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Criteria for Limiting Condition of Metal Alloys under Biaxial Low-Cycle Fatigue

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ABSTRACT: *The paper is devoted to development of the evaluation methods for limiting condition of metal alloys under biaxial proportional and non-proportional low-cycle fatigue. To investigate the multiaxial fatigue behaviour of two titanium alloys (BT9 and BT1-0) and stainless steel (08X18H10T) strain controlled tests under uniaxial loading and under combined in- and out-of-phase axial and shear strains were carried out. Experimental data are compared with the results of calculations carried out by the several criteria of fatigue. Validity of Pisarenko-Lebedev's and Coulomb-Mohr's criteria to predict fatigue lives is demonstrated for proportional loading. In case of non-proportional loading the good conformity between experienced and predicted results is received by use both of energy approach of Garud and Novozhylov-Rybakina's damage model.*

Notation

A,α	constants in Garud's energy	a	amplitude
a,b	constants in equation of regression	c	cycle
C,S	constants in Coulomb-Mohr	eq	equivalent strain
E	Young's modulus of elasticity	f	failure
N	number of cycles	fs	fatigue strength at N cycles
R	ratio of minimum and maximum	CM	Coulomb-Mohr
P	probability of survival	M	von Mises
P_i	microstress intensity	pr	predict
W	plastic work per unit volume	PL	Pisarenko-Lebedev
γ	shear strain	R	Rankine
ε	axial strain	s	secant modulus
φ	biaxiality factor	T	Tresca
ω	angular frequency	t,e,p	total, elastic, plastic
θ	phase angle between axial and shear	1,2,3	principal values
ν	Poisson's ratio		
Ω	damage parameter		

Introduction

Many structural members and machine components in service are subject to multiaxial fatigue stress conditions. The prediction of multiaxial fatigue life is often based on simple laboratory tests under uniaxial loading and the appropriate fatigue criterion. The latter is essentially a prescription for reducing the complex multiaxial loading to the "equivalent" uniaxial loading. Regardless of damage parameters used as equivalent ones (stress, strain, energy) it is possible to classify the limit conditions by the number of material constants. The most common one-parameter (one constant being determined experimentally) fatigue criteria are essentially the classical static failure theories extended to the fatigue case. However, their application even in a case of proportional loading does not provide satisfactory results. In practice, for a case of proportional loading the two-parameter (two constants at two types of stress state) criteria appear to be effective. For a case of non-proportional loading more promising ones are approaches that use integral-differential equations of state to determine the equivalent parameters. The paper gives the examples of such calculations that use of Garud's energy approach and the Novozhylov-Rybakina's damage model together with the Mruz yield theory. The objective of this paper is to discuss the applicability of multiaxial strain and energy parameters.

Experimental procedure

Specimens and experimental apparatus

The strain controlled low-cycle fatigue tests were carried out by electromechanical machine capable of combined axial and torsional loading designed by the author. The procedure and the testing equipment for uniaxial and biaxial tests are described in detail in (1). The specimen geometry is shown in Fig. 1. Table 1 lists the dimensions of test specimens. The chemical composition and mechanical properties of titanium alloys and 08X18H10T steel are given in Tables 2 and 3, respectively. All the materials tested were of commercial quality.

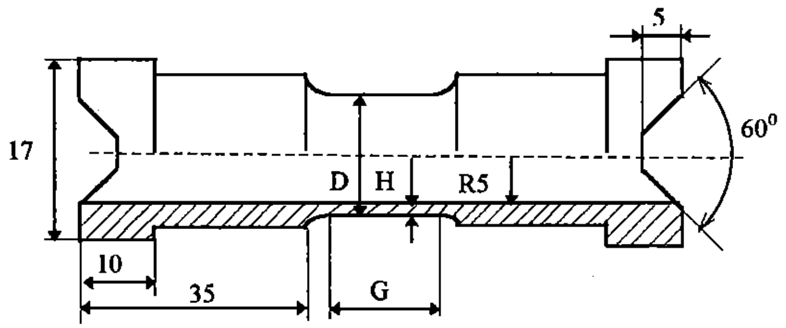

Fig. 1 Thin-walled tubular specimen

Table 1. Dimensions of test specimens

	08X18H10T	BT1-0	BT9
Gage length G, mm	15	20	20
Outside diameter D, mm	11	12	11
Thickness H, mm	0.5	0.75	0.5

