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BIAXIAL FATIGUE: AN ANALYSIS OF THE COMBINED BENDING/TORSION LOADING CASE

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Fatigue, Biaxial Fatigue, Combined Loading.

ABSTRACT: Many structural mechanical components like crank and drive shafts are subjected to combined bending/torsion loading that may cause abrupt fatigue failures. The fatigue process under complexes states of stresses generated in these situations is known as Multiaxial Fatigue, and its assessment is fundamental to correct design and safe operational life. Although there have been many important developments over 100 years of multiaxial fatigue research, large factors of safety are still being employed to guard against multiaxial fatigue failures. In general, the proposed theories to predict fatigue behaviour under multiaxial stress states have been developed from three different approaches: a) the equivalent stress or strain; b) the plastic work and energy and; c) the critical planes. The present work reports an investigation on the applicability of these different approaches to the problem of combined bending/torsion loading. None of them seems to consider all the aspects and variables involved in the problem but the results showed that the equivalent stress/strain approach, apart from being simple and uncomplicated to implement, gives satisfactory predictions of fatigue strength or life for the problem studied.

Introduction

Most of the structural mechanical components are frequently subjected to variable loading that can lead to sudden fatigue failure. Crank and drive shafts, pressure vessels, blade/rotor junctions, bolted junctions and many aeronautical components are usually operating under combined loads, which can still be out of phase and in different frequencies, generating complexes biaxial or triaxial states of stresses. The fatigue process under such states of stresses is known as Multiaxial Fatigue whose consideration is very important for the component correct design, life assessment and its operational reliability.

Although many important developments have been made over more than a hundred years of research on the subject, many designers still resort to large factors of safety to guard structural components against multiaxial fatigue failures. The first attempts to investigate problems of multiaxial fatigue go back to 1886 when Lanza (1) published the first results of tests concerning combined bending/torsion loading. In the early decades of this century other investigators (2-5) presented new experimental data and raised a theoretical hypothesis to explain the phenomenon. The attempts made to develop theories that can be used to predict fatigue behaviour under multiaxial stress states are generally based on parameters which can be obtained in basic uniaxial reversed bending fatigue tests. Most of the proposed models fall into three basic approaches: a) the equivalent stress methods (6); b) the critical plane methods (7-12) and; c) the energy and plastic work methods (13-17). Most of the methods included in these categories have been reviewed by Garud (18). In the present work, the main objective was to select a model appropriate to analyse a practical problem involving in-phase cyclic bending/torsion combined loading with superimposed mean stress. The fundamental principles of each approach and some of the models most referred in the literature were examined and their applicability to the case in study checked in order a suitable model for the problem under analysis could be chosen.

The Equivalent Stress or Strain Approach

The first attempts to predict fatigue failure under combined loading consisted basically in the extension of the failure theories for static multiaxial state of stress to multiaxial states of cyclic stresses. In these theories, an uniaxial stress amplitude which would produce the same fatigue life as the multiaxial cyclic stress states is calculated and used to predict fatigue life from conventional S-N curves, obtained from reverse bending tests.The Maximum Shearing Stress Theory of Fatigue Failure and the Distortion Energy Multiaxial Theory of Fatigue Failure (19) are extensions of the Tresca and Von Mises theories, respectively, where the stress amplitudes are substitutes for the static principal stresses and the reversed fatigue strength or fatigue limit replaces the yield stress. The experimental evidence showed these methods are very conservative. One of the most important developments was made by Sines (6) who analysing experimental data observed a linear relation between the effect of the mean (static) stresses and the stress amplitude if the stresses do not exceed the yield strength. A general criterion was then postulated to account for the effect of different combinations of alternating stress with static stresses. The criterion is expressed in terms of octahedral-shear stress as a linear function of the sum of the orthogonal normal static stresses. It is mathematically expressed as:

$$\frac{1}{3}\left\{(\sigma_{1a}-\sigma_{2a})^2+(\sigma_{2a}-\sigma_{3a})^2+(\sigma_{1a}-\sigma_{3a})^2\right\}^{\frac{1}{2}} \leq A-\alpha(\sigma_{xm}+\sigma_{ym}+\sigma_{zm}) \quad (1)$$

