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Probabilistic Design Methods for Multiaxial, High-cycle Fatigue

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ABSTRACT The purpose of this paper is to present high-cycle fatigue strength prediction methods. First, fatigue criteria are discussed, and the relative advantages and disadvantages of Goodman's, Haigh's, Dang Van's and other criteria are presented. Secondly, it is shown how probabilistic methods such as the Monte-Carlo method associated with importance sampling methods can be used to provide fatigue fracture probabilities. Last, an application of a fatigue fracture probability evaluation is given for an automotive suspension arm, and comparison with experimental results is made.

1 Introduction

In the restricted area of high-cycle fatigue of metallic structures, there remains at least three major problems for which the design engineer lacks adequate solutions.

(1) The definition of a multiaxial fatigue criterion and the choice of a safety factor, neither too severe nor too low.
(2) The evaluation of damage due to variable amplitude fatigue.
(3) The evaluation of the effect of residual stresses and surface conditions on the fatigue behaviour of structures.

The purpose of this paper is to present some methods recently developed to address the first problem.

Since the general tendency nowadays is to evaluate potential fatigue problems in the very early design stages of the component, the proposed algorithm has been computerized and gathered in a fatigue routine associated with a finite-element code. This fatigue routine named SOLSTICE (SOLlicitations, STructures Industrielles et Criteres d'Endurance) has deliberately been made as user-friendly as possible, and can be used by any designer who does not need any special knowledge of fatigue theories to use the results produced by the algorithm.

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The goals sought in this paper are

- to give a method for the evaluation of the fatigue damage associated with multiaxial, high-cycle, constant-amplitude fatigue;
- to give a method for the probabilistic evaluation of the fatigue fracture risk;
- to present an example of application on a car suspension arm.

2 Criteria for Constant Amplitude Load

2.1 *Problems linked to Haigh's and Goodman's criteria in the case of multiaxial loads*

Experience shows that Haigh's and Goodman's criteria, as ancient as they may be, are still the most popular for fatigue design because of their simplicity and the satisfactory results they provide on sample testing. Such has been their success that they are now used often in areas and for purposes which their authors did not intend. Although engineers have learned with time and usage the degree of trust they can give to these two criteria, and the necessary safety factors which need to be used, some utilizations especially in the area of multiaxial fatigue seem somewhat hazardous and unjustified. Examples will make these problems more clear.

Let us consider a sample undergoing 90° out-of-phase push-pull and torsion. Experiments **(8)** have shown that the fatigue limit is considerably lower than that of push-pull alone or reverse torsion alone, but Haigh's and Goodman's criteria do not account for multiaxial stresses. Let us now assume a pure shear loading for which the stress tensor is

$$\boldsymbol{\sigma} = \begin{bmatrix} 0 & \tau & 0 \\ \tau & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{1}$$

or, in a diagonalized referential

$$\boldsymbol{\sigma}' = \begin{bmatrix} \tau & 0 & 0 \\ 0 & -\tau & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{2}$$

It is known from Sines **(9)** that a mean shear stress does not affect fatigue resistance of the structure. Since Haigh's or Goodman's criteria can process only one stress component, if such a criterion were to be applied on component $\sigma_{11} = \tau$ of $\boldsymbol{\sigma}'$, interpreted as a local tension, then the mean stress would be taken into account and a shear stress τ would be compared to a push-pull fatigue resistance.

In any case, finite-element software delivers full six-component stress tensors, which cannot be processed correctly with uniaxial fatigue criteria.

1.2 *Proposed criteria: Sines, Crossland* (**10**), *Dang Van, Papadopoulos* (**11, 12**), *Double-diameter* (**13, 14**).

This deficiency has been detected and mentioned in the past by many authors, and fatigue specialists have proposed various methods to solve the problem. Our approach is based on the theoretical work of Dang Van (**1**). Only a brief summary of the method will be given here.

In Dang Van's method, assumption is made that the structure is subjected to high-cycle constant amplitude fatigue which means implicitly that the overall behaviour remains elastic. The main hypothesis of the authors is that a fatigue crack will initiate if and only if the material does not reach locally elastic shakedown. The local stress tensor $\boldsymbol{\sigma}$ is computed through a Hill–Mandel approach, and further hypotheses are made to make $\boldsymbol{\sigma}$ accessible directly from the macroscopic stress tensor. The local equivalent shear stress τ_c is then computed with either Tresca's or von Mises' plastic yield criterion, from which are derived respectively either Dang Van's or Papadopoulos' criterion.

