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A Stress-Based Approach for Fatigue Assessment under Multiaxial Variable Amplitude Loading

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ABSTRACT: *This paper presents a new multiaxial variable amplitude fatigue life prediction method. The main steps of this stress-based approach are the definition of a so-called counting variable to identify and extract cycles from the multiaxial random stress history, the use of both multiaxial criteria which are extended from endurance to finite fatigue lives, in order to calculate the life of each extracted cycle by Rainflow counting and then the use of damage rules to assess the life of the material submitted to the multiaxial stress sequence. The different steps are detailed : the choosen counting variable is justified, the global approach and critical plane approach criteria are presented and both linear Miner and non linear Lemaitre & Chaboche damage laws may be used. Several biaxial variable amplitude fatigue tests that were carried out in the Technical University of Opole (Poland) allows to validate the proposed method with both damage rules.*

Introduction

The economical constraints which become stronger and stronger nowadays induce the trend of an important weight reduction of structures or components. This purpose imposes to reduce dimensions and thickness of thin walled components and, as a consequence, to submit structures to generally higher stress levels. This results in pointing out the fatigue phenomenon as a problem of great priority.

The aim of this paper is to present a stress-based approach (Kenmeugne (1)) which allows engineering designers to assess the fatigue life of a component solicited by a multiaxial variable amplitude loading which is the most general case of solicitations. The method described in this paper contains different steps as shown on the following flow chart (Figure 1).

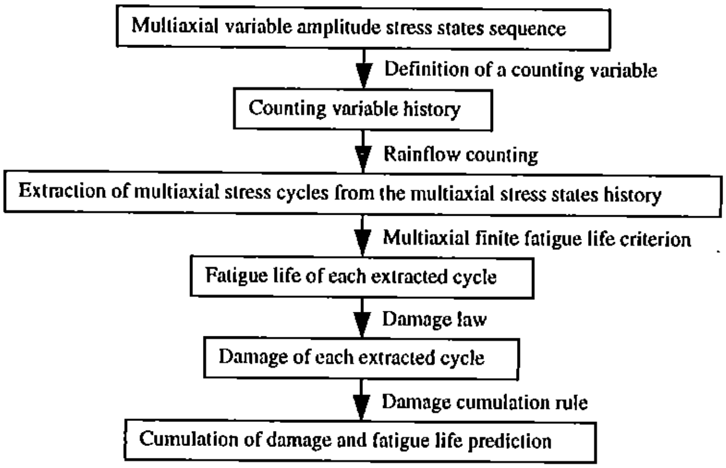

Fig. 1 Flow chart of the proposed stress-based fatigue life prediction method.

The proposed stress-based approach derives from an extension of criteria from endurance (infinite lives) to finite fatigue lives (Robert (2)). The procedure keeps then the main steps of uniaxial fatigue life prediction methods (Robert and Bahuaud (3)).
Each step of the method is explained and justified in the following sections.

Counting variable definition

One of the main problems with multiaxial variable amplitude stress is the definition of a cycle. A cycle is generally defined by a hysterisis loop of the material uniaxial stress-strain response (Figure 2).

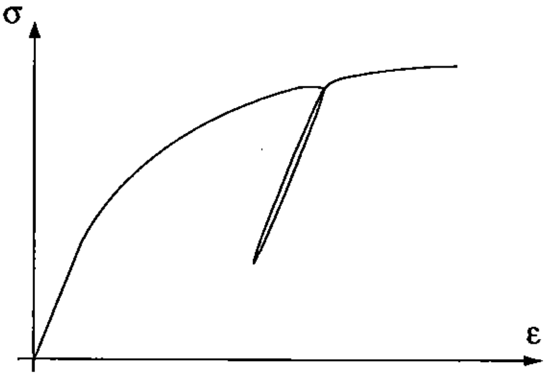

Fig. 2 Hysterisis loop of the material stress-strain response.

The Rainflow counting is a simple and well-established technique (Amzallag (4)) to identify uniaxial stress cycles represented by these hysterisis loops.

