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A mesoscopic approach for fatigue life prediction under multiaxial loading

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ABSTRACT : This paper deals with the presentation of a high cycle multiaxial fatigue life prediction method for metallic materials. By means of the mesoscopic approach introduced by Dang Van and developed by Papadopoulos, accumulated plastic strain due to external loading is estimated at a scale on the order of a grain or a few grains. Its evaluation requires the use of a critical plane type fatigue criterion. As soon as the accumulated plastic mesostrain, considered as the damage variable, reaches a critical value, a crack is considered to be initiated. The complex and combined cases of loading (multiaxial and variable amplitude) can be analysed with this new method. Particular attention is given to a description of the detrimental effect of out-of-phase loadings. A good agreement has been found between the predicted and experimental results for in-phase and out-of-phase sinusoidal constant amplitude loadings by examining a large amount of experimental data.

Notation

Macroscopic quantities :

$\underline{\underline{\Sigma}}$ macroscopic stress tensor

$\underline{\underline{E}}$ macroscopic strain tensor

$\underline{C}$ macroscopic shear stress vector

$\underline{\mathcal{T}}$ macroscopic resolved shear stress vector acting on an easy glide direction

$\mathcal{T}_a$ amplitude of macroscopic resolved shear stress

P macroscopic hydrostatic stress

Mesoscopic quantities :

$\underline{\underline{\sigma}}$ mesoscopic stress tensor

$\underline{\underline{\varepsilon}}$	mesoscopic strain tensor
$\underline{\tau}$	mesoscopic resolved shear stress vector acting on an easy glide direction
γ^p	mesoscopic shear plastic strain
τ_y	shear yield limit of a crystal
Γ	accumulated plastic mesostrain
T_σ	measure proportional to an upper bound of the plastic mesostrain accumulated on an elementary material plane Δ, also average value of $\mathcal{T}_a$
T_Σ	maximum value of T_σ
H	phase-difference coefficient

Introduction

The mesoscopic approach introduced by Dang Van (1) and developed by Papadopoulos (2-3) forms the basis of this study. Multiaxial endurance criteria built according to this theory has been successfully (4) used to predict fatigue behaviour of mechanical components. Nevertheless, these methods only permit to differentiate a damaging cyclic loading from a non damaging one. When a failure event is to be predicted, it is important to know how many cycles must be applied to reach it. Most of the fatigue life prediction methods proposed with this aim in view are built by extending, to the limited fatigue life regimes, endurance criteria expressed in terms of macroscopic mechanical parameters (5). In the high cycle fatigue field, crack initiation is a phenomenon taking place at the scale of a few grains. Consequently, it seems natural to introduce a damage variable computed at this scale. Papadopoulos (2) used the accumulated mesoscopic plastic strain. We will make the same choice to propose a method producing fatigue life prediction for multiaxial constant or variable amplitude loading.

Overview of the mesoscopic approach

To depict fatigue crack initiation phenomenon in polycrystalline metallic materials, two scales of description of a material are distinguished : the usual macroscopic scale and a mesoscopic one. The macroscopic scale is defined with the help of an elementary volume V determined at any point O of a body as the smallest sample of the material surrounding O that can be considered to be homogeneous. Usually, engineers use stresses and strains measured or estimated at this scale. V is containing a large number of grains (crystals) and the mesoscopic scale is defined as a small portion of this volume. In the high cycle fatigue regime, some grains undergo local plastic strain while the rest of the matrix behaves elastically (the overall plastic strain is negligible). It seems, therefore, legitimate to use the scheme of an elastoplastic inclusion submitted to uniform plastic strain $\underline{\underline{\varepsilon}}^{p}$ and embedded in an elastic matrix, both having the same elastic coefficients. If the total strains of the matrix $\underline{\underline{E}}^{e}$ and of the inclusion $\underline{\underline{\varepsilon}}^{e}+\underline{\underline{\varepsilon}}^{p}$ are supposed to be the same (Lin-Taylor hypothesis), it follows that (1) :

$$\underline{\underline{\sigma}} = \underline{\underline{\Sigma}} - 2\mu\underline{\underline{\varepsilon}}^{p} \quad (1)$$

where $\underline{\underline{\Sigma}}$ and $\underline{\underline{\sigma}}$ are the macroscopic and the mesoscopic stress fields, $\underline{\underline{\varepsilon}}^{p}$ is the plastic mesostrain and μ is the shear modulus.