The authors also introduce, on an experimental basis, hydrostatic pressure as an important factor influencing fatigue crack initiation. The final expression of the criterion is

$$\tau_c + \alpha.p < \beta \tag{3}$$

where p_{max} is the maximum hydrostatic pressure reached during the cycle, and α and β two material coefficients which can be identified from two separate fatigue test results such as reversed shear and alternate bending.

Sines' and Crossland's criteria, based on phenomenological observations, calculate τ_c as

$$\tau_c = \max_{(t_1,\, t_2)} [\text{Von Mises}(s(t_1) - s(t_2))] \tag{4}$$

where (t_1, t_2) are two instants of the fatigue cycle, and s is the deviatoric part of the stress tensor

$$S = \boldsymbol{\sigma} - \text{tr}(\boldsymbol{\sigma}).1 \tag{5}$$

The criteria are expressed by

$$\text{Sines:} \qquad \tau_c + \alpha.p_{av} < \beta \tag{6}$$

$$\text{Crossland:} \qquad \tau_c + \alpha.p_{max} < \beta \tag{7}$$

where p_{av} is the average hydrostatic pressure over the fatigue cycle.

The 'double-diameter' formulation is another extension of these criteria. After τ_c is calculated as in Crossland's criterion, and if t_1 and t_2 are the two instants for which τ_c is maximum τ_c' is defined as

$$\tau_c' = \max_{(t_1,\, t_2)} [\mathit{Von\ Mises}(s'(t_1) - s'(t_2))] \tag{8}$$

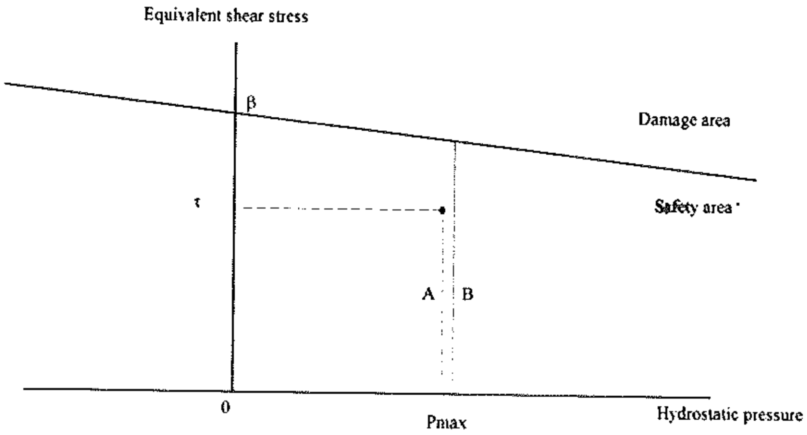

Fig 1 **(p, τ) diagram.**

where $s'(t)$ is the deviatoric part of $\boldsymbol{\sigma}'(t)$ defined as the projection of the stress tensor $\boldsymbol{\sigma}(t)$ on a sub-space perpendicular to $\boldsymbol{\sigma}(t_2) - \boldsymbol{\sigma}(t_1)$.

The expression of the criterion is

$$\sqrt{\tau_c^2 + \tau_c'^2} + \alpha . p_{max} < \beta \tag{9}$$

Details about the algorithms required to compute τ_c from Σ have been given in Ref. (14). A convenient way to understand these criteria is to plot the point $P(p_{max}, \tau_c)$ in an equivalent shear stress vs. hydrostatic pressure diagram. Fatigue will occur at the specified number of cycles if point P is above the limiting line defined (Fig. 1) by term

$$\tau + \alpha . p = \beta \tag{10}$$

2.3 *Algorithm for the evaluation of the fatigue criteria*

The algorithm requires the stress tensor for at least two instants in the cycle, since fatigue can occur only if the stress is fluctuating.