The most complex counting problem concerns the case where any of the six different components of the stress tensor varies independently from each other. The figure 3 shows for instance that a cycle may occur on one channel (σ_{22}) but during the same time interval [t_1, t_2] none of the other channels experiences exactly a cycle.

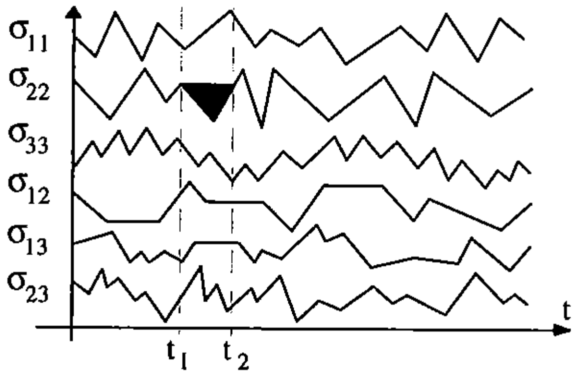

Fig. 3 Problem of the identification of cycles inside a multiaxial sequence.

The proposed way to overcome this difficulty is to define a counting variable V(t), which must be good representative both of the stress states and of their evolution versus time. Cycles are then Rainflow-counted by the use of the french AFNOR recommended procedure (5).

The first idea was to choose as the counting variable the projection of the octahedral shear stress onto the deviatoric plane (2) (6) to build the counting variable sequence. But Kenmeugne (1) showed that this variable does not allow to identify the real period of cycles in some particular cases of stress states. Then he proposed to use the normal stress acting on a fixed (relatively to the body) physical plane as the counting variable (Figure 4). The physical plane is represented by its unit normal vector $\vec{h}$ and the normal stress that is acting on it is denoted σ_{hh}. This counting variable avoids the periodicity mistakes of the particular cases of stress states that make fail the first counting variable. The only difficulty which appears is to find the right fixed plane where the more conservative fatigue life result is obtained. This point will be discussed further, when the multiaxial stress sequences will be considered.

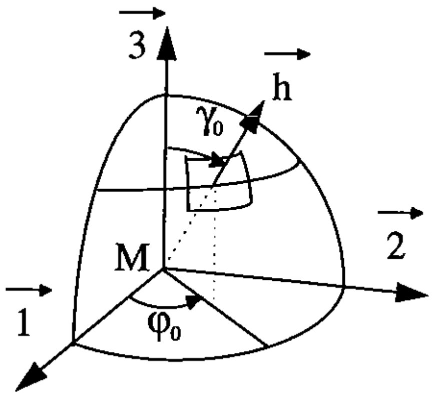

Fig. 4 Location of a physical plane with respect to the body frame.

A validity condition is induced by the Rainflow counting procedure. It consists in the fact that the counting variable must not remain constant when the stress tensor is varying. A preliminary test, previously to the cycles counting, is thus realized for the choice of the counting plane.

When a cycle is identified within the counting variable sequence, the multiaxial time-corresponding stress states are extracted from the multiaxial sequence and are considered as a multiaxial stress cycle (Figure 5).

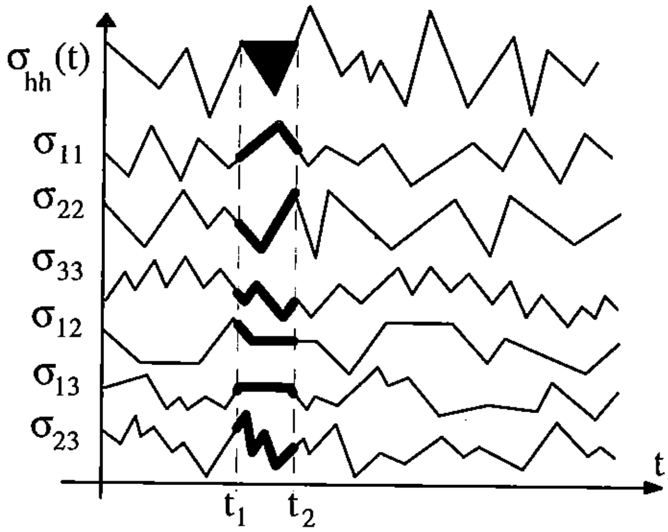

Fig. 5 Extraction of a multiaxial stress cycle corresponding to a cycle experienced by $\sigma_{hh}(t)$.