By assuming that only one glide system (defined by a normal vector $\underline{n}$ to a plane and a direction $\underline{m}$ on this plane) is active per every plastically deforming grain of the metal, Papadopoulos (3) established from the last relation a macro-meso passage for a glide system activated in a flowing crystal :

$$\underline{\tau} = \underline{\mathcal{T}} - \mu\gamma^{p}\underline{m} \quad (2)$$

where $\underline{\tau}$ and $\underline{\mathcal{T}}$ are the mesoscopic and macroscopic resolved shear stresses acting along the slip direction $\underline{m}$:

$$\underline{\tau} = \left(\underline{m}.\underline{\underline{\sigma}}.\underline{n}\right)\underline{m} \quad (3)$$

$$\underline{\mathcal{T}} = \left(\underline{m}.\underline{\underline{\Sigma}}.\underline{n}\right)\underline{m} \quad (4)$$

γ^{p} is the magnitude of the plastic mesoscopic shear strain.

A critical plane type fatigue criterion

The ability of a loading to create a macrocrack will be checked here through an endurance criterion based on a critical plane approach and presented by Papadopoulos in reference (3). We have just seen that, during a high cycle fatigue test, some plastically less resistant grains (mesoscopic scale) of V are subjected to plastic glide. The fatigue limit can therefore be related to some characteristic quantities of an elastic shake-down state reached by these plastically deforming crystals. A parameter, denoted as T_σ and proportional to an upper bound of the plastic mesostrain accumulated in some crystals of V, has been introduced (3). It has been shown that the limit to apply on this parameter depends on the maximum value P_{max} that reaches the mesoscopic (equal to the macroscopic) hydrostatic stress during a loading cycle. The criterion is written as :

$$\max_{\theta,\varphi}\left(T_\sigma(\theta,\varphi)\right)+\alpha P_{max}\leq\beta \tag{5}$$

T_σ is a function of the orientation of a material plane Δ through the angles θ and φ, spherical co-ordinates of the unit normal $\underline{n}$ to the plane Δ :

$$\underline{n}=\begin{pmatrix}\sin\theta\cos\varphi\\ \sin\theta\sin\varphi\\ \cos\theta\end{pmatrix} \tag{6}$$

$T_\sigma(\theta,\varphi)$ is estimated by an integration carried out through the whole area of the plane Δ :

$$T_\sigma(\theta,\varphi)=\sqrt{\int_{\psi=0}^{2\pi}\mathcal{T}_a^{\,2}(\theta,\varphi,\psi)d\psi} \tag{7}$$

Where $\mathcal{T}_a$ is the amplitude of the macroscopic resolved shear stress acting on a line of the plane Δ directed by $\underline{m}$ (figure 1). This line is located by the angle ψ that makes with an arbitrary but fixed axis in Δ.

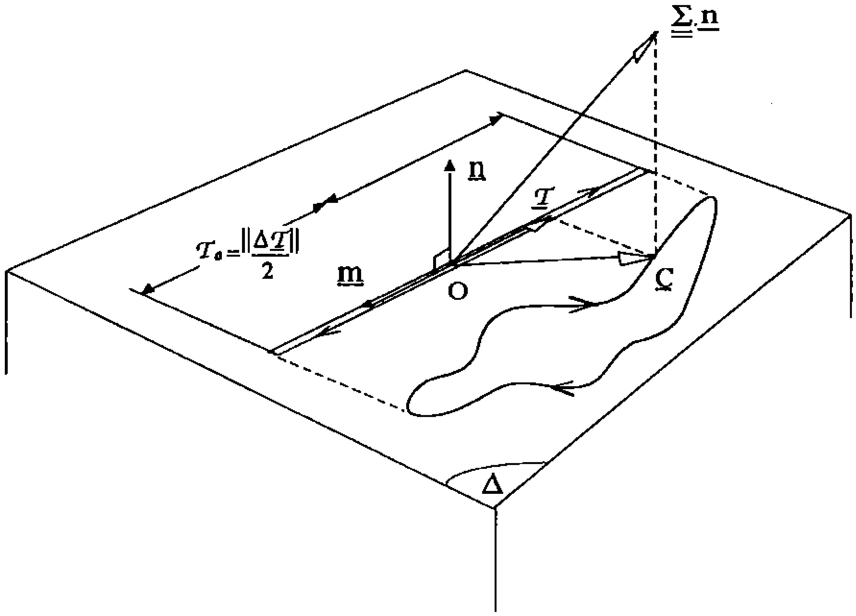

Fig.1 Path of the macroscopic shear stress C acting on a material plane Δ and corresponding path of the macroscopic resolved shear stress 𝒯 acting on an easy glide direction.