Let $\boldsymbol{\sigma}(t_i, l)$ be the stress tensor at time t_i, $i \in [1, m]$, computed at node or element l of the mesh ($l \in [1, L]$). Let t be the fatigue limit in reverse shear and f the fatigue limit in rotating bending. The algorithm requires no other data to proceed. Compute material coefficients α and β as

$$\alpha = \frac{t - f/\sqrt{3}}{f/3}$$

$$\beta = t \tag{11}$$

Compute the hydrostatic pressure p_i at each instant t_i. p_{max} is defined as

$$p_{\max} = \max_{i \in [l,\, m]} (p_i) \tag{12}$$

Compute the equivalent shear stress τ_c with the algorithm corresponding to whichever criterion was chosen. Full details are given in reference **(14)**.

The value of the criterion can be made personal to the user. The authors have found that the quantity

$$C_s = \frac{\tau_c + \alpha p_{\max} - \beta}{\beta - \alpha p_{\max}} = \frac{A - B}{B} \tag{13}$$

is convenient. Negative values mean safety and are viewed in blue on the majority of postprocessing softwares, whilst the opposite, positive values, mean danger and are viewed in red on the same devices.

3 Probabilistic Evaluation of Fatigue Fracture Initiation

3.1 *Safety factor and reliability*

There are two main problems in the high-cycle fatigue area which require the use of probabilistic methods: the choice of the safety factor in design and the calculation of the reliability of structures.

The safety factor problem

The safety factor C_s is in itself a problem for designers. It is usually defined as a reducing factor for the stress σ with respect to the fatigue limit σ_y

$$C_s \sigma < \sigma_y \tag{14}$$

Safety factors are supposed to account for

- computation errors in the stress evaluation;
- scatter on the material resistance;
- scatter on the dimensions;
- scatter on the loading.

Since safety factors are usually adopted once and for all, it is important to give their choice the greatest possible care: a too large safety factor means taking penalties in weight and costs; a too low safety factor means taking risks of fatigue fracture of the structure.

A probabilistic approach taking into account the scatter on the parameters mentioned above can give a good idea of the safety factors which ought to be taken into consideration in fatigue design.

Reliability

Let us assume that the fatigue resistance decreases with the number of cycles, as is usual with materials like steel. A computation of the probability for fatigue

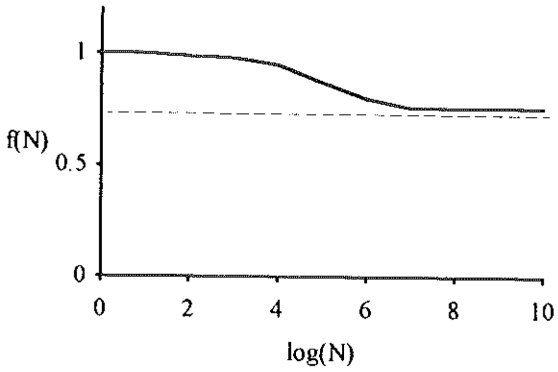

Fig 2 Reliability.

fracture initiation can be made for the different number of cycles, leading to a reliability curve such as the one presented in Fig. 2.

If the fatigue strength is constant and equal to the fatigue limit for a number of cycles above 10^7, which is the case for steel, the reliability is also constant above this value. This result is somewhat inconsistant with Weibull life distributions currently used to describe reliability of specimens.

3.2 *Monte-Carlo method and importance sampling methods*

Probabilistic approach

From the considerations above, it seems reasonable to use statistical approaches to determine the probability of a given structure for fatigue crack initiation. With the recent increase of computer performances, a tool such as the Monte-Carlo method has become particularly appropriate to compute probabilities when the analytical approach is too complex. In the present case, the distributions of the random variables α, β, τ_c, cannot be computed analytically.

Let us recall briefly the principle of the method when it is applied to our problem: it consists in simulating a large population of individuals, with characteristics (load, dimensions, mechanical resistance) randomly distributed according to the known data (if they exist), and in evaluating on this sample population the value of a given result.

The Monte-Carlo method

Let X_1, X_2, ..., X_p be the random variables of the problem such as load, geometry, material characteristics or other. Let us assume that the probability densities of the variables X_i are known and equal to $f_1(x), \ldots, f_p(x)$.