Fatigue life assessment of multiaxial cycles

When a multiaxial cycle is identified and extracted from the multiaxial sequence by Rainflow application, the second main step consists to calculate its fatigue life by the way of a multiaxial fatigue criterion. Then the damage is assessed by the way of a damage law.

A multiaxial criterion is a fatigue function E that allows to compare any multiaxial stress cycle with the N cycles-fatigue strength of the material. E equals the unit value when the stress cycle reaches the fatigue strength; E is smaller (respectively greater) than the unit value if the fatigue strength is not reached (respectively is exceeded). From this point of view the applicability domain of the fatigue criteria has been extended from the infinite fatigue lives (endurance domain) to finite ones (2).

A criterion can be written as :

$$E\left(\sigma_{ij}(t), \sigma_{-1}(N), \tau_{-1}(N), \sigma_0(N)\right) = 1 \qquad (1)$$

The fatigue function depends on the cycle stress states tensor and some material fatigue data which are three S-N curves $\sigma_{-1}(N)$, $\tau_{-1}(N)$ and $\sigma_0(N)$. These give the material fatigue strengths corresponding to N cycles for respectively a reversed tensile test ($R = -1$), a reversed torsion test ($R = -1$) and a zero to maximum tensile test ($R = 0$). When E is equal to unit, the fatigue life of the cycle $\sigma_{ij}(t)$ is equal to N cycles. It is calculated directly from equation (1) by the way of an implicit algorithm. Two formulations of criteria have been proposed by the INSA Laboratory of Solids Mechanics.

The first one proposed by Fogue (7) in 1987 is a Global Approach (GA). Fogue defines a fatigue indicator E_h for any physical plane P which unit normal vector is $\vec{h}$:

$$E_h = \frac{1}{\sigma_{-1}(N)}\left[a(N)\tau_{ha} + b(N)\sigma_{hha} + d(N)\sigma_{hhm}\right] \qquad (2)$$

where a(N), b(N) and d(N) are criterion parameters depending on $\sigma_{-1}(N)$, $\tau_{-1}(N)$ and $\sigma_0(N)$. They are determined by stating that the criterion is checked ($E=1$) for these three basic N cycles fatigue strengths.

τ_{ha} is the shear stress amplitude acting on the plane $P(\vec{u},\vec{v})$, σ_{hha} is the normal stress amplitude, σ_{hhm} is the mean normal stress acting on P during the cycle. τ_{ha} is obtained by building the surrounding circle to the loading path, i.e. the tip of the shear stress vector acting on the considered plane P during the whole cycle (Figure 6). τ_{ha} is the radius of this surrounding circle.

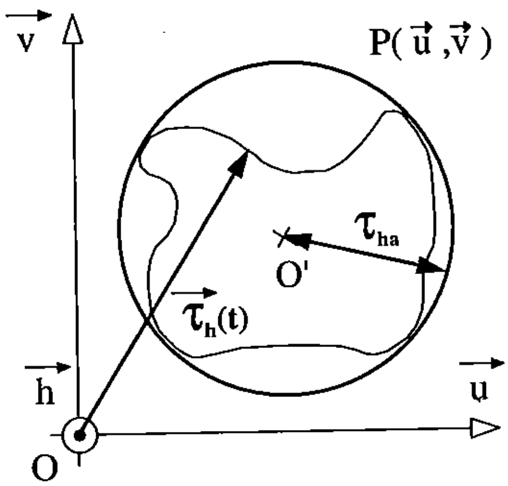

Fig. 6 The surrounding circle to the loading path.

