loading cycles, $N*$, is often taken as the stress to-rupture [8], i.e.

$$t* = TN*$$

where T is the time of realization of one cycle (period).

It is necessary to keep in mind that the real test duration (in terms of time), $\bar{t}*$, which corresponds to the test duration (in terms of number of cycles), $N*$, can differ considerably from $t*$. Unfortunately, in the literature there are no recommendations on the scientifically justified definition of $t*$ and, consequently, on the calculation of the adequate values of stress-to-rupture.

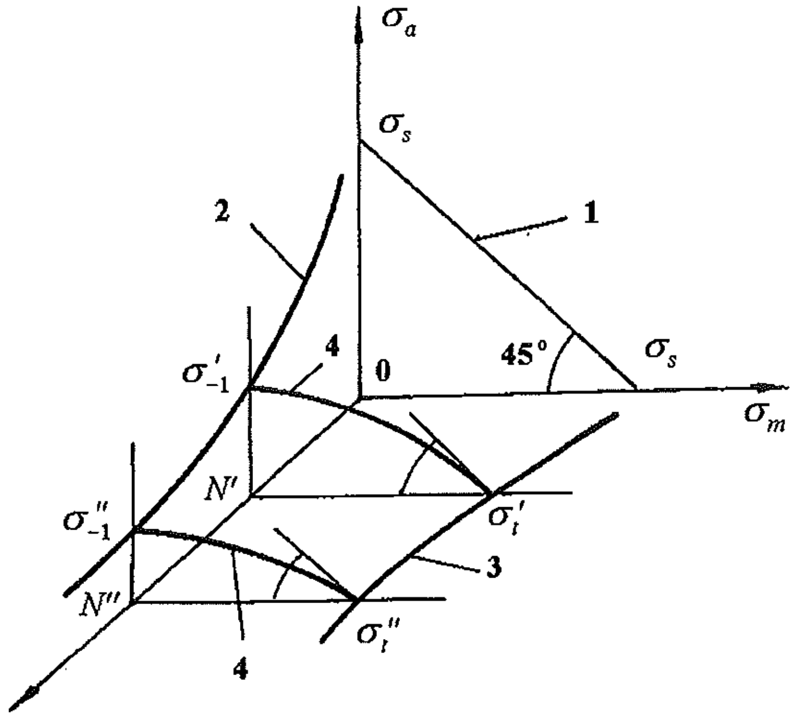

Fig. 2. The surface of limit stresses: 1 - line of static fracture; 2 - fatigue curve; 3 - long-term strength curve; 4 - curves of limit stresses for unsymmetrical cycles.

On the basis of the analysis of a great body of experimental data it was shovn in Ref. [5] that the limit curve $\sigma_a(\sigma_m)$ can be described to sufficient accuracy by the following equation:

$$\frac{\sigma_a}{\sigma_s} = \left(1 - \frac{\sigma_m}{\sigma_s}\right)\left[A_0 + \gamma\left(1 - A_0\right)\right] \tag{3}$$

where γ is some function. For steel smooth specimens

$$\gamma = \frac{\sigma_m}{3\sigma_s}\left(2 + \frac{\sigma_m}{\sigma_s}\right).$$

The complexity of Eqn (3) makes its extension to the cases of multiaxial loading difficult. Therefore, the following more simple equation is proposed

$$A\frac{\sigma_a}{\sigma_s}+\left(\frac{\sigma_m}{\sigma_s}\right)^A=1,\quad A=\frac{\sigma_s}{\sigma_{-1}} \tag{4}$$

which yields essentially the same diagrams of limit stresses as does Eqn (3). The comparison of those diagrams for $A=2.0$ is shown in Fig. 3.

It should be noted that many of previous approximations (e.g. ellipse, parabola) are inconsistent, because they do not satisfy the boundary conditions: for the domain of high σ_m values they fall outside the region bounded by the straight line $\sigma_a/\sigma_s+\sigma_m/\sigma_s=1$, which corresponds to an improbable state of the material wherein the endurance limit exceeds the ultimate strength (see the line 2 at $\sigma_m/\sigma_s>0.6$ in Fig. 3).

By generalizing Eqn (1) to the cases of the oscillating loading at multiaxial stress state and approximating the limit stress curves using Eqn (4) we get

$$A\frac{\sigma_{eq(a)}}{\sigma_s}+\left(\frac{\sigma_{eq(m)}}{\sigma_s}\right)^A=1 \tag{5}$$

where the equivalent stress amplitude

$$\sigma_{eq(a)}=\chi_a\sigma_{u(a)}+(1-\chi_a)\sigma_{1(a)} \tag{6}$$

and the average equivalent stress

$$\sigma_{eq(m)}=\chi_m\sigma_{u(m)}+(1-\chi_m)\sigma_{1(m)}\,. \tag{7}$$

In expressions (6) and (7)

$$\chi_a=1.37\left(\frac{\sigma_{-1}}{\tau_{-1}}-1\right);\quad \chi_m=1.37\left(\frac{\sigma_{\lim}}{\tau_{\lim}}-1\right). \tag{8}$$