Let X be the random vector with coordinates ${}^t(X_1, X_2, \ldots, X_p)$. Its probability density is

$$P(\mathrm{d}x) = \prod_{i=1}^{p} f_i(x_i).\mathrm{d}x_i = p(x).\mathrm{d}x_1 \ldots \mathrm{d}x_p \tag{15}$$

The problem to solve is the determination of the probability for $Y > 0$, where Y is one of the fatigue criteria mentioned above.

If $Z(\mathbf{X})$ is defined as

$$\begin{cases} Z(X) = 1 & \text{if} \quad Y \geqslant 0 \\ Z(X) = 0 & \text{if} \quad Y < 0 \end{cases} \tag{16}$$

then the problem is to find $\bar{Z} = P_{\text{fracture}}$.

Hereafter, $\bar{Z}$ or $\langle Z \rangle$ will denote the expected value of the random variable Z

$$\bar{Z} = \int Z(\mathbf{x}).p(\mathrm{d}\mathbf{x}) \tag{17}$$

Let us carry out n calculations, the results of which is a sample of n values of $X: X_1, \ldots X_n$.

Let us define $\bar{Z}_n$ as

$$\bar{Z}_n = \frac{1}{n} \sum_{i=1}^{n} Z(\mathbf{X}_i) \tag{18}$$

$\bar{Z}_n$ is also itself a random variable, and it can be shown that $\bar{Z}_n$ is an unbiased estimation of $\bar{Z}$ and if $\sigma(Z)$ is the standard deviation of Z, then

$$\sigma^2(\bar{Z}_n) = \frac{\sigma^2(Z)}{n} = \frac{\langle Z^2 \rangle - \langle Z \rangle^2}{n} \tag{19}$$

Bienaymé–Tchebychev's formula yields

$$P(|\bar{Z}_n - \bar{Z}| \geqslant \varepsilon) \leqslant \frac{\sigma^2(Z)}{n\varepsilon^2} = \tau \tag{20}$$

$(1 - \tau)$ is the level of confidence and ε is the precision required on $\bar{Z}$.

It is interesting to deduce ε from the preceding formula

$$\varepsilon = \frac{\sigma(Z)}{\sqrt{n\tau}} \tag{21}$$

which means that for a given level of confidence $(1 - \tau)$ the precision increases as the square root of the size of the sample: to increase precision by one order of magnitude, the size of the sample will have to be multiplied by 100. This is the most serious disadvantage of the Monte-Carlo method. However, for a given τ and ε, it is possible to reduce the size of the sample by reducing the standard deviation $\sigma(Z)$. Such methods, called 'importance sampling methods', will be detailed in the next paragraph.

Remark: A rough estimation of the precision on $\bar{Z}_n$ can be deduced from the fact that $n\bar{Z}_n$ is distributed with a probability density equal to a binomial law $b(.; n, \bar{Z})$. Bienaymé–Tchebychev's formula thus becomes

$$P(|\bar{Z}_n - \bar{Z}| \geqslant \varepsilon) \leqslant \frac{\bar{Z}(1-\bar{Z})}{n\varepsilon^2} \leqslant \frac{1}{4n\varepsilon^2} \tag{22}$$

Let us assume that $P_{\text{fracture}} = \bar{Z} \approx 10^{-3}$, that $\varepsilon = 10^{-4}$, and $(1-\tau) = 90\%$; the formula above yields

$$n \approx 10^6 \tag{23}$$

3.3 *Importance sampling methods*

As mentioned above, the idea of these techniques is to reduce the size of the sample by favouring those events which are the less probable but the most influential on the final result, and to disfavour the others, providing adequate correction coefficients are introduced.

The probability density of X can also be written

$$P(\mathrm{d}x) = \prod_{i=1}^{p} \frac{f_i(x_i)}{f_i^*(x_i)} f_i^*(x_i).\mathrm{d}x_i \tag{24}$$

If the random variables $X_1, \ldots, X_p$, are each picked out with probability densities f_i^*, X will not be distributed according to its 'natural' probability density $P(\mathrm{d}x)$, but according to another probability density $P^*(\mathrm{d}x)$

$$P^*(\mathrm{d}\mathbf{x}) = \prod_{i=1}^{p} f_i^*(x_i).\mathrm{d}x_i = p^*(\mathbf{x}).\mathrm{d}x_1 \ldots \mathrm{d}x_p \tag{25}$$