Table 2. Chemical composition of test materials (weight percent)

Material	Element								
	Al	Mo	Si	Fe	Zr	H_2	N_2	C	Ti
BT9	6,50	3,40	0,30	0,081	1,58	0,006	0,018	0,06	balance
BT1-0	0,45		0,01	0,02					balance
08X18	Si	Mn	Cr	Ni	Ti				Fe
H10T	0,64	1,65	20	11	0,2				balance

Table 3. Static tensile properties

Nomenclature	Material		
	08X18H10T	BT1-0	BT9
Young's module E, 10^3 MPa	203	116	118
0.2% Yield Stress $\sigma_{0.2}$, MPa	320	490	865
Ultimate Strength σ_u, MPa	690	565	970
Elongation δ, %	40	26	17
Reduction of Area Ψ, %	55	57	45

Experimental conditions

The present cyclic tests were performed at room temperature. For all tests the frequency was 1 cycle/min., and the strains were completely reversed: $R_\varepsilon = R_\gamma = -1$. Wave forms of the axial and torsional strain cycles under loading specimens of 08X18H10T steel were triangular. In biaxial cyclic loading of titanium alloys' specimens, axial and torsional strains were given as (sinusoidal wave form)

$$\varepsilon_x(t) = \varepsilon_a \cdot \sin(\omega \cdot t), \qquad \varepsilon_y(t) = \varepsilon_z(t) = -\frac{1}{2}\varepsilon_x(t),$$
$$\gamma_{xy}(t) = \gamma_a \cdot \sin(\omega \cdot t + \theta), \quad \gamma_{xz}(t) = \gamma_{yz}(t) = 0. \tag{1}$$

The parameter ω is the angular frequency and θ the phase angle. The strain state is triaxial and the stress state is biaxial. The state of strain was defined by the biaxiality factor

$$\varphi = \arctan\left(\frac{\gamma_a}{\sqrt{3}\varepsilon_a}\right). \tag{2}$$

The definition used for equivalent strain amplitude is the following

$$e_{eq} = \max\left[\left(\varepsilon_x^2 + \frac{\gamma_{xy}^2}{3}\right)^{1/2}\right]. \tag{3}$$

The failure life N was defined as the number of cycles to the formation of a crack of approximately 1.0 mm long. The failure cycles N were in the range of 10^2 cycles to around 4×10^3 cycles. Each strain-life curve was obtained by several tests on at least three strain levels to enable a statistical evaluation of mean values with the probability of survival of P=50%. During each test the load-deformation curves were plotted periodically. A list of the experiments performed in the present study is shown in Table 4.

Table 4. Test programme

Material	e_{eq},%	ω , degree	θ , degree
08X18H10T	0.8, 1.0, 1.2	0, 14, 45, 76, 90	0, 45, 90
BT9	0.7, 1.0, 1.3	0, 45, 90	0
BT1-0	0.8, 1.0, 1.1, 1.2	0, 37, 45, 90	0

Experimental results

After the basic constant amplitude fatigue test under pure axial strain and pure torsional shear strain had been over, fatigue tests were conducted under in- and out-of-phase strains. The tests conducted and the resulting fatigue lives are given in Tables 6 and 7. From this tables it can be seen that the strain path has a significant influence on the fatigue life. For the same equivalent strain amplitude e_{eq}, fatigue life is longest for the case of pure torsion and shortest for pure axial straining. Analysis results also show that out-of-phase loading tends to be more damaging than in-phase loading. More in-depth discussion of the experiments appears elsewhere (2, 3).

The relationships between total strain amplitude and fatigue life were obtained by means of linear regression analysis using least squares method. The equation of regression was postulated in following form:

$$\lg N_f = a + b \cdot \left(\lg \varepsilon_a - \overline{\lg \varepsilon_a}\right). \qquad (4)$$

The Table 5 summarizes the relevant constants for the testing materials under uniaxial loading.