where σ_{1a} , σ_{2a} and σ_{3a} are the alternating principal stress on the directions 1,2 and 3; σ_{xm} , σ_{ym} and σ_{zm} are the normal mean stress on the directions x, y and z; A and α are material constants, being A proportional to the reversed fatigue strength and α gives the variation of the permissible range of stress with static stress. For a biaxial state of stress, like the one generated by combined bending and torsion, and calculating the values of A and α, equation (1) reduces to:

$$\left\{(\sigma_{1a}^2+\sigma_{2a}^2)-(\sigma_{1a}\sigma_{2a})\right\}^{\frac{1}{2}} \leq \sigma_{rf}-\left[\left(\frac{\sigma_{rf}}{\sigma_{rf}'}-1\right)\left(\sigma_{xm}+\sigma_{ym}\right)\right] \quad (2)$$

where σ_{rf} is the amplitude of the reversed stress which would cause fatigue failure at a desired cyclic life and σ_{rf}' is the amplitude of fluctuating stress that would cause fatigue failure at the same life as σ_{rf} . Equation (2) is the equation of an ellipse whose size depends on the sum of the static (mean) stresses ($\sigma_{xm}+\sigma_{ym}$). The region inside the ellipse is the safe region and any combination of loads which produce alternate stresses within the area enclosed by the ellipse will not have premature failure. The right side of equation (2) gives the equivalent amplitude of stress:

$$\sigma_{aeq}=\left\{(\sigma_{1a}^2+\sigma_{2a}^2)-(\sigma_{1a}\sigma_{2a})\right\}^{\frac{1}{2}} \quad (3)$$

whose limit is the permissible amplitude of stress given by:

$$\sigma_a = \sigma_{rf} - \left[\left(\frac{\sigma_{rf}}{\sigma_{rf}^{\prime}} - 1\right)\left(\sigma_{xm} + \sigma_{ym}\right)\right] \quad (4)$$

The applicability of these equations is limited to situations in which the principal axes do not rotate during cyclic loading, *i.e.*, they are fixed in the body. To overcome this restriction, Fuchs (20) proposed a modification in the first term of equation (1):

$$\frac{1}{6}\left[\left(\Delta S_{11} - \Delta S_{22}\right)^2 + \left(\Delta S_{22} - \Delta S_{33}\right)^2 + \left(\Delta S_{33} - \Delta S_{11}\right)^2 + 6\left(\Delta S_{12}^2 + \Delta S_{23}^2 + \Delta S_{31}^2\right)\right]^{\frac{1}{2}} + \quad (5)$$
$$+\alpha\left(\sigma_{xm} + \sigma_{ym} + \sigma_{zm}\right) = A$$

where ΔS_{ij} are the differences of the stress components at times t_1 and t_2:

$$\Delta S_{ij} = \sigma_{ij}(t_1) - \sigma_{ij}(t_2) \quad (6)$$

Similar methods were proposed using the equivalent alternating strain as independent variable, instead of stress, and then used for low cycle fatigue by entering an ε-N curve. These methods did not consider the dependence of the fatigue process on the stress/strain response of the material.

The Critical Plane Approach

The equivalent stress or strain approaches are of difficult application to situations involving nonproportional loading where the principal stress axes rotate during the load cycle. Some researchers who carried early investigations on multiaxial fatigue (21-23) questioned how the behaviour of cracking mechanisms would influence the fatigue process. Fatigue cracks initiate in planes of maximum shear and propagate through the grains whose irregular surfaces would difficult the crack growth due to mechanical interlocking and friction effects. But normal stresses and strains acting upon the crack planes would open the crack, allowing it to grow. From this point of view the stresses and strains on the most severely loaded planes in the material would govern the fatigue process. Considering the multiaxial fatigue problem from the *critical plane approach*, Brown and Miller (7) proposed a theory based on the fatigue crack mechanisms. According this theory, which applies to situations of fixed principal strain axes and in-phase straining, failure under multiaxial fatigue is governed by the maximum range of shear strain and the range of normal strain acting on the plane where the maximum shear strain occurs. Based on it a number of proposals were been made. Kandil *et al.* (24) developed the model expressed by the following equation:

$$\frac{\Delta\gamma}{2}+\Delta\varepsilon_n=C \tag{7}$$

where the first term represents the shear strain amplitude on the maximum shear strain plane, the second the tensile strain normal to this plane and C is a material constant. According this model equivalent fatigue lives will result from equivalent values of C. Brown and Miller (7) stated that the critical plane is the plane of maximum range of shear strain and that the cracks could grow on these planes in two different ways: Type A cracks would propagate along the surface and Type B cracks would propagate away from the surface. Thus, two different relationships would be necessary to account for the fatigue process in a given material and given life, one for Type A cracks and other for type B cracks. The occurrence of Type A or Type B cracks depends on the type of loading, magnitude of strain and the materials characteristics. As the way the cracks will grow is not known in advance it is necessary to consider both situations making calculations for the two possible modes of cracking. Socie (9) proposed a model for the case of cracks that grow in planes of high tensile stress (mode I), expressed as:

$$\sigma_{max}\varepsilon_{1a}=\sigma_f'\varepsilon_f'(2N_f)^{c+b}+\left(\frac{\sigma_f'}{E}\right)(2N_f)^{2b} \tag{8}$$

where ε_{1a} is the amplitude of the principal strain and σ_{max} is the maximum (mean + alternate) stress acting on the plane of ε_{1a} . The right side of equation (8) is the description of a ε-N curve and the left side represents the loading variables for the plane of the greatest amplitude of normal strain (principal strain). For situations where the cracks grow on planes of high shear stress (mode II) Fatemi and Socie (10) suggested the following relationship:

$$\gamma_{ac}\left(1+\frac{\alpha\sigma_{max}}{\sigma_y'}\right)=\frac{\tau_f'}{G}(2N_f)^b+\gamma_f'(2N_f)^c \tag{9}$$

where γ_{ac} is the largest amplitude of shear strain for any plane; $\sigma_{max\,c}$ is the peak tensile stress normal to the plane of γ_{ac}, occurring any time during the γ_{ac} cycle; α is an empirical constant and; σ_y' is the cyclic yield strength. The terms of the left side of equation (9) represent the loading variables and τ_f', b, γ_f' and c defines the strain-life curve from completely reversed tests in pure shear. Other theories are reviewed by Leese and Socie (25) and Kussmaul *et al.* (26).

The physical interpretation of the multiaxial fatigue behaviour in terms of the cracking process, relating prediction of fatigue life to what is experimentally observed, is the great appeal of this approach. However, the definition of critical plane is still matter of controversy. Bannantine and Socie (27) and Socie (9) showed that the most probable planes for crack growth could be the planes of maximum amplitude of normal strain or maximum amplitude of shear strain. Moreover, for more complex loading situations like out-of-phase, non-proportional and variable amplitude loading, the ratios between the principal stresses change as well as the principal directions change during the load cycle and/or from cycle to cycle making difficult to define a critical plane. One of the effects which appears in these situations is an additional cyclic hardening of the material, which will require the use of fairly sophisticated plasticity theories in the models.

The Energy and Plastic Work Approaches

More recently, proposals (13-17) have been done to approach the fatigue problem under multiaxial stress states using energy as the correlation parameter. Garud (13) suggested to correlate fatigue life to crack initiation to the plastic work, done in each cycle, W_c, defined as:

$$W_c = \int_{cycle}(\sigma_x.d\varepsilon_{xp} + \sigma_y.d\varepsilon_{yp} + \sigma_z.d\varepsilon_{zp} + \tau_{xy}.d\gamma_{xyp} + \tau_{yz}.d\gamma_{yzp} + \tau_{zx}.d\gamma_{zxp}) \quad (10)$$

where σ_x, σ_y, σ_z, τ_{xy}, τ_{yz}, τ_{zx} are the stress components at a particular instant and $d\varepsilon_{xp}$, $d\varepsilon_{yp}$, $d\varepsilon_{zp}$, $d\gamma_{xyp}$, $d\gamma_{yzp}$, $d\gamma_{zxp}$ are the plastic normal and shear strain increments for a small increment of load. From the known load cycle and through a step-by-step procedure the parameter W_c is calculated and related to fatigue life through the equation:

$$N_f = F(W_c) \tag{11}$$

where N_f is the number of cycles to initiate a crack and F is a functional relation determined experimentally. The application oh this theory depends on the stress-strain response of the material under multiaxial loading and the stable cyclic stress-strain curve is the one to be used for the calculations of the plastic work per cycle. This approach has been extended in terms of total work energy released per cycle, including the elastic work done (16) and the effect of hydrostatic stresses (15).