Equation (17) can be rewritten as

$$\bar{Z} = \int Z(x) \prod_{i=1}^{p} \frac{f_i(x_i)}{f_i^*(x_i)}.P^*(\mathrm{d}x) \tag{26}$$

which shows that the correcting weight to be introduced in calculations of expected values is

$$\prod_{i=1}^{p} \frac{f_i(x_i)}{f_i^*(x_i)} \tag{27}$$

To see what has been gained with this change, let us suppose that n calculations are carried out with the X_i picked out with the f_i^* as probability density functions. The result is a set of n vectors $X_1^*, \ldots, X_n^*$, and n values of $Z: Z_1^*, \ldots, Z_n^*$.

Let us introduce

$$\bar{Z}_n^* = \frac{1}{n}\sum_{i=1}^{n} Z_i^* = \frac{1}{n}\sum_{i=1}^{n}\left[Z(x_i).\prod_{k=1}^{p}\frac{f_k(x_{ki})}{f_k^*(x_{ki})}\right] \tag{28}$$

where $\bar{Z}_n^*$ is another estimation of $\bar{Z}$. However, its standard deviation is now

$$\sigma^2(\bar{Z}_n^*) = \frac{1}{n}\left(\left\langle\left(\frac{Zp}{p^*}\right)^2\right\rangle - \left\langle\frac{Zp}{p^*}\right\rangle^2\right) \tag{29}$$

Bienaymé–Tchebychev's formula can once more be applied to give

$$P(|\bar{Z}_n^* - \bar{Z}| \geqslant \varepsilon) \leqslant \frac{\sigma^2(Z^*)}{n\varepsilon^2} \tag{30}$$

It can be seen that with a correct choice of p^*, $\sigma^2(Z^*)$ can be greatly reduced. In fact, $\sigma^2(Z^*)$ is zero for

$$p^*(x) = \frac{Z(x).p(x)}{\bar{Z}} \tag{31}$$

$\bar{Z}$ is unknown, but can be estimated, so as to give in turn suitable distributions for p^* and the f_i^*.

4 Application

4.1 *Application to finite-element computations*

For each element of the finite-element mesh, random values of α, β and $\boldsymbol{\sigma}(x, t_i)$ ($i \in [1, Q]$), are computed, representing as many individual specimens. For each of them, the equivalent stress τ_c and the 'resistance' $R = \beta - \alpha p_{max}$ of the element are computed.

The failure will occur if $Y > 0$, with

$$Y = \frac{\tau_c - \beta + \alpha p_{max}}{\beta - \alpha p_{max}} \tag{32}$$

The probability $P(Y > 0)$ for each element of the mesh can be computed by the Monte-Carlo method previously detailed.

From a practical point of view, the distribution of fatigue resistance is usually approached by staircase methods. Since the mechanical properties of the tested specimens will probably be around the average, and since the weaker components are the most critical in the simulation, it is necessary to make an overall hypothesis on the distribution of the mechanical properties in the metal. A gaussian probability density is not satisfactory: mechanical properties do not extend to infinite values, and they are not usually distributed symmetrically around the mean value. Truncated two-parameter Weibull distributions seem more adequate to model physical properties. Such questions have already been discussed elsewhere (**15**).

Some dimensional distributions are available through statistical process control (SPC). Since components with dimensions out of requirements are discarded, the distributions measured can be used directly in the Monte-Carlo simulations without any corrections. However the most interesting parameters, such as curvature radiuses or local thickness will probably require specific measurements.

Load distributions are less well known but can be approximated, measured or guessed.

Since it is not possible to realize a finite-element computation for each sample, a possibility is to resort to 'sensitivity analysis' (nowadays provided by all major finite-element software suppliers) which gives for instance the derivative of the stress with respect to a given dimension, and uses Taylor developments around its mean value. For instance, if a single random load $F(t_i)$ is applied at instant t_i to the structure, with mean value $F_0(t_i)$, and if x is a dimensional random variable (i.e. a tangential radius) with mean value x_0.

$$\sigma(x, t_i) = \frac{F(t_i)}{F_0(t_i)} \cdot \left[\sigma(x_0, t_i) + \frac{\partial\sigma(x_0, t_i)}{\partial x} . (x - x_0) \right] \tag{33}$$

All in all, only two finite-element results are required for each instant of the fatigue crack, to obtain $\sigma(x_0, t_i)$ and $\partial\sigma(x_0, t_i)/\partial x$.