Table 5. Regression coefficients

Coefficient	08X18H10T	BT9	BT1-0
a	2.838	2.445	2.615
b	-3.461	-2.019	-3.632
$\overline{\lg \varepsilon_a}$	2.313	2.075	2.028

For biaxial proportional loading, equation (4) can be written as

$$\lg N_f = a + b \cdot \left(\lg \varepsilon_{eq} - \overline{\lg \varepsilon_a}\right), \qquad (5)$$

where ε_{eq} is an appropriate equivalent strain amplitude.

Review of life prediction methods

Proportional loading

Most design guidelines have been on the reduction of a multiaxial cyclic strain situation to an equivalent uniaxial strain, from which a life estimate can be made. Many strain based multiaxial fatigue theories have been proposed in the literature. The advantage of these types of theories is its simplicity.

An important example is the octahedral equivalent strain amplitude. In terms of principal strains $\varepsilon_1, \varepsilon_2$ and ε_3, this may be written as

$$\varepsilon_{eqM} = \frac{1}{\sqrt{2}\cdot(1+\nu)}\left\{(\varepsilon_{1a}-\varepsilon_{2a})^2+(\varepsilon_{2a}-\varepsilon_{3a})^2+(\varepsilon_{3a}-\varepsilon_{1a})^2\right\}^{1/2}, \tag{6}$$

where $\varepsilon_{1a} \le \varepsilon_{2a} \le \varepsilon_{3a}$ - principal strain amplitudes; ν - elasto-plastic value of Poisson's ratio. Clearly equation (6) can be associated with the von Mises yield criterion, for a strain level equal to cyclic yield.

Another equivalent strain formula is based on the Tresca or maximum shear criterion, giving

$$\varepsilon_{eqT} = \frac{1}{(1+\nu)}(\varepsilon_{1a}-\varepsilon_{3a}). \tag{7}$$

For elastic conditions, the maximum principal stress is given by

$$\sigma_1 = E\{\varepsilon_1(1+\nu_e)+\nu_e(\varepsilon_1+\varepsilon_2+\varepsilon_3)\}/\{(1+\nu_e)(1-2\nu_e)\} \tag{8}$$

where E and ν_e are Young's modulus and the elastic Poisson's ratio. For proportional loading conditions, equation (8), may be extended to elastic-plastic conditions by replacing E with the secant modulus, E_s, and ν_e with the elasto-plastic Poisson's ratio, ν. Thus an equivalent strain may be defined, equal to σ_1/E_S, where

$$\varepsilon_{eqR} = \frac{1}{(1+\nu)}\left\{\ \varepsilon_{1a}+\frac{\nu}{1-2\nu}(\varepsilon_{1a}+\varepsilon_{2a}+\varepsilon_{3a})\ \right\}. \tag{9}$$

The Poisson's ratio has to be obtained in the elastic-plastic range from the following relation (4):

$$\nu = 0.5 - 0.2 \cdot \frac{\sigma_{0.2}}{E \cdot \varepsilon_{eq}} . \tag{10}$$

Note that equations (6), (7) and (9) have been derived for the case of fully reversed strain controlled cycling.

The failure condition in the above criteria can be expressed as

$$\varepsilon_{eq} \le \varepsilon_{fs} . \tag{11}$$

In these criteria there is only one material constant; i.e., uniaxial fatigue limit amplitude ε_{fs}. Therefore, they may be called one-parameter criteria. The value of ε_{fs} comes from the uniaxial strain-life curve.