The fatigue function (E_{GA}) of the criterion is given by the root mean square of its fatigue indicator E_h all over the possible planes through the calculation point M :

$$E_{GA} = \sqrt{\frac{1}{S}\int_S E_h^2 dS} \tag{3}$$

Where S is the area of a sphere surrounding M and which radius is equal to unit ($S = 4\pi$).
The second criterion proposed by Robert (2) is based on the Critical Plane Approach (CPA). The fatigue indicator $E_h(t)$ is time dependent and is a linear combination of the components of the stress vector acting at time t on the surface element which unit normal vector is $\vec{h}$.

$$E_h(t) = \left\|\vec{\tau}_{ha}(t)\right\| + \alpha(N)\sigma_{hha}(t) + \beta(N)\sigma_{hhm} \tag{4}$$

Where $\alpha(N)$, $\beta(N)$ and $\theta(N)$ are criterion parameters depending on $\sigma_{-1}(N)$, $\tau_{-1}(N)$ and $\sigma_0(N)$, $\left\|\vec{\tau}_{ha}(t)\right\|$ is the alternate shear stress versus time t acting on the plane $P(\vec{u},\vec{v})$ (Figure 7), $\sigma_{hha}(t)$ is the alternate normal stress versus time t, σ_{hhm} is the mean normal stress during the cycle.

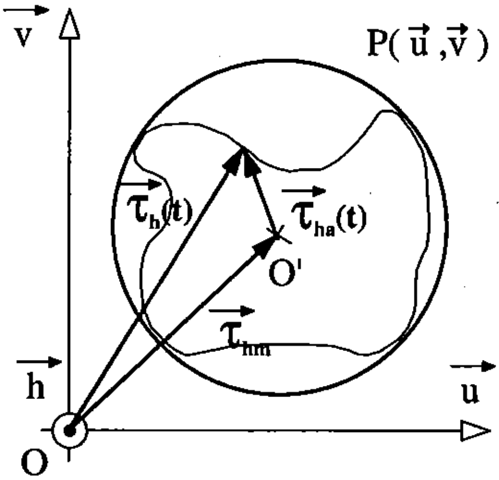

Fig. 7 Definition of the alternate shear stress vector $\vec{\tau}_{ha}(t)$.

The fatigue indicator corresponding to the considered plane is :

$$E_h = \frac{1}{\theta(N)} \max_t \left[E_h(t)\right] \tag{5}$$

The concept of the criterion is to search the material plane where the fatigue indicator $E_h(t)$ is maximum. The fatigue function is hence obtained by:

$$E_{CPA} = \max_h \left[E_h\right] \tag{6}$$

A previous work (Robert, Fogue & Bahuaud (8)) has shown that the criterion based on the Critical Plane Approach is especially suitable when principal stress directions remain fixed during the cycle relatively to the body, as the most activated slipping plane is always the same. The Global Approach gives the best description of the fatigue behaviour of the material when principal stress directions rotate during the cycle, relatively to the body because in that case several slipping plane are activated. The root mean square which makes a quadratic average of the fatigue indicator is a way to take into account that physical phenomenon.

Figures 8 and 9 give the distributions of the fatigue indicator E_h of the Global Approach criterion for respectively a fixed principal stress directions cycle and a rotating principal stress directions cycle. The distributions concerns all the possible physical planes which unit normal vector $\vec{h}$ may be defined by two angles φ and γ (see figure 4 where these angles are denoted φ_0 and γ_0 respectively). The distributions obtained for the Critical Plane Approach criterion which are not given here have similar shapes. The most important points are that, a finite number of physical planes are critical in the first case (fixed principal directions), whereas a large set of physical planes are equally critical in the second case (rotating principal stress directions).

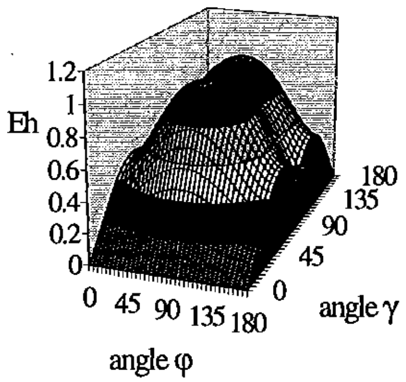

Fig. 8 Distribution of the fatigue indicator E_h - Fixed principal stress directions.