Critical Analysis and Selection of the Models for the Bending/Torsion Case

Having the general description of the different approaches for the problem of multiaxial fatigue, some remarks can be done in order to select the models to be evaluated for the analysis of a problem involving in-phase cyclic bending/torsion combined loading with superimposed mean stress. The Sines model expressed by equation (1) is of easy implementation and easy graphical and analytical interpretation but it lacks to consider the physical stress-strain response of material in terms of crack nucleation and growth, a decisive part of the fatigue process. The model predicts that fatigue life is controlled by the amplitude of the octahedral shear stress therefore the fatigue cracks could be expected to grow on the octahedral planes what it does not happen as showed by Socie (9). Another disadvantage is that its applicability is limited to the cases in which the principal axes of the alternating components are fixed to the body.

The critical plane approaches offer a physical interpretation of the fatigue process but they are more difficult to implement as most of them require a number of parameters to be determined experimentally from different types of fatigue tests. If plastic deformations are involved, as in low cycle fatigue, the characterization of the plastic behaviour of the materials have to be done through a plasticity theory which adds an extra complication in the implementation of the method. The two types of possible crack growth require to consider both possibilities in the analysis and some models do not distinguish the two types of cracks losing the relation with the physical interpretation which is the great appeal of this approach. Gough *et al.* (5) observed that under the various systems of combined stress tested, the directions of the cracks showed such diversity and irregularity that no relation with the applied stressing system could be established. Other models which are based on strain do not give good results for high cycle fatigue as, in this situation, the fatigue process is controlled by elastic stresses. All these aspects, associated with the difficulty to identify the critical plane in complex loading situations, generate uncertainties on the applicability of the methods to more general loading situations.

The energy and plastic work approaches consider the interaction between stress and strain (hysteresis loop) during the fatigue process, reflecting the dependence of the damage process on the material response to the applied loads. The main objection raised against this approach is that energy is a scalar quantity while fatigue failures occur on preferential planes of crack initiation and growth thus, being impossible to differentiate between the two types of cracks observed by Brown and Miller (7). Another difficulty arises from the need to use sophisticated models of cyclic plasticity, especially under complex situations such as the non-proportional loadings. For high cycle fatigue cases the precision of the method is questionable as the plastic strains are very small or even unexistent. According Tipton and Nelson (28) a small variation in the value of the calculated plastic work may result in large discrepancies on the predicted fatigue life.

The case under analysis circumscribed very clearly a high cycle fatigue situation under non-proportional loading. The presence of superimposed mean stresses causes a rotation of the planes of maximum shear and normal stresses along the cycle. The approaches of energy and plastic work appeared not to fit the case as only elastic stresses were present. Despite the various theories based on the critical plane approach are of non-complicated implementation for proportional loading, the same is not true for non-proportional loading like the case of combined bending and torsion with superimposed mean stress. In this case is difficult to define the critical plane and, consequently, to make the calculations to determine the parameters involved in the models. The equivalent stress theories which are extensions of the static failure theories to multiaxial states of cyclic stresses were analysed by Araujo and Balthazar (29) and the results confirmed, as expected, their conservativeness. Thus, for its simplicity and lack of restrictions in its applicability to non-proportional loading the Sines's model was chosen to be evaluated.

Discussion

In order to evaluate its applicability to the case of in-phase combined bending/torsion loading with superimposed mean stress the predictions given by the Sines'Model were compared with experimental results published in the literature. Gough *et al.* (5) present the results of a set of fatigue tests which are frequently taken by many authors to validate multiaxial fatigue models. In this work the fatigue strength of a NiCrMoV alloy steel under combined bending/torsion loading was determined. The chemical composition of the material tested is shown in table 1 and its mechanical properties in table 2.

