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Multiaxial Fatigue Assessment of Automotive Chassis Components on the basis of Finite-Element Models

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ABSTRACT A two-stage strain-life approach to the fatigue assessment of automotive chassis components is proposed. The first stage involves preliminary global fatigue calculations to identify critical regions and the critical events in the loading sequence. The second stage involves characterization of local stress-state variations and application of appropriate methods of damage calculation. The emphasis is on practical engineering techniques which can be used to support a vehicle development program, from the concept design stage through to the final release.

1 Introduction

There is increasing pressure within the automotive industry to produce designs which are strong and safe, while having good fuel economy, and therefore minimum weight. This is motivating a move from stress-based assessments involving standard load cases, to life-based calculations with realistic load histories which are representative of vehicle sign-off tests. The greatest need here is not for further fundamental research, but for practical and efficient design methods and software. It is desirable that lifetime estimates and optimization of components and structures be carried out analytically, minimizing the need for testing, and thus reducing development costs and product lead times.

A key feature of the modern durability design process is the use of computer-based finite-element methods. This is especially true of fatigue life assessments where FE methods permit concept-level life estimates. There are 3 basic steps in any life assessment. The first step is to define the load environment of the component or structure, and in many cases this will be the most difficult part of the whole process. Having done this it remains to establish the load–strain relationship and finally the strain–life relationship, and it is these two steps which are the main focus of this paper.

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1.1 *Load–strain relationship*

The transfer functions between applied loads and elastic stresses and strains are readily defined using FE methods. The local strain approach however, requires elastoplastic strain histories. When the loading, that is the variation of the local stress-strain state, is proportional there are two alternatives: to use notch correction procedures (**1**, **2**), or to use nonlinear FE analysis. The use of the Neuber approximation (**1**) is widely accepted for uniaxial loadings and its limitations, including a tendency to give conservative results for sharp notches, are well understood. A procedure (**3**) which extends the Neuber method to situations where the loadings are proportional and multiaxial is accomplished by replacing axial stress and strain with equivalent parameters based on the von Mises yield criterion. This procedure also includes a means of accounting for net-section yielding, as does that due to Dittmann (**4**). Of course such situations can also in theory be dealt with by means of stepwise nonlinear FE analysis, but the computational overhead for FE models of significant size and realistic load histories consisting of thousands of time steps would be considerable.

The methods described above assume that hardening is isotropic. When the loading is nonproportional, that is the orientation and/or the ratio of the principal stresses varies, this assumption is no longer reasonable and something more sophisticated than methods based on simple extensions of static yield theories is required. Under these conditions, kinematic hardening models (**5-8**) have been shown to provide a better description of stress–strain behaviour. Bannantine incorporated one such model into a fatigue analysis program, but this program processes measured data from strain gauge rosettes, and cannot work from elastic FE results. Such a hardening model has also been incorporated into a fatigue analysis procedure, based on a point-by-point nonlinear FE analysis, and using a substructuring technique (**9**). The authors are not aware that this technique has been applied to chassis components subject to realistic load histories. An attractive and computationally more efficient alternative would be some kind of nonproportional multiaxial notch correction. Now, the Neuber method (**1**) requires only stresses and strains at turning points; when the loading is nonproportional such methods are no longer applicable, as the hardening becomes path-dependent. However, an incremental formulation of the Neuber or Glinka methods (**1**, **2**) in conjunction with a suitable hardening model might be appropriate (**7**).

1.2 *Strain-life relationship*

When the load–strain relationship has been established, the next element in the process is the strain–life relationship. Bannantine (**10**) has written a useful review of the various ways in which the local strain approach has been extended in attempts to deal with multiaxial loadings. Where loadings are proportional, effective stress–strain methods, based on extensions of static yield theories, are

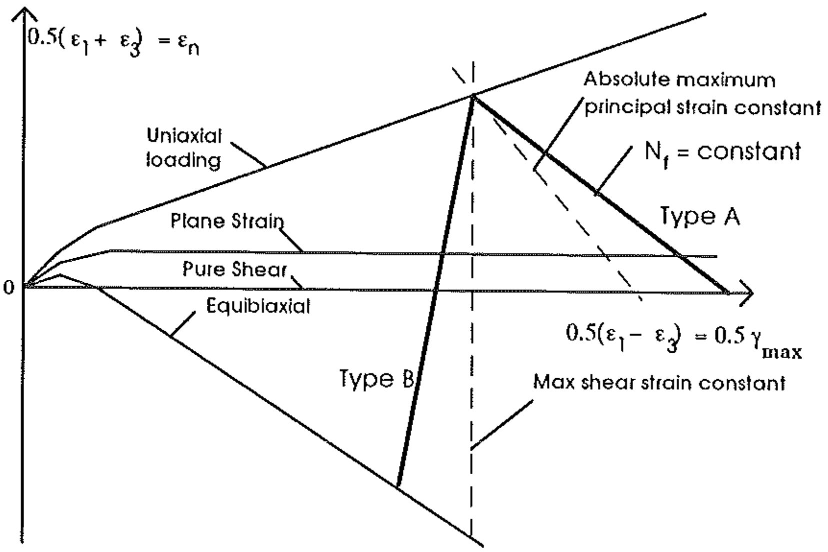

Fig 1 Schematic of typical Γ-plane plot.

quite widely used, despite certain limitations. These include methods based on maximum principal, von Mises and Tresca strains. They have the advantages of being easy to implement and relating multiaxial conditions to uniaxial cases for which much more fatigue data is available. Drawbacks include the fact that these failure criteria do not take into account the directional nature of fatigue damage, nor do they account for the effect of mean stresses. When loadings are nonproportional it becomes more difficult to justify the use of such parameters as the maximum principal stress, because the principal axes of stress may no longer be fixed in orientation. Since fatigue damage consists of cracking and is therefore directional, it is not meaningful to simply sum damage which should be assigned to particular planes.