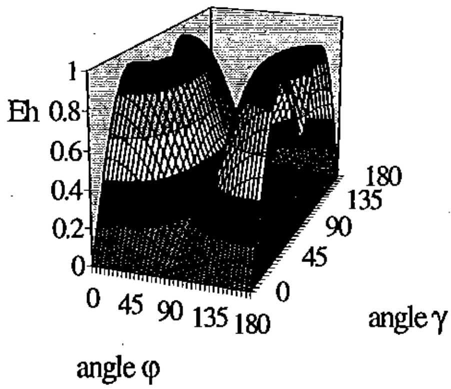

Fig. 9 Distribution of the fatigue indicator E_h - Rotating principal stress directions.

Linear and non linear damage laws

For this third step of the method, two damage and cumulation rules may be used. The first one is the linear Miner's rule (9). The second one is the non linear Lemaitre and Chaboche damage law (10, 11).

Linear Miner's rule

The damage d_k induced by a cycle extracted from the multiaxial sequence is obtained from its fatigue life N_k as :

$$d_k = \frac{1}{N_k} \tag{7}$$

The damage cumulation is defined by the summation of the damage d_k of all the cycles (equation (8)). The fatigue life N of the whole sequence corresponds to the number of repetitions of this stress history up to crack initiation and is obtained by equation (9).

$$D = \sum_k d_k \tag{8}$$

$$N = \frac{1}{D} \tag{9}$$

Non linear Lemaitre & Chaboche's rule

This law can not be used with its initial formulation as it is designed by its authors for uniaxial stress states sequence only. Its necessary adaptation to multiaxial cycles is hereafter proposed. A multiaxial fatigue criterion gives the fatigue life N_k of any cycle and so gives the corresponding fatigue strength $\sigma_{-1}(N_k)$. The procedure defines by this way an uniaxial cycle equivalent to the initial one from the fatigue life point of view. Then the non linear law can be applied.

The Lemaitre and Chaboche's rule which is developed further is regarded as having many advantages as taking into account the cycles occurance order, considering cycles below and over the so-called fatigue limit in different manners, and presenting a non linear damage cumulation. The differential expression of the law gives the increase of damage δD due to δN identical uniaxial stress cycles defined by their amplitude σ_a and their mean value σ_m, as follows :

$$\delta D = \left[1-(1-D)^{\beta+1}\right]^{\alpha}\left[\frac{\sigma_a}{M_0(1-b\sigma_m)(1-D)}\right]^{\beta}\delta N \tag{10}$$

with

$$\alpha = 1 - a\left\langle\frac{\sigma_a - \sigma_A(\sigma_m)}{R_m - \sigma_a - \sigma_m}\right\rangle \tag{11}$$

$\sigma_A(\sigma_m)$ is the fatigue limit for a non-zero mean stress σ_m, corresponding to the endurance constant life diagram that describes the dependency between σ_m and σ_a according to a linear model (Figure 10) :

$$\sigma_A = \sigma_{-1}(1 - b\sigma_m) \tag{12}$$

b, β, a and M_0 are material coefficients. R_m is the ultimate tensile strength. σ_{-1} is the fatigue limit of the material for a reversed tensile test (R = -1).
The symbol < > is defined as <g>=0 if g≤0 and <g>=g if g>0. It gives the expressions of damage for both large amplitude cycles ($\sigma_a > \sigma_A$) and small amplitude cycles ($\sigma_a \leq \sigma_A$).

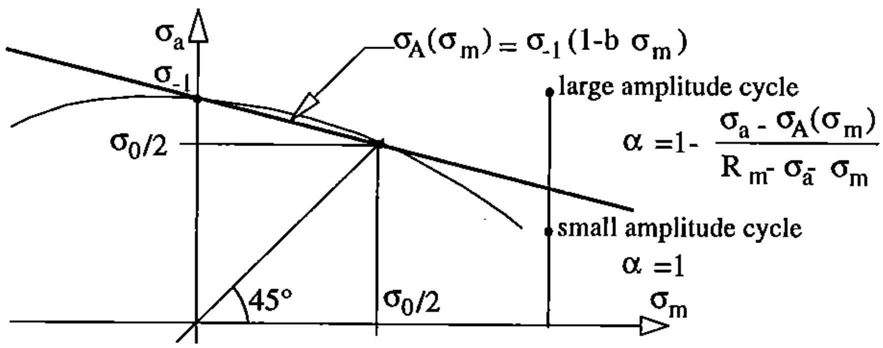

Fig. 10 Endurance constant life diagram.