Another equally interesting method to reduce the number of computations is to resort to experience plans, which can be transposed without any major difficulties to 'numerical experiments'.

After some preliminary trial calculations, it has turned out that the choice of uniform functions for the f_i^* is the best compromise between efficiency and universality.

5 Experimental Verification

5.1 *Component, material and loading*

The automobile component calculated and tested is a front suspension arm. The loading is applied through a ball joint with a force system following x and y directions

$$F_x = Fa \sin \omega t$$
$$F_y = Fa \left(-\frac{1}{2} + \cos \omega t \right) \tag{34}$$

The material is a forged steel 5MCB4 (to the AFNOR standard) with UTS = 1000 MPa and YS = 850 MPa. The component has been tested as forged, without sand blasting nor shot peening. The fatigue limits of the material have been determined in repeated tension $R = 0.1$, alternate tension $R = -1$ and repeated compression, on specimens with the forge skin, machined from the components. The fatigue limits for these different conditions and the corresponding values of p and τ_a are given in Table 1.

Table 1 Fatigue limits

Loading mode	*Compression*	*Push–pull*	*Tension*
R ratio	133	-1	0.1
fatigue imit min/max (MPa)	$-730/-5{,}4$	$-300/+300$	61,8/618
τ_a (MPa)	181	150	139
τ_a^* (MPa)	208	173	160
p (MPa)	1.8	100	206

Note that the local equivalent shear stress amplitude is calculated either for Treca's, τ_a, or von Mises', τ_a^*, plastic yield criterion, from which are derived Dang Van's criterion for the former or Papadopoulos' or the Double-Diameter criterion for the latter.

The fatigue criteria are

Dang Van's approach: $\tau_a + 0.20\, p < 180$ (35)

Papadopoulos and Double-Diameter: $\tau_a^* + 0.23\, p < 208$ (36)

5.2 *Numerical analysis*

The numerical model is made of about 3000 elements and 4000 nodes. The elements are essentially 8-node brick elements, with beam elements and rigid

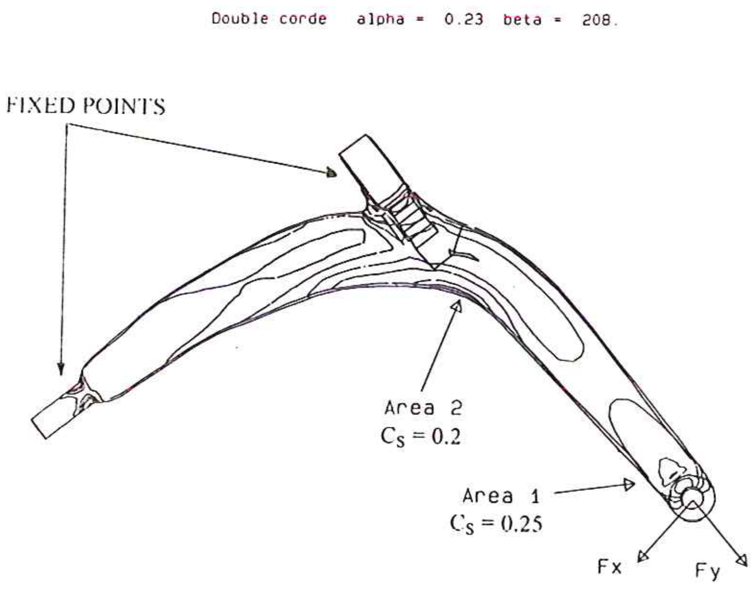

Fig 3 Isovalues of the Double-Diameter criterion for $Fa = 8\,500$ N.