The material parameters α and β can be related to the fatigue limits of two standard fatigue tests, for example fully reversed tension-compression, s, and fully reversed torsion, t :

$$\alpha = \sqrt{\pi}\,\frac{t - \frac{s}{2}}{\frac{s}{3}} \qquad \beta = \sqrt{\pi}\; t \tag{8}$$

Hereafter, in order to make relations less cumbersome, maximum value of T_σ will be denoted as T_Σ:

$$T_\Sigma = \max_{\theta,\varphi}\left(T_\sigma(\theta,\varphi)\right) \tag{9}$$

Life prediction assessment procedure

Sinusoidal constant amplitude loading case

We first consider the synchronous sinusoidal loadings defined by :

$$\Sigma_{ij}(t) = \Sigma_{ijm} + \Sigma_{ija} \sin\left(\omega t - \beta_{ij}\right) \qquad i,j = x,y,z \qquad (10)$$

where Σ_{ija} and Σ_{ijm} are amplitudes and mean values of the (i,j) stress components and β_{ij} the phase differences between (i,j) stress component and a reference stress Σ_{xx} (β_{xx}=0). On the critical material plane Δ_c related to the maximum measure T_Σ, the loading path described by the shear stress vector is elliptic and the corresponding amplitude of the shear stress, defined as half of the longest chord of the closed curve, is denoted as C_A.

Definition of a multiaxial limit loading

A particular multiaxial loading can be defined according to the endurance limit concept applied to the criterion (T_Σ, P_{max}). If sinusoidal constant amplitude loadings are defined by the same mean values Σ_{ijm} and the same phase angles β_{ij} between the stress components and a simple multiplicative coefficient is applied to all the amplitudes then, in the plane (T_Σ, P_{max}), they are displayed by points lying on a same line (figure 2). These loadings are said to be "similar". The multiaxial limit loading is defined as one of these loadings. It is displayed by a point which belongs to the threshold endurance line delimiting the domain of safe operation against fatigue. The corresponding mechanical parameters are denoted as $T_{\Sigma lim}$ and $(P_m + P_a)_{lim}$ where $T_{\Sigma lim}$ is function of the ratio $\frac{T_\Sigma}{P_a}$ and of the mean of the hydrostatic pressure P_m :

$$T_{\Sigma lim} = \frac{-\alpha P_m + \beta}{\alpha + \frac{T_\Sigma}{P_a}} \frac{T_\Sigma}{P_a} \qquad (11)$$

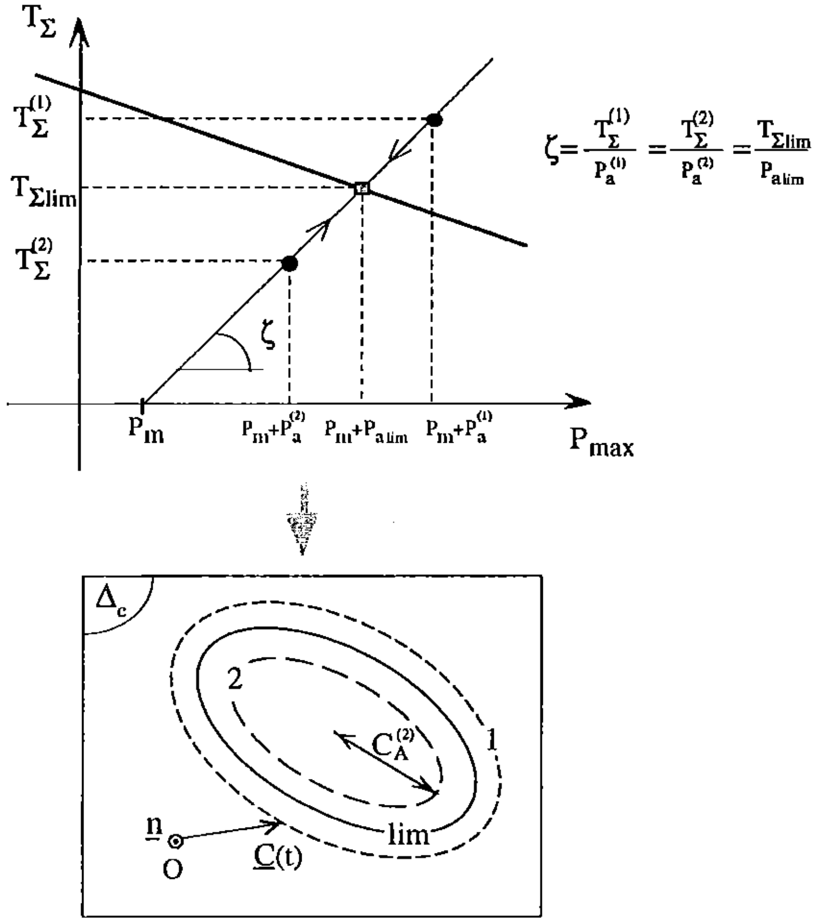

Fig.2 Determination of the limit loading characteristics in the plane of the endurance criterion (T_Σ, P_{max}) from two similar loadings and corresponding elliptic paths on the critical material plane Δ_c.