Equation (10) is integrated. In case of n_j successive small amplitude cycles (σ_{aj}, σ_{mj}, $\alpha = 1$),the damage increases from D_i to D_j. Stating $X_i = 1-(1-D_i)^{\beta+1}$,it can be expressed as :

$$X_j = X_i e^{n_j(\beta+1)\left[\frac{\sigma_{aj}}{M_0(1-b\sigma_{mj})}\right]^\beta} \tag{13}$$

In the case of large amplitude cycles, the damage increase due to n_j cycles (σ_{aj}, σ_{mj}) is expressed as :

$$X_j^{1-\alpha_j} - X_i^{1-\alpha_j} = n_j(1-\alpha_j)(\beta+1)\left[\frac{\sigma_{aj}}{M_0(1-b\sigma_{mj})}\right]^\beta \tag{14}$$

X_i is a parameter that allows to follow the damage evolution even if the corresponding damage D_i is not directly known. From equation (14) can be derived the fatigue life N_{fj} of a reversed tensile test (σ_m=0, D_i=0 and D_j=1) :

$$N_{fj} = \frac{1}{aK_j(\beta+1)}\left[\frac{M_0(1-b\sigma_{mj})}{\sigma_{aj}}\right]^{\beta} \quad \text{with} \quad 1-\alpha_j = aK_j \qquad (15)$$

Both relations (13) and (14) are simultaneously utilized as the continuous damage function X_i^a and the damage D_i are together equal to zero when the material is not damaged and equal to the unit value when the crack occurs. Two parameters $aM_0^{-\beta}$ and β are needed to calculate X_i^a. They are obtained with the $\sigma_{-1}(N)$ S-N curve. The following algorithm (Figure 11) summarizes all the possibilities of the X_i^a function increases. It stops when the damage function X_i^a reaches the unit value.

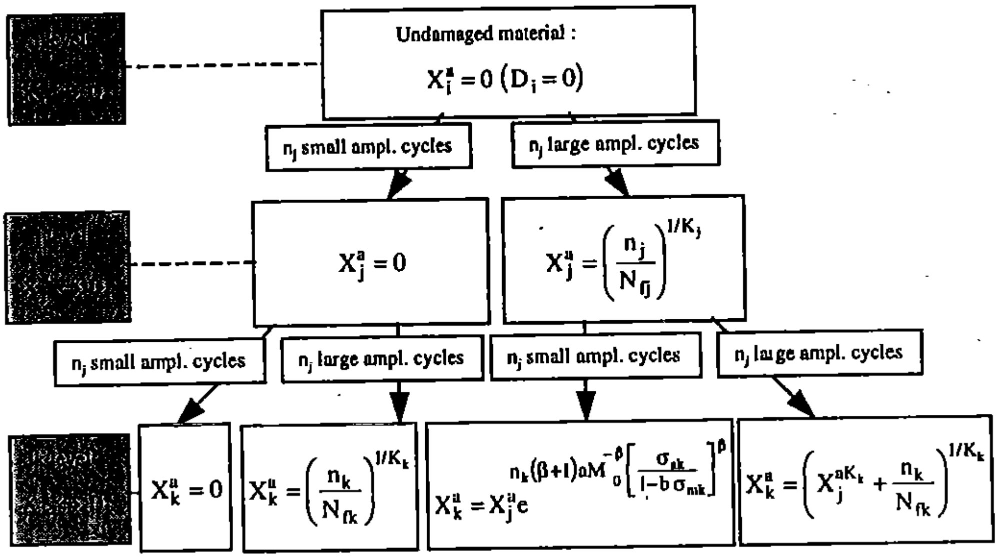

Fig. 11 Algorithm of the non linear damage function.

An optimization of Lemaitre and Chaboche's law algorithm (12) has provided a strong calculation time reduction. The non linear law is now not more time consumming than the linear one.