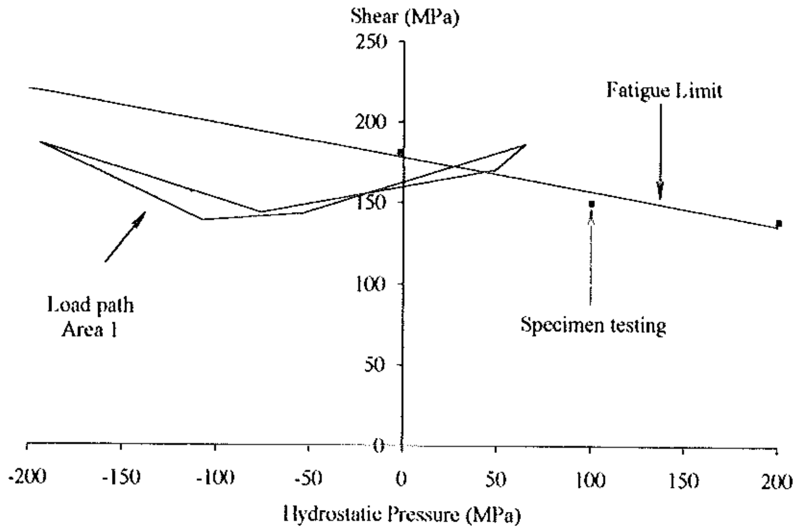

Fig 4 Load path at the critical point (Dang Van's criterion).

elements to link the arm to the fixations. The representativity of the boundary conditions has been verified by comparison of the stresses calculated at four nodes on current parts of the model, to the stresses measured by strain gauges on the component at the same locations. The calculation has been performed in elasticity with MSC/NASTRAN. The material constants are E = 200 000 MPa and $\nu = 0.3$.

5.3 *Fatigue life prediction*

The fatigue criterion requires that the fatigue cycle be described at several instants. In the present example six values of ωt have been chosen $\omega t = 0, \pi/2, \pi, 3\pi/2$ and the two intermediate values such that $F_X = F_Y$. The corresponding stress tensors are then calculated by FEM before stress results are analysed with the fatigue routine. The calculated values of the criterion displayed on graphic output, Fig. 3, is the quantity C_s. Negative values mean safety, whilst positive values mean danger. The example is given for $Fa = 8500$ N. For both criteria, two critical areas appear in the component:

- area 1, close to the ball joint;
- area 2, at the middle of the arm in intrado.

The load path at the most critical point in area 1 is given in Fig. 4, for Dang Van's criterion. In the present case $C_s = 0.12$. Nevertheless in area 2 this criterion is slightly more severe, $C_s = 0.2$.

Using the Double-Diameter criterion, area 1 becomes the most critical (C_s area 1 = 0.25, Fig. 5, C_s area 2 = 0.2).

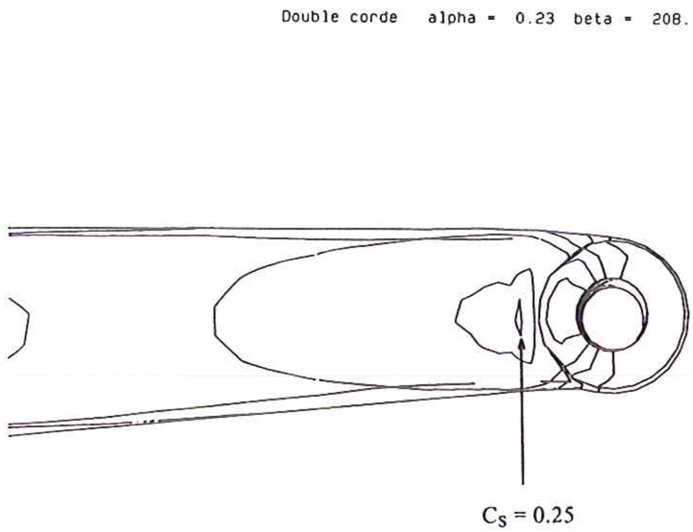

Fig 5 Isovalues in area 1 for the Double-Diameter criterion.

This reflects the fact that the loading path in the stress space is complex (out-of-phase) in area 1. In these conditions the formulation of the Double-Diameter leads to a higher measure of the shear amplitude (see Section 3). The fatigue limit ($C_s = 0$) is found for $Fa = 8000$ N, Fig. 6.