Table 1 - Chemical Composition of the NiCrMoV Alloy Steel

Element	C	Si	Mn	S	P	Ni	Cr	Mo	V	Fe
%	0.24	0.20	0.57	0.004	0.015	3.06	1.29	0.54	0.25	balance

Table 2 - Monotonic and Fatigue Mechanical Properties

Yield Strength, S_y	931,00 MPa
Ultimate Tensile Strength, S_u	984,00 MPa
Young Modulus, E	200.00 GPa
True Fracture Ductility, ε_f	23.5 %
Reduction in Area, RA	67.0 %
Shear Yield Stress, S_{sy}	703.00 MPa
Shear Modulus, G	79.00 GPa
Fatigue Limit (reverse bending), S_e	569.00 MPa
Fatigue Limit (reverse torsion), S_{se}	326.00 MPa

The experimental program carried by Gough *et al.* (5) comprised a series of tests combining different ratios of the stress amplitudes σ_{xa} / τ_{xya} to three levels of superimposed bending mean stress and three levels of torsion stress. For each combination of mean stresses up to five ratios of amplitude of bending stress, σ_{xa}, to amplitude of torsion stress, τ_{xya}, were tested. These ratios varied from σ_{xa} / τ_{xya} = ∞ (alternating bending only) to σ_{xa} / τ_{xya} = 0 (alternating torsion only) .

The results for the 29 fatigue tests under combined bending/torsion loading are showed in table 3. The fatigue limits showed correspond to the applied amplitude stresses for a life of 10^7 cycles.

Table 3 - Results of Fatigue Tests Under Combined Bending/Torsion Loading (5).

Mean Stress		σ_{xa} /	Fatigue Limit		Mean Stress		σ_{xa} /	Fatigue Limit	
Bendi	Torsio	τ_{xya}	σ_{xa}	τ_{xya}	Bendi	Torsio	τ_{xya}	σ_{xa}	τ_{xya}
0	0	∞	574.0	0	524.0	167.0	0	0	276.0
262.0	0	∞	544.0	0	262.0	338.0	0	0	304.0
524.0	0	∞	524.0	0	524.0	338.0	0	0	281.0
0	0	0	0	365.0	0	0	3.5	538.0	153.0
0	167.0	0	0	325.0	0	0	1.5	383.0	255.0
0	338.0	0	0	337.0	0	0	0.5	166.0	330.0
0	167.0	∞	541.0	0	262.0	167.0	3.5	475.0	135.0
0	338.0	∞	532.0	0	262.0	167.0	1.5	368.0	245.0
262.0	167.0	∞	547.0	0	262.0	167.0	0.5	158.0	316.0
262.0	338.0	∞	532.0	0	524.0	338.0	3.5	395.0	111.0
524.0	167.0	∞	462.0	0	524.0	338.0	1.5	310.0	207.0
524.0	338.0	∞	465.0	0	524.0	338.0	0.5	125.0	248.0
262.0	0	0	0	307.0	262.0	0	1.5	380.0	252.0
524.0	0	0	0	279.0	0	167.0	1.5	377.0	251.0
262.0	167.0	0	0	279.0					

The data of table 3 was compared with the Sines' Criterion and the results can be seen in Figures 1 to 4. Figure 1 show the plot for bending mean stress, σ_{xm} = 0. The trace-point line corresponds to the alternating torsion loading case where $\sigma_{1a} = \sigma_{2a}$. Figures 2 and 3 show the comparisons for $\sigma_{xm} = 262$ MPa and $\sigma_{xm} = 524$ MPa, respectively. Except for the higher value of σ_{xm}, whose points felt inside the ellipse, the agreement obtained was quite satisfactory. The non-conservative situation obtained with the higher mean stress reflects, possibly, the influence of some degree of plasticity as the maximum stresses approaches the yield stress values. This would confirm the inappropriateness of the Sines's model to low cycle fatigue. Figure 4 show the superposition of all results of table 3.