Validation with biaxial variable amplitude sequences

The first validation of the method has been obtained by the use of biaxial tension-compression variable amplitude tests results. Ten biaxial random stress histories are considered. They are composed of 177,000 up to 190,413 events. These tests were carried out in the laboratory of Professor Macha (Bedkowski (13)) in Opole (Poland). The specimens have a cruciform shape as shown on figure 12.

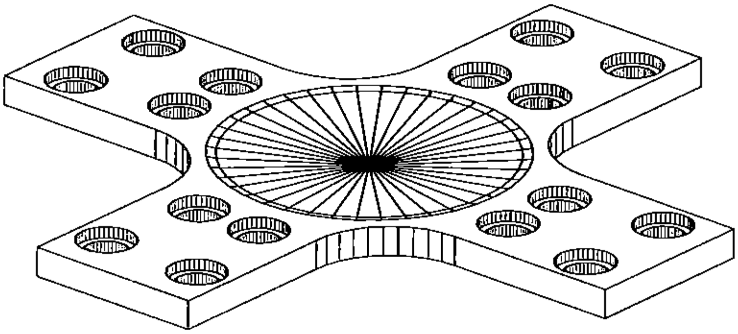

Fig. 12 Cruciform specimen description.

They are made of a low carbon steel 10HNAP (see chemical composition in table 1). Mechanical static properties are presented in table 2.

Table 1. Chemical composition

Elements	C	Mn	Si	P	S	Cr	Cu	Ni
Content [%]	0.115	0.71	0.41	0.082	0.028	0.81	0.30	0.50

Table 2. Mechanical static properties

σ_e [MPa]	R_m [MPa]	ν	E [MPa]
418	566	0.29	215 000

The required material fatigue data are the three S-N curves $\sigma_{-1}(N)$, $\tau_{-1}(N)$ and $\sigma_0(N)$. These give the material fatigue strengths versus the number N of cycles for respectively a reversed tensile test (R = -1), a reversed torsion test (R = -1) and a zero to maximum tensile test (R = 0). Bedkowski and Macha obtained these following S-N curves that can be described as shown on figures 13, 14 and 15.

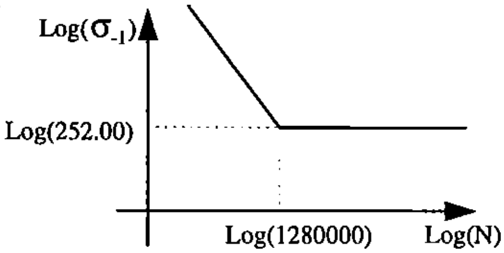

Fig. 13 Reversed tensile S-N curve $\sigma_{-1}(N)$.

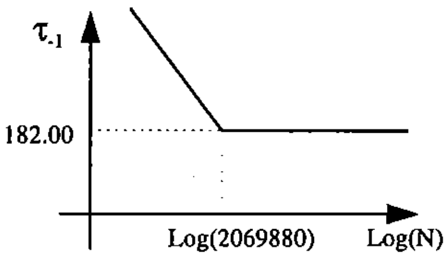

Fig. 14 Reversed torsion S-N curve $\tau_{-1}(N)$.

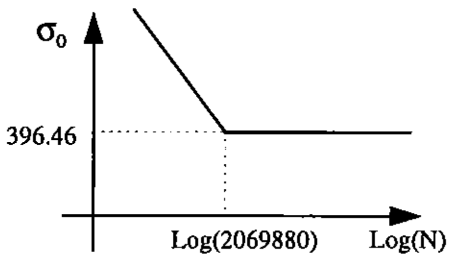

Fig. 15 Zero to maximum tensile S-N curve $\sigma_0(N)$.

As the biaxial variable amplitude fatigue tests involve fixed principal stress directions, the criterion based on the Critical Plane Approach is here the most suitable one for predicting fatigue lives.

Experimental lives of the ten biaxial sequences are compared with the theoretical ones (assessed by using linear and non linear damage models). They are expressed as the number of repetitions of the sequence up to crack initiation. The table 3 summarizes experimental and numerical results which are plotted on the figure 16.