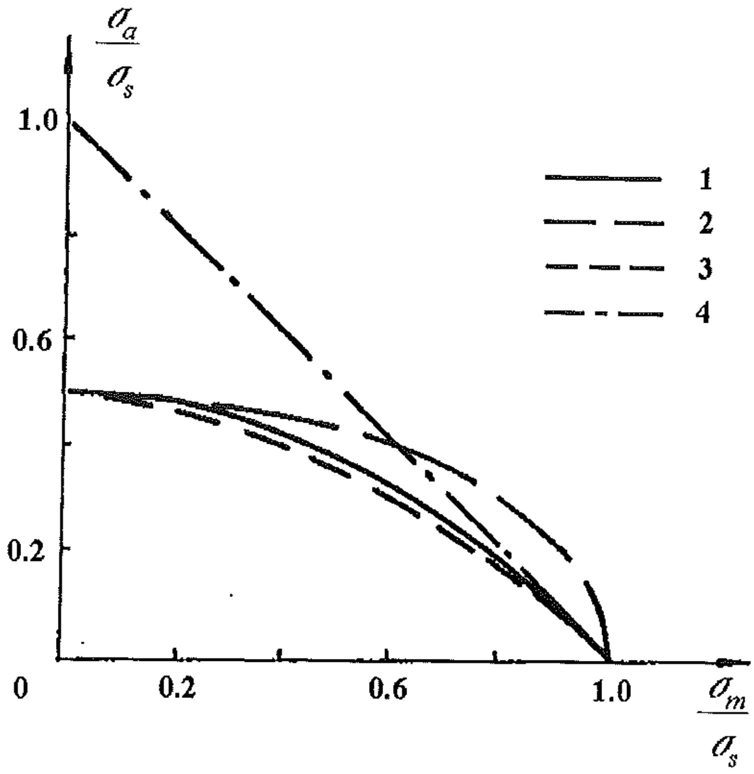

Fig. 3. Diagrams of limit stresses for unsymmetrical cycle: 1 - according to Eqn (4); 2 - according to Marin (an ellipse); 3 - according to Eqn (3); 4 - the boundary of theoretically possible amplitudes.

As has already been mentioned, $\sigma_{-1}/\tau_{-1} \approx \sigma_{\lim}/\tau_{\lim}$. Therefore, in order to reduce the number of additional tests we can set $\chi_a = \chi_m$ and use either of expressions (8) depending on the availability of one or another of mechanical characteristics.

Taking into account Eqns (6) and (7), the validity of Eqn (5) is illustrated by the data of Fig. 4 which presents the test results for steels such as nimonic (Kh18N9T) and low-alloy steel 30KhGSA (normalized and quenched with a high and low-temperature tempering) under conditions of plane stress state with an oscillating load cycle.

A comparison between those data and the calculation results using Eqn (5) shows that the experimental points have only a slight deviations from the calculated curve.

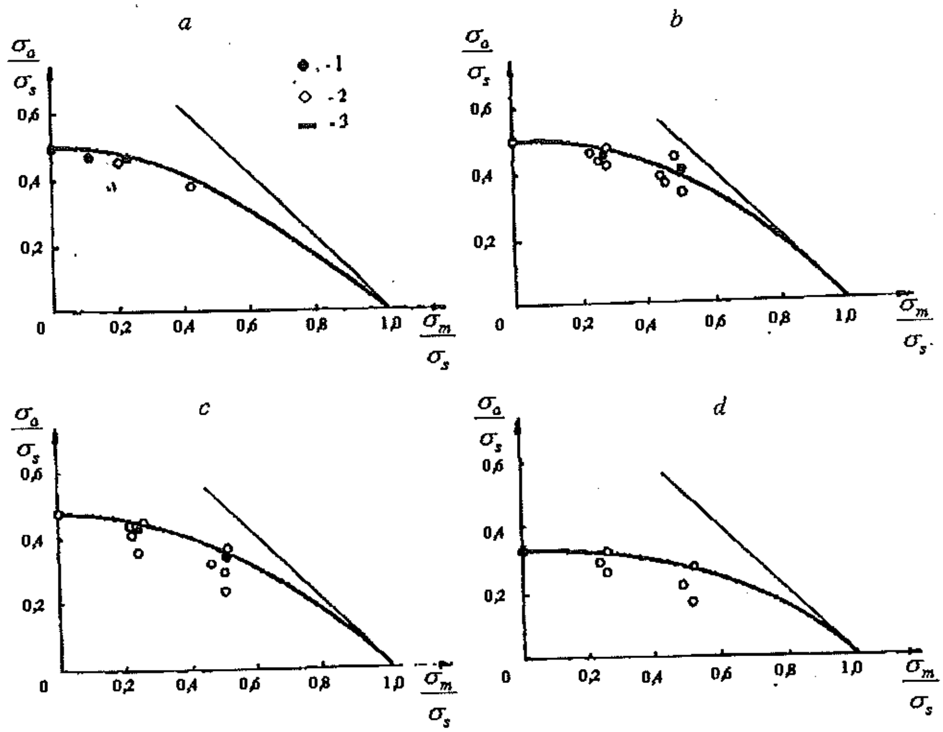

Fig. 4. Diagrams of limit stresses for steels Kh18N9T and 30KhGSA: a - steel Kh18N9T; b - steel 30KhGSA (normalized); c - steel 30KhGSA (oil-quenched at 890°, tempered at 500°C); d - steel 30KhGSA (oil-quenched at 890°, tempered at 200°C). 1 - under uniaxial loading (bending + torsion); 2 - under biaxial loading (bending + torsion + normal pressure); 3 - according to condition of (4) taking into account Eqns (5) and (6).

The parameters χ_a and χ_m are determined from the test results for tubular specimens in symmetrical bending and torsion. The acceptability of the criterion under consideration to describe the material ultimate state under conditions of repeated static (low-cycle) loading has been verified by test results for chrome-nickel steel at plane stress state. The investigations carried out have revealed that the parameter χ and, consequently, the shape of limiting curves under conditions of low-cycle fatigue depend upon the test duration and the deviation of the calculated data from the experimental ones for stresses is no more than 6%. Similar results, including those for creep and long-term strength, have been obtained by other authors [4, 5, 12].

Thus, Eqn (4) enables describing, to sufficient accuracy, the material ultimate state corresponding to the nucleation of a macrocrack the propagation regularities for which are established by way of additional calculations with the fracture mechanics apparatus being invoked.

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