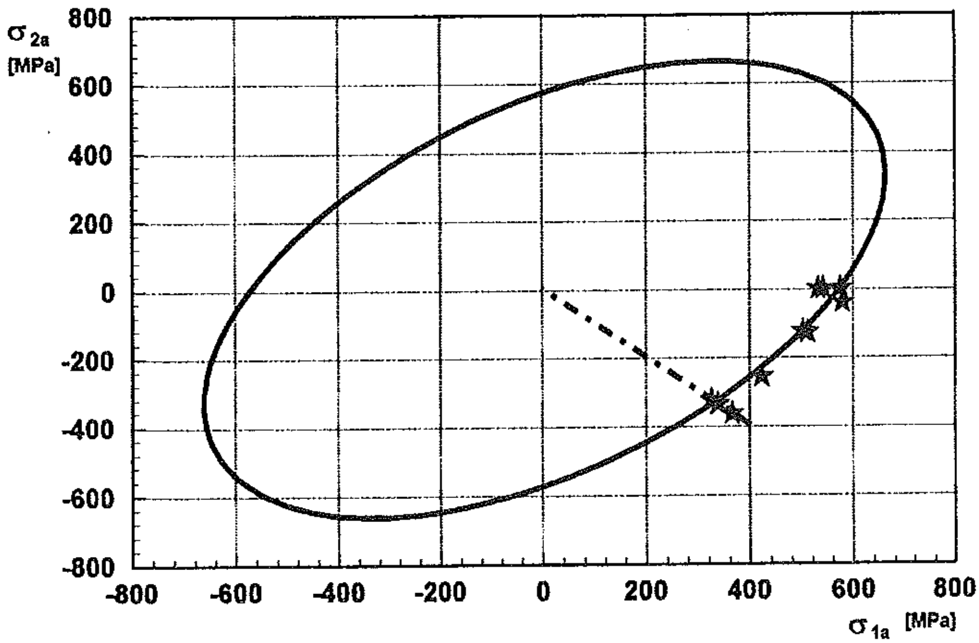

Fig.1 - Sines' Theory and Fatigue data for $\sigma_{xm} = 0$.

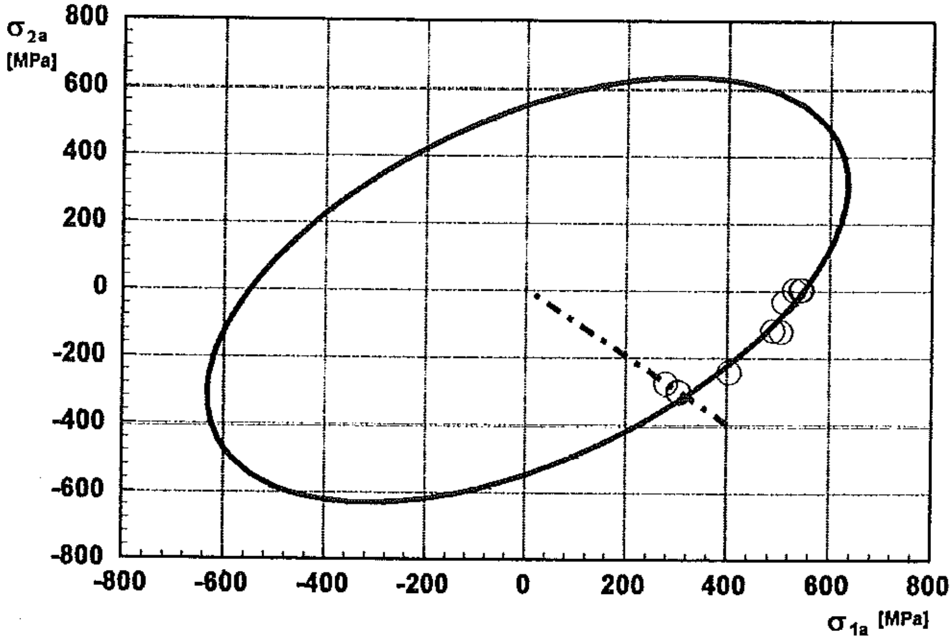

Fig.2 - Sines' Theory and fatigue data for σ_{xm} = 262 Mpa.