Table 4. Experimental and numerical lives

Sequences	Experimental fatigue life	Numerical fatigue life [number of sequences]	
	[number of sequences]	Miner	Lemaitre and Chaboche
GP9302	3273	5142	650
GP9305	287	189	130
GP9307	398	652	416
GP9308	875	505	369
GP9310	1301	593	390
GP9312	2468	971	610
GP9313	1664	672	548
GP9314	848	322	239
GP9315	267	28	19
GP9619	342	308	167

The ratio between experimental lives and predicted ones varies from 0.6 to 9.5 for the linear damage rule and 1.0 to 14.0 for the non linear damage law. The average ratio is 2.4 and 4.0 respectively. The difference between these assessed lives is induced by the fact that small amplitude cycles contribute to the damage and make decrease the fatigue lives in the case of the non linear law.

The lives are very conservative for one case (sequence GP9315).

The lives calculations are established by using a counting plane defined by angles $\varphi_0=\pi/\sqrt{7}$, $\gamma_0=\pi/\sqrt{19}$ (in radians). A study of the influence of the counting plane has been realized and has shown that the choosen plane was suitable for all the considered stress sequences as it closely corresponds to the one that gives the most conservative fatigue lives.

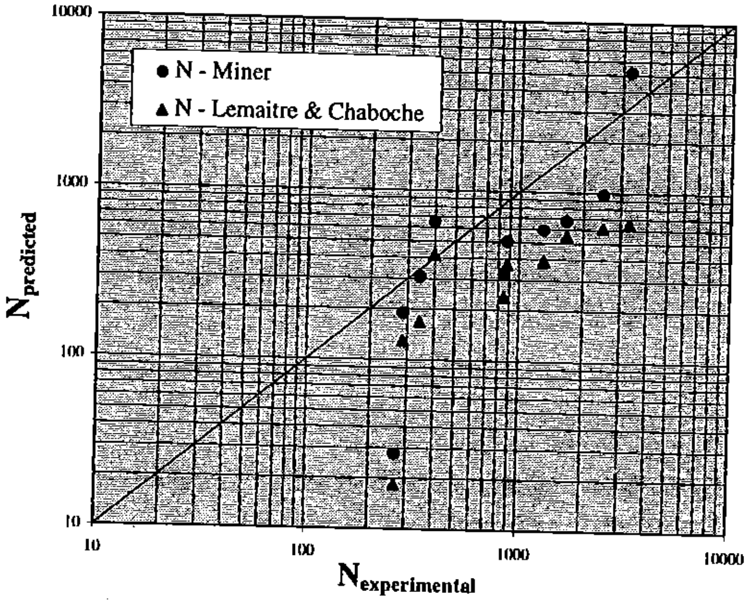

Fig. 16 Assessed lives against experimental ones.

The problem of the determination of the most conservative counting plane is a work currently underway. The purpose is to avoid to repeat the calculations for any possible counting plane. The attention is focussed on the most damaged material plane, determined by a so-called plane per plane damage cumulation (14). All the material planes are not equally damaged during any stress cycle, depending on their orientation. This physical damage distribution all over the planes, induced by the distribution of stresses acting on these planes, leads to the concept of the plane per plane damage cumulation. A possible way to select the counting plane is to take the most damaged plane as the reference counting one.

Conclusions and perspectives

A fatigue life prediction method has been proposed for multiaxial variable amplitude stress histories. It is based upon a counting variable representative of the stress states and of their evolution versus time in order to identify and extract multiaxial cycles. Then finite fatigue lives criteria are used to established the life corresponding to any multiaxial cycle. Two linear and non linear damage laws are usable for the fatigue assessment.

Ten biaxial random stress histories issued from tests carried out on cruciform specimens allow to validate the suitability of the proposed method. The ratio between experimental lives and expected ones has an average conservative value of 2.4 for Miner's rule and 4.0 for Lemaitre and Chaboche's law.

Future work will concern the determination of the most representative counting plane and the influence of the stress gradient effect on the fatigue behaviour of materials.

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