The Coulomb-Mohr and the Pisarenko-Lebedev criteria are well known for static loading. In contrast to one-parameter criteria they involve two parameters to be determined by experiment. The strain analog of the Coulomb-Mohr criterion in the linear form coincides with the two-parametric modification of the Brown-Miller criterion:

$$\frac{\gamma_{max}}{2} + S \cdot \varepsilon_n = C , \tag{12}$$

where $\frac{\gamma_{max}}{2}$ - maximum shear strain amplitude; ε_n - strain amplitude normal to the maximum shear strain plane; S and C are material constants. As the result of equation (12) application to the cases of uniaxial tension/compression and torsion the S and C constants can be determined as follows:

$$S = \frac{1}{(1-\nu)} \left\{ \frac{\gamma_{fs}}{\varepsilon_{fs}} - (1+\nu) \right\}, \qquad C = \frac{\gamma_{fs}}{2} , \tag{13}$$

where γ_{fs} is shear strain amplitude under conditions of pure torsion for given fatigue life, and ε_{fs} is axial strain amplitude under conditions of tension/compression for the same fatigue life. Thus

$$\varepsilon_{eqCM} = \frac{\varepsilon_{fs}}{\gamma_{fs}} \cdot \gamma_{max} + \frac{2}{(1+\nu)} \cdot \left\{ 1 - (1+\nu) \cdot \frac{\varepsilon_{fs}}{\gamma_{fs}} \right\} \cdot \varepsilon_n . \tag{14}$$

From the stress factor expression of the Pisarenko-Lebedev criterion (4) with transition to its strain expression, the following equation is easy to obtain:

$$\varepsilon_{eqPL} = \chi_\varepsilon \cdot \varepsilon_{eqM} + (1-\chi_\varepsilon)\cdot \varepsilon_{eqR} \tag{15}$$

where

$$\chi_\varepsilon = \frac{1}{\sqrt{3}-1}\cdot\left\{2\cdot(1+\nu)\cdot\left(\frac{\varepsilon_{fs}}{\gamma_{fs}}\right)-1\right\}. \tag{16}$$

Biaxial in-phase fatigue test results and predicted lives using one- and two-parameter criteria are shown in Table 6. It is seen that in all cases two-parameter criteria are much closer representation of experimental results than one-parameter criteria.

Table 6. Comparison between experimental and predicted lifetimes under proportional loading

	Loading conditions		Test results	Predicted lives N_{pr} by equetions:				
			N_f,	(9)	(7)	(6)	(14)	(15)
Material	ε_a, %	γ_a, %	cycles	cycles	cycles	cycles	cycles	cycles
	0.495	0.857	1248	455	292	346	537	518
	0.707	1.225	344	226	150	176	224	224
	0.919	1.592	147	136	91	107	116	120
	0.460	0	1019	943	943	943	943	943
	0.660	0	403	455	455	455	455	455
	0.700	0	386	404	404	404	404	404
BT9	1.000	0	222	197	197	197	197	197
	1.300	0	111	116	116	116	116	116
	0	1.212	3773	918	233	303	3763	3763
	0	1.576	1738	540	146	189	1402	1402
	0	1.732	498	446	123	160	555	555
	0	1.957	449	349	98	128	408	408
	0	2.252	189	264	76	99	184	184
$S=\left\{\left[\sum_{i=1}^{13}\left(N_{pri}-N_{fi}\right)^2\right]/13\right\}^{1/2}$				888	1120	1093	223	228