A newly defined phase-difference coefficient

For a similar loadings group described above, one can show (6) that the ratio $\frac{T_\Sigma}{C_A}$ remains constant. It will constitute a phase-difference coefficient denoted as H :

$$H = \frac{T_\Sigma}{C_A} \tag{12}$$

The more the elliptic path is open, the higher is the coefficient H (figure 3). For a proportional loading, H is equal to $\sqrt{\pi}$. In the case of a particular circular path, H reaches the maximum value $\sqrt{2\pi}$. The linear path and the circular one lead to two bounds of the coefficient H. Since H is the same for these two similar loadings, it follows that :

$$\frac{T_{\Sigma}}{C_A} = \frac{T_{\Sigma \lim}}{\tau_{\lim}} \tag{13}$$

where $\tau_{\lim}$ is the amplitude (on the critical plane) of the macroscopic shear stress for the limit loading. From the last relation, expression of $\tau_{\lim}$ can be deduced :

$$\tau_{\lim} = \frac{T_{\Sigma \lim}}{H} \tag{14}$$

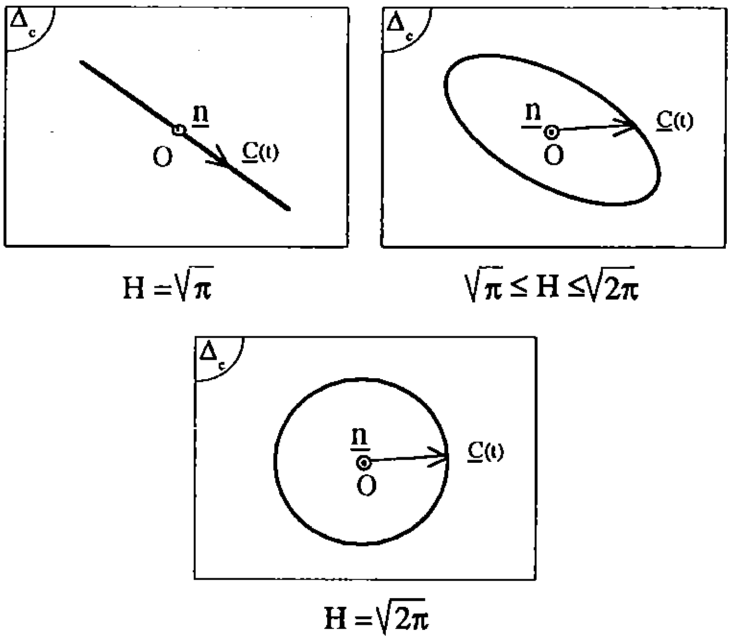

Fig.3 Different paths and corresponding phase-difference coefficient H values.

For a constant value $T_{\Sigma lim}$, τ_{lim} decreases while H increases. The coefficient H is able to reflect the influence of the path shape on the parameter τ_{lim} related to the endurance limit. If we imagine that each direction on the critical plane Δ_c is related to an easy glide direction of a crystal and that only one glide system is active per every plastically deforming crystal, then the use of the parameter T_σ (considering all the directions of a material plane) in τ_{lim} estimation can be understood as a precise description of the contribution of many grains to damage mechanism. When the crystals are equally stressed (circular path), τ_{lim} reaches its minimum value because H is maximum.

Damage estimation and initiation criterion

Initiation of fatigue cracks in metals is known to be a consequence of cyclic plastic strain localisation (7). The cumulative plastic mesostrain will then be considered as the principal cause of damage accumulation. In the same way as in Papadopoulos' work (3), the crystal is assumed to follow a combined isotropic and kinematical rule when flowing plastically and the initiation of slip in the crystal is determined by Schmid's law. A crystal starts to deform plastically when the shear stress acting on the slip plane in the slip direction, reaches a critical value denoted as τ_y. Three successive linear isotropic hardening rules are adopted to describe the crystal behaviour from initial yield to failure. The yield limit starts to increase in the initial hardening phase, remains constant in the saturation phase (represented by τ_s) and then decreases in the softening phase (figure 4 b)). The crystal is said to be broken as soon as the yield limit becomes negligible. By using the macro-meso passage of the relation (2) and linear isotropic and kinematical hardening rules, damage (accumulated plastic mesostrain) evolutions in the three phases can be drawn (6) (2) (figure 4a)).
It is assumed that crack initiation occurs by the breaking of the most stressed grains along the plane experiencing T_Σ (maximum value of T_σ) and that only one glide system operates in them (8). Consequently, it seems natural (on this critical plane) to be interested in these plastically less resistant grains whose easy glide directions coincide with the direction leading to the maximum value of the macroscopic resolved shear stress C_A. Once the accumulated plastic mesostrain Γ along this