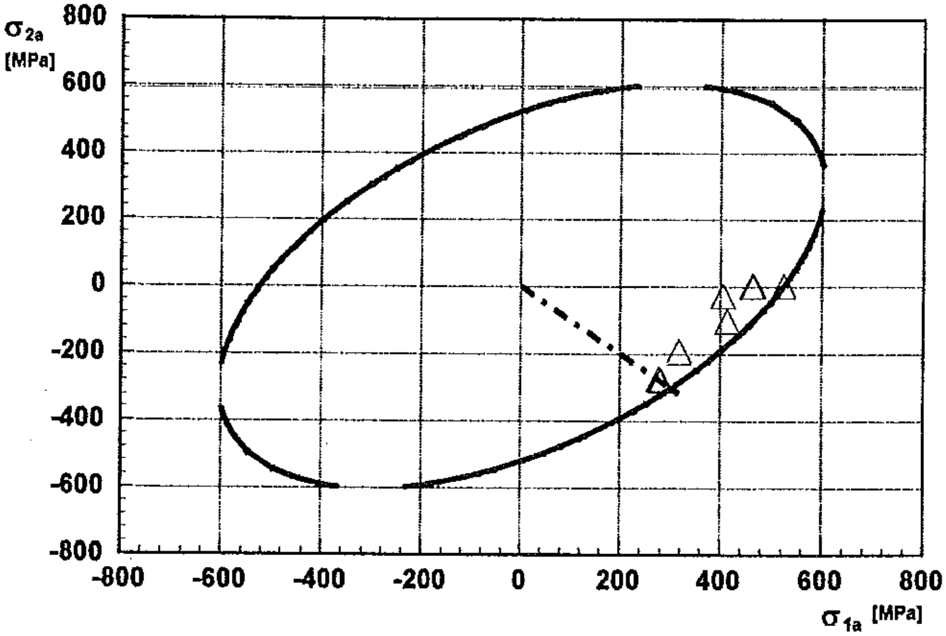

Fig.3 - Sines' Theory and fatigue data for σ_{xm} = 524 MPa.

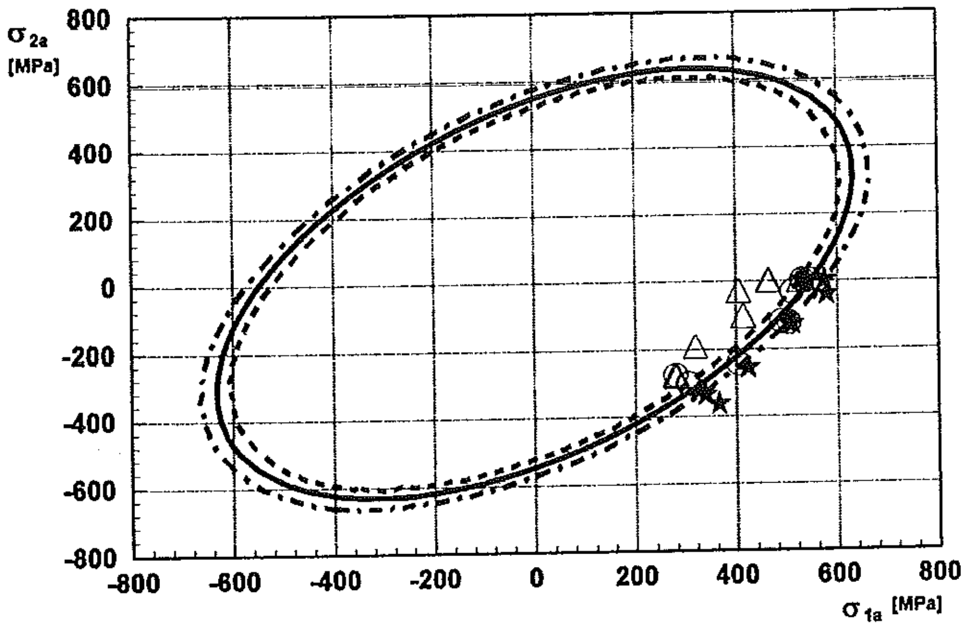

Fig.4 - Sines' Theory and fatigue data for $\sigma_{xm} = 0$ Mpa (◊ – · – · –), $\sigma_{xm} = 262$ Mpa (O ——) and $\sigma_{xm} = 524$ Mpa (◁ – – – –).

Conclusions

A critical analysis of the different approaches of the multiaxial fatigue problem, as well as comparison of experimental data with the Sines' Theory, indicated that the case involving combined in-phase bending/torsion loading with superimposed mean stress can be satisfactorily analysed with the Sine's Theory. The quality of the predictions and facility of implementation were the main factors in its favour and the difficulty of interpretation in terms of the physical stress-strain response of the material is its main weakness. The critical analysis done indicates that further investigation is still necessary in order theories suitable to appraise the fatigue multiaxial problem in a more universal way could be developed. The attraction of the critical plane approach, its relation with the physical observations of the cracking process, seems not to give a satisfactory response for the non-proportional loading cases. The plastic work and energy approaches seem promising and of general applicability if some relation with the physical damaging process could be established.

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