Table 6. Continued

Material	Loading conditions ε_a, %	γ_a, %	Test results N_f, cycles	Predicted lives N_{pr} by equations: (9) cycles	(7) cycles	(6) cycles	(14) cycles	(15) cycles
	0.760	0.000	1235	884	884	884	884	884
	0.780	0.000	1063	804	804	804	804	804
	0.820	0.000	550	670	670	670	670	670
	0.840	0.000	442	614	614	614	614	614
	0.940	0.000	335	408	408	408	408	408
	1.760	0.000	46	42	42	42	42	42
	0.778	1.336	315	325	160	212	256	269
	0.819	1.065	452	369	223	277	307	321
BT1-0	0.778	1.004	500	447	271	336	371	388
	0.717	0.936	920	591	355	441	475	500
	0.594	1.032	1031	831	402	536	602	644
	0.000	1.230	2045	4850	517	836	1492	1492
	0.000	1.340	1072	3725	387	629	1312	1312
	0.000	1.470	787	2787	283	461	1023	1023
	0.000	1.720	693	1687	165	270	599	599
	0.000	2.120	394	850	80	132	329	329
$S=\left\{\left[\sum_{i=1}^{16}\left(N_{pri}-N_{fi}\right)^2\right]/16\right\}^{1/2}$				1132	531	418	262	254
	0.388	0.168	1645	1393	1292	1340	1392	1393
	0.425	0.735	565	466	238	312	458	460
0	0.122	0.840	1291	1857	345	544	1853	1854
8	0.485	0.210	730	645	599	621	644	644
X	0.283	0.490	2163	1853	937	1230	1849	1850
1	0.146	1.008	886	1020	187	297	992	995
8	0.582	0.252	458	343	319	331	343	343
H	0.354	0.613	1360	868	441	578	857	860
1	0.097	0.672	3586	3858	724	1137	3999	3991
0	0.570	0.000	374	398	398	398	398	398
T	0.550	0.000	470	450	450	450	450	450
	0.500	0.000	640	626	626	626	626	626
	0.450	0.000	804	902	902	902	902	902
	0.380	0.000	1717	1619	1619	1619	1619	1619
	0.000	1.039	1790	1797	182	295	1788	1788
$S=\left\{\left[\sum_{i=1}^{15}\left(N_{pri}-N_{fi}\right)^2\right]/15\right\}^{1/2}$				240	993	850	254	252

Non-proportional loading

Two integral theories are now discussed and compared with the experimental results. These are the plastic work theory of Garud (6) and the micro-damage concept of Novozhylov-Rybakina (7, 8).

The plastic work approach of Garud uses both stress and strain, being based on hysteresis loop area. Garud has used the plastic work per cycle as a measure of damage:

$$W_c = \int_{cycle} s_{ij} \cdot de_{ij}^p = \sum_{cycle} \Delta W_p . \tag{17}$$

For the case of tension-torsion, his general expression of plastic work reduces to

$$W_c = \int_{cycle} \sigma \cdot de^p + r \cdot \int_{cycle} \tau \cdot d\gamma^p , \tag{18}$$

where r is a weighting factor determined in test (for 08X18H10T steel r =0.5). The following relation is postulated between the plastic work per cycle and fatigue life N_f :

$$N_f = F(W_c) = A \cdot (W_c)^\alpha . \tag{19}$$

The cyclic constants used were those obtained from thin-walled tube specimens under axial cyclic loading. These values for 08X18H10T steel are: A = 5365 and α = -1.913. Predictions were made using the von Mises type of yield function. A good correlation between the plastic work per cycle and fatigue life was obtained for a set of tests under axial and torsional strains with different phase angles.

The incremental damage approach proposed by Novozhylov V.V. and Rybakina O.G. (5) was applied to the low cycle fatigue. Following this approach, the material damage is determined by Ω parameter that depends on the whole previous loading history. At initial undamaged state $\Omega = 0$, and at the moment of macrocrack initiation $\Omega = 1$.
The suggestion made later (8) states that microstress is the determining failure factor. Thus, the following equation was obtained:

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$$\frac{d\Omega}{dL} = B \cdot P_i , \tag{20}$$

that relates Ω parameter and permanent microstress intensity $P_i = \sqrt{3/2\rho_{ij}\cdot\rho_{ij}}$. In (18) L is plastic strain curve length; B is material constant. The uniaxial loading mode calculation of 08X18H10T steel demonstrated that B is not a constant but a liner strain function:

$$B = B(\varepsilon_a) = 0.00266\cdot\varepsilon_a \ . \tag{21}$$

In (2) this approach was applied to the case of multiaxial low cycle fatigue. For a cyclically stable material the damage equation is as follows:

$$N_f \cdot \int_{cycle} B(\varepsilon_{eq})\cdot\left(\frac{3}{2}\rho_{ij}^{*}\cdot\rho_{ij}^{*}\right)^{1/2} dL = 1\,, \tag{20}$$

where

$$\rho_{ij}^{*} = \begin{bmatrix} \rho_{11} & k\cdot\rho_{12} & 0 \\ k\cdot\rho_{21} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

is the modified deviator of the microstress tensor , k is weighted factor, determined from the experiment (for 08X18H10T steel k =0.25).