particular gliding system reaches a critical value Γ_R, these grains are said to be broken and an analytical expression of the number of cycles to initiation can be achieved (6) (2) :

$$\Gamma = \Gamma_R \quad \Rightarrow \quad N_i = p \ln\left(\frac{C_A}{C_A - \tau_{lim}}\right) + q\,\frac{\tau_{lim}}{C_A - \tau_{lim}} - \frac{r}{C_A} \qquad (15)$$

where p, q and r are functions of the hardening parameters of the three phases defined above.

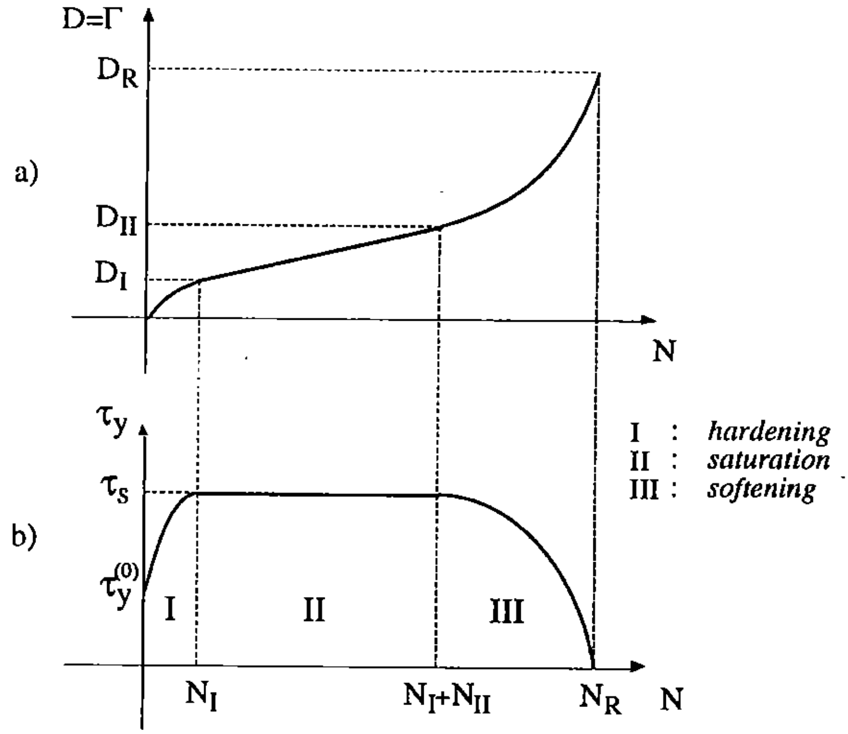

Fig.4 a) Damage and b) yield limit evolutions (mesoscopic scale) in the three behaviour phases (hardening, saturation and softening) when a cyclic loading is applied.

In the last relation, the detrimental effect of out-of-phase loadings are introduced through τ_{lim}. As the coefficient H increases, τ_{lim} as well as N_i decrease so more damage is accumulated. The identification of the model parameters requires two endurance limits (parameters α and β of the endurance criterion) and a single S-N curve (parameters p, q and r). The location of the critical plane by means of the measure T_σ is the first step of the life prediction procedure. C_A, T_Σ and the phase-difference coefficient H are computed on this plane. Afterwards, τ_{IIm} and $T_{\Sigma\text{IIm}}$ estimations are carried out according to the endurance criterion (T_Σ, P_{max}). Finally, N_f is readily deduced from (15).

Applications

A number of data concerning multiaxial (out-of-phase or in-phase) constant amplitude fatigue loading has been found in the scientific literature (9-12). The data are from bending-torsion tests, tension-torsion tests and from tests on thin-walled tubes under internal pressure and axial load. Figure 5 shows the comparison between estimated and measured lives.