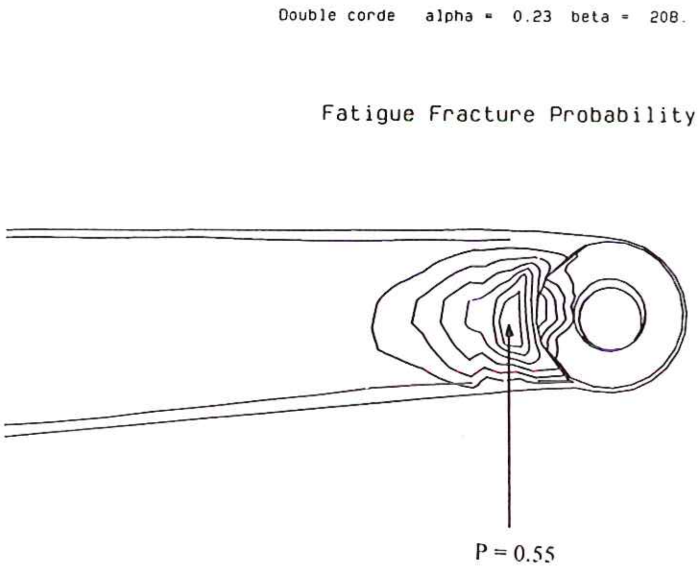

Fig 6 Fatigue fracture probabilities in area 1 for the Double-Diameter criterion for $Fa = 8000$ N.

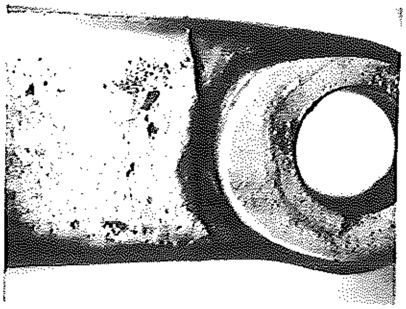

Fig 7 Crack initiation area.

5.4 *Probabilistic fatigue life prediction*

To compute the probabilistic approach, gaussian distribution appears for the fatigue limits with a standard deviation equal to 15 MPa, and for the loading *Fa* with a standard deviation equal to 20%.

The results are expressed in terms of probability of crack initiation, Fig. 7.

5.5 *Fatigue testing*

Fatigue tests have been performed on suspension arms, withloading on he ball joint given in equation (34).

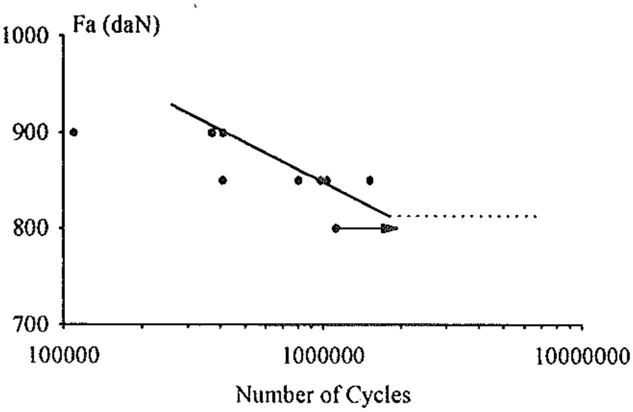

Fig 8 Fatigue tests results.

Fatigue test results are given in a diagram *Fa* vs *N*, Fig. 8. The number of cycles to failure is defined by the first visible crack. Below the calculated fatigue limit, 8000 N, no fatigue cracks were detected at the end of the test after 1.2×10^6 cycles. For loading above the fatigue limit, the failure of the components occurs in area 1, at the most critical point identified by the Double-Diameter criterion. For this condition, the mean value of the fatigue strength at 10^6 cycles is estimated at $Fa = 8500$ N.

6 Conclusions

The aims of this study were first to check the efficiency of the predictions of standard fatigue criteria, then to develop methods for the probabilistic evaluation of fatigue fracture risks. However, in most cases, the deterministic evaluation will be sufficient, and probabilities will need to be computed only for the dependability requirements, or for components with very short safety margins. Another possibility is to use probabilistic evaluations for the computation once and for all of a safety factor, which will be used afterwards for all fatigue calculations.

All in all, the fatigue rountines which were developed have been integrated in a single FORTRAN 77 routine easily accessible, with interfaces written for the post-processing of results issued from major finite-element softwares, which is a major insurance for reliable, reproductible results, and overall quality in design.

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