The results shown in the Table 7 present the comparison between the experimental and the calculated data.

Table 7. Comparison between experimental and predicted lifetimes of 08X18H10T steel under non-proportional loading

No	Loading conditions			Test results	Predicted lives (cycles) by equations:			
		ε_a, %	γ_a, %		(6)	(7)	(8)	(9)
				cycles	cycles	cycles	cycles	cycles
1	0.8	14	0	1645	919	898	1392	1393
2	1.2	45	0	565	517	406	458	460
3	1.0	76	0	1291	1861	1540	1853	1854
10	1.0	14	0	730	496	487	644	644
11	0.8	45	0	2163	1583	1254	1849	1850
12	1.2	76	0	886	1124	921	992	995
19	1.2	14	0	458	304	293	343	343
20	1.0	45	0	1360	847	679	857	860
21	0.8	76	0	3586	3608	2877	3999	3991
$S=\left\{\left[\sum_{i=1}^{9}\left(N_{pri}-N_{fi}\right)^2\right]/9\right\}^{1/2}$					420	530	324	322
4	1.2	14	45	143	259	274		
5	1.0	45	45	221	161	212		
6	0.8	76	45	2094	2686	2240		
13	0.8	14	45	655	796	857		
14	1.2	45	45	176	85	116		
15	1.0	76	45	1079	1390	1091		
22	1.0	14	45	403	429	461		
23	0.8	45	45	462	348	462		
24	1.2	76	45	801	810	609		
$S=\left\{\left[\sum_{i=1}^{9}\left(N_{pri}-N_{fi}\right)^2\right]/9\right\}^{1/2}$					237	117		
7	1.0	14	90	276	415	435	627	627
8	0.8	45	90	259	225	309	1354	1356
9	1.2	76	90	437	764	621	1755	1755
16	1.2	14	90	248	250	260	334	334
17	1.0	45	90	191	104	148	626	626
18	0.8	76	90	1102	2484	2246	7136	7135
25	0.8	14	90	627	772	806	1357	1357
26	1.2	45	90	116	56	84	333	333
27	1.0	76	90	855	1279	1089	3298	3297
$S=\left\{\left[\sum_{i=1}^{9}\left(N_{pri}-N_{fi}\right)^2\right]/9\right\}^{1/2}$					500	403	2266	2266

Table 8. Correlation of experimental data with criteria of failure

Criterion of failure	Material	In phase: Conservative N_f / N_{pr}	In phase: Non conservative N_{pr} / N_f	Out-of phase: Conservative N_f / N_{pr}	Out-of phase: Non conservative N_{pr} / N_f
Renkine	08X18H10T	1.6	1.4		5.8
	BT1-0	1.5	3.4		
	BT9	4.2	1.4		
Tresca	08X18H10T	5.0		2.6	5.2
	BT1-0	5.0			
	BT9	16.0			
Mises	08X18H10T	3.2		2.6	5.2
	BT1-0	3.0			
	BT9	12.6			
Coulomb-Mohr	08X18H10T	1.6	1.4		6.5
	BT1-0	1.9	1.4		
	BT9	2.3			
Pisarenko-Lebedev	08X18H10T	1.6	1.4		6.5
	BT1-0	1.6	1.4		
	BT9	2.4			
Garud	08X18H10T	1.8	1.4	2.1	2.3
Novozhylov-Rybakina	08X18H10T	2.0	1.2	1.5	2.0

Conclusions

According to the data in the Tables 6, 7 and 8 the best correlation between the experimental and calculated data at proportional loading is achieved when calculations are made according to two-parameter criteria of Pisarenko-Lebedev and Coulomb-Mohr. For a case of non-proportional loading the best results are obtained for the integral criteria of Garud and Novozhylov-Rybakina. The differences between last two approaches are negligible since in both cases the damage parameters are calculated using the Mruz plastic yield theory.

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