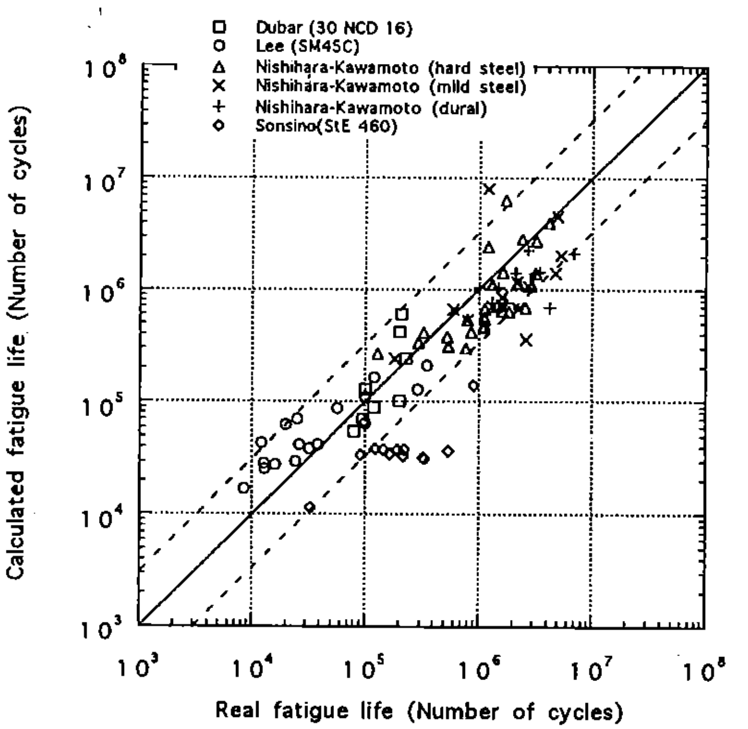

Fig.5 Calculated fatigue life versus fatigue life from multiaxial constant amplitude fatigue tests.

The diagonal represents a perfect agreement between the model and experimental results. The dashed line determine an interval of agreement by a factor of three. It should be noted that the eight points related to Dubar's data have been evaluated through a statistical analysis on 200 tests. Every other point on the graph represent a failed specimen. The theoretically predicted lives using the proposed model and the experimentally measured ones are in fairly good agreement except for Sonsino data. It appears through this case (where only a few S-N curve data are available) that accurate statistical evaluation of the fatigue limits used in the model is an essential condition to good life predictions. In conclusion, more experimental work is needed to fully assess the model's behaviour and accuracy.

Variable amplitude loading case

In general, service loadings which is applied to mechanical components vary irregularly with time. For instance, suspension arms of a car, are exposed, during their service life, to a large number of cycles of variable amplitudes, caused by external forces and resulting in the possibility of fatigue cracking. The prevention of such fatigue cracks and failures therefore requires relevant fatigue life prediction method. An extension of the previously described method to variable amplitude multiaxial stress history can be proposed by using statistical parameters.

Damage accumulation

With the present method, damage accumulation is still carried out by adopting three successive linear isotropic hardening rules of crystal behaviour (figure 4). Besides, the mechanical parameters representative of the loading and used for damage accumulation are the macroscopic resolved shear stress on a particular gliding system and the hydrostatic stress. The yield limit acts as a filter that defines the part of a transition leading to damage. Figure 6 shows, for a complex loading, the evolution of the macroscopic resolved shear stress $\mathcal{T}(t)$ on a particular gliding system and the yield limit $\tau_y^{(i)}$ reached at the i^{th} extremum. The segment length denoted as $\Omega_{i\to i+1}$ is proportional to the plastic mesostrain $\Gamma_{i\to i+1}$ accumulated during the transition from i to i+1 :

$$\Gamma_{i\to i+1} \propto \Omega_{i\to i+1} \tag{16}$$

and

$$\Omega_{i\to i+1} = |\mathcal{T}_{i+1} - \mathcal{T}_i| - 2\tau_y^{(i)} \tag{17}$$

where $\mathcal{T}_i$ and $\mathcal{T}_{i+1}$ are values of the extrema i and i+1.