This objection is addressed by a variety of 'critical plane' methods which resolve stresses and strains onto particular planes, to which damage is then assigned by use of a variety of models. Many such models are based on the work of Brown and Miller (**11**) who noted that cracks tend to nucleate on planes of maximum shear strain, and that the damaging effect of the shear strains is modified chiefly by the normal strain (or stress) on the plane of maximum shear. This led them to plot contours of constant endurance on a graph of normal strain ε_n, plotted against $\frac{1}{2}\gamma_{max}$, the maximum shear strain amplitude, for constant proportional biaxial loaings. This plot is known as the Γ-plane. A schematic of a typical Γ-plane plot is shown in Fig. 1.

Two distinct regimes of cracking behaviour are observed, type A and type B. In the early stages of crack initiation, type A cracks, which are formed when

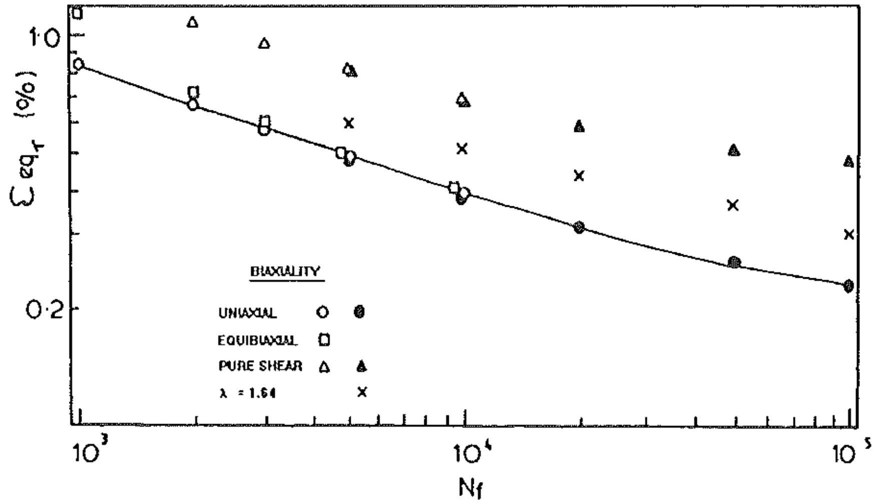

Fig 2 Biaxial strain data showing correlation with Tresca criterion (11).

the loading is between uniaxial and pure shear, are essentially driven in mode II along the surface, on maximum shear planes that intersect the free surface at right angles. On the other hand type B cracks, formed when the loading is between uniaxial and equibiaxial, are driven through thickness on maximum shear planes that make an angle of 45° with the free surface, and these cracks are more damaging than type A. It is interesting to plot lines of constant maximum principal strain and maximum shear strain on the same plot. A comparison between these lines and the case A and B life contours suggests that a maximum principal strain-based method might be expected to give reasonable and somewhat conservative life predictions for proportional multiaxial conditions lying between uniaxial and pure shear. Loadings which are between uniaxial and equibiaxial are likely to be better correlated by a more conservative method using a maximum shear strain-based criterion. In general this observation agrees well with published proportional biaxial test results; results for negative biaxialities (uniaxial to pure shear) are correlated reasonably well by maximum principal strain, while the Tresca parameter is better for positive biaxialities (uniaxial to equibiaxial) and approximately describes a lower bound for proportional loadings, as illustrated in Fig. 2 (reproduced with permission from (12)).

It is a basic premise of the current work that there is a range of complexity of fatigue analysis problems; these are methods that are well-established and whose limitations are well understood for dealing with uniaxial loadings, and there are sensible modifications that can be made to these methods to obtain life estimates where the loadings are proportional and multiaxial. There are also much more complex, computationally intensive and less well tried methods for dealing with nonproportional loadings, and attempts have been made to produce models that will correlate fatigue damage under any loading conditions. Previous work by the authors (**13**, **14**) indicates that although a component may be subject to a complex set of multiaxial out-of-phase loads, local loadings at many locations on the surface of the component may approximate uniaxial or proportional multiaxial loading conditions. Socie notes in his summary and interpretation of the SAE's biaxial testing program (**15**) that no single approach to life prediction appeared superior. We therefore propose a two-stage assessment procedure. In the first stage, critical events and locations are identified through preliminary life estimates and examination of stress ranges, and summary statistics are obtained concerning the behaviour of the (elastic) stress tensor. A more detailed analysis of the critical events and locations is then carried out, with the simplest credible method appropriate to the prevailing stress–strain states being used to assess life.

2 Summary of Process

The two-stage approach to fatigue analysis described in this paper forms part of the overall durability engineering process. The way that it fits into this process is summarized in Fig. 3. The inputs to the process are an FE model of the component or structure, a set of cyclic material properties and a set of representative load time histories. The stiffness and mass of the component considered here, a steering knuckle, are such that the problem can be treated as quasistatic. The process is iterative, that is the inputs and analysis are progressively refined in the light of the information available. For example, for a new component on a new vehicle, measured load histories may not be available, and the first-pass analysis might use standard load cases, or load histories calculated using a commercial vehicle dynamics modelling program. When the first prototype is tested, the measurements should yield more realistic loading histories. Similarly, it may be necessary to refine the FE model in the light of preliminary analysis. The present paper is concerned chiefly with the fatigue analysis component of this process.

3 Two-stage Approach to Fatigue Analysis

3.1 *Initial assessment*

As an example of the proposed approach, an analysis carried out on a motor vehicle steering knuckle is briefly described. An FE