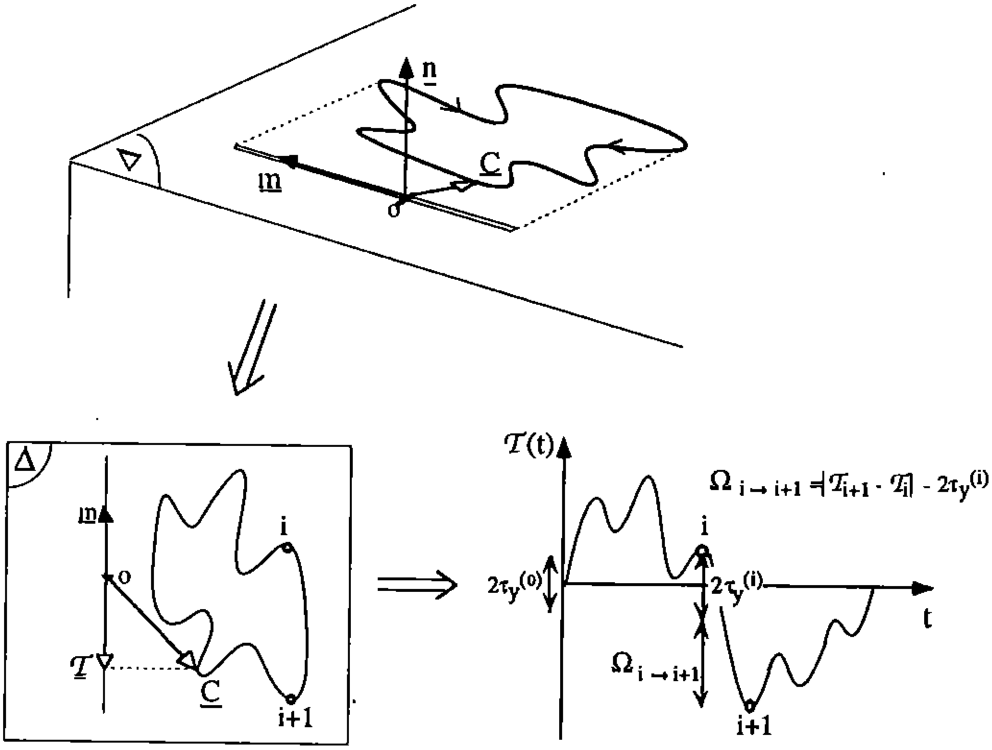

Fig.6 Accumulation of plastic mesostrain when a complex loading is applied.

If this sequence is applied successively until failure, the yield limit will first increase in the hardening phase, remain constant during saturation and decrease in the softening phase. With the present way of accumulating damage, no counting method is required. Indeed, damage is deduced step by step from the hardening rules. This fact is quite new because most of the fatigue life prediction methods in the literature apply successively a counting method and a damage law without any links between them.

Statistical analysis

Complete knowledge of hardening rules is achieved when the saturation phase yield limit is known. Its estimation requires the introduction of statistical parameters. The mechanical factors T_σ and C_A are no more relevant when loadings vary irregularly with time. At a first approximation, the root mean square value of the macroscopic resolved shear stress denoted as $\mathcal{T}_{rms}$ seems more convenient in this case :

$$\mathcal{T}_{rms} = \sqrt{\frac{1}{N}\sum_i \left(\mathcal{T}_i - \mathcal{T}_{mean}\right)^2} \tag{18}$$

where $\mathcal{T}_i$ are the peak and valley values of $\mathcal{T}(t)$ evolution, N their number and $\mathcal{T}_{mean}$ the mean value defined by :

$$\mathcal{T}_{mean} = \frac{1}{N}\sum_i \mathcal{T}_i \tag{19}$$

In the expression of T_σ previously mentioned, $\mathcal{T}_a$ is now replaced by $\mathcal{T}_{rms}$ and, by this way, a new parameter denoted as $T_{\sigma rms}$ can be defined :

$$T_{\sigma rms}(\theta,\varphi) = \sqrt{\int_{\psi=0}^{2\pi} \mathcal{T}_{rms}^2(\theta,\varphi,\psi)d\psi} \tag{20}$$

Like T_σ, $T_{\sigma rms}$ is a function of the orientation of the material plane Δ through the angles θ and φ, spherical co-ordinates of the unit normal $\underline{n}$ to a plane Δ. The maximum value of $T_{\sigma rms}$ will be denoted as $T_{\Sigma rms}$:

$$T_{\Sigma rms} = \max_{\theta,\varphi}\left(T_{\sigma rms}(\theta,\varphi)\right) \tag{21}$$

The critical material plane on which damage estimation will be carried out corresponds to the measure $T_{\Sigma rms}$. On this plane denoted as Δ_c, a variable amplitude loading generates a macroscopic shear stress vector path of complex shape. A "global" phase-difference coefficient, denoted as H, can be introduced to take into account the "out-of-phase content" of this complex loading sequence. H is equal to the ratio between $T_{\Sigma rms}$ and C_{rms} where C_{rms} is the maximum value of $\mathcal{T}_{rms}$ on the critical plane :

$$H = \frac{T_{\Sigma rms}}{C_{rms}} \tag{22}$$

As in the previous case of sinusoidal constant loading, H is bounded by $\sqrt{\pi}$ and $\sqrt{2\pi}$.

From this point, $T_{\Sigma lim}$ and τ_{lim} computations follow the same procedure as previously, T_Σ and P_a being replaced respectively by $T_{\Sigma rms}$ and P_{rms} (P_{rms} is the root mean square value of the hydrostatic pressure).

Fatigue life prediction steps (figure 7)

Required material fatigue characteristics are two endurance limits, a S-N curve and a particular two constant amplitude blocks loading test. The first step of the fatigue life prediction procedure is once again the location of the critical material plane through a maximization of the parameter $T_{\sigma rms}$. After $T_{\Sigma rms}$, P_{rms}, $T_{\Sigma lim}$ and τ_{lim} computations, damage accumulation can be estimated from the evolution of the macroscopic resolved shear stress $\mathcal{T}(t)$ on a direction of the critical plane. The number of sequences to crack initiation is deduced from a calculation on the direction leading to the highest accumulated plastic mesostrain.

Discussion

Difficulty in the application of a linear damage rule (Miner rule) results from improper characterization of damaging events and from not taking the interaction effect into consideration. With our model, some sequence effects are considered. In fact, yield limit evolution in the hardening and softening phases leads to a non linear damage accumulation. Sequence effects depend on the material behaviour phase reached in the plastically deforming crystals. On the contrary, in the saturation phase, no sequence effect occurs and damage accumulation is linear.

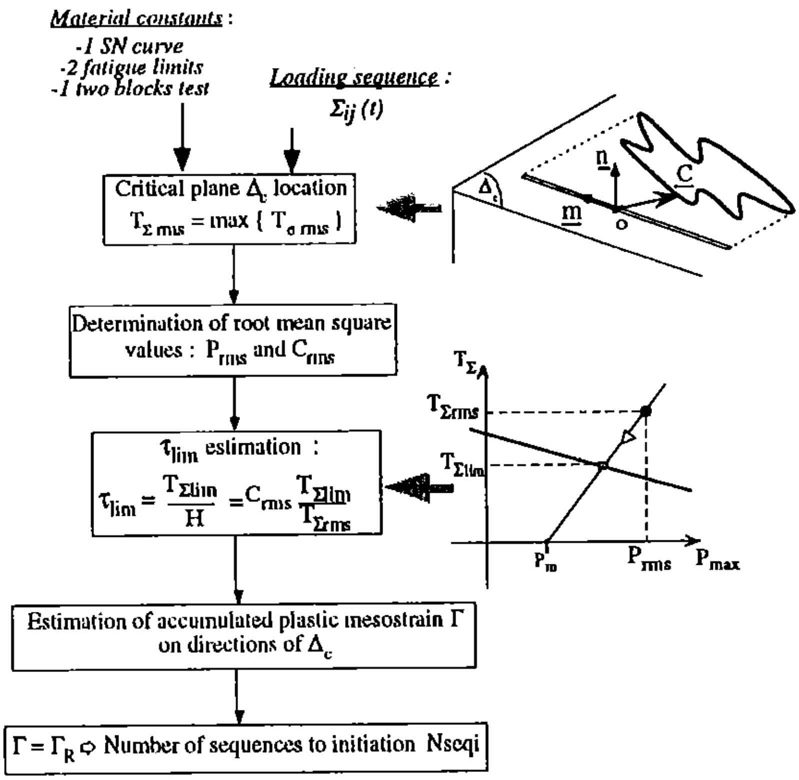

Fig.7 Diagram of algorithm for evaluation of fatigue life of metals under (uniaxial or multiaxial) variable amplitude loading.

The rainflow counting method first proposed by Endo (13) was initially applied to the case of low cycles fatigue. The author considered that a successive application of plastic strain was the principal cause of fatigue crack initiation. Load cycles directly linked to the apparition of closed hysteresis loops in the stress-plastic strain plane was then extracted. In the proposed method, we stay closed to the first idea of Endo. Although the macroscopic strain is purely elastic in the high cycle fatigue field, a loading range responsible for plastic mesostrain can be defined with the help of the macro-meso passage.

Conclusions

A new fatigue life prediction method that can be applied to any kind of loading (multiaxial and variable amplitude) is presented in this paper. A crack is supposed to initiate by failure of some plastically deforming grains following three successive phases of behaviour : hardening, saturation and softening. The damage variable chosen is the plastic strain accumulated at a mesoscopic scale and its estimation requires the location of the plane subjected to maximum damage. This original proposal has been built to take into account all the damage influencing parameters. A newly defined coefficient is introduced to represent the effect of phase difference on damage accumulation. Besides, sequence effects are reflected by the non linear damage accumulation in two of the three behaviour phases adopted.

The predicted results for in-phase and out-of-phase cyclic loading have been compared with the experimental results and the agreement is found to be rather good.

A critical plane type fatigue criterion has been used in the paper but an extension to a volume type approach is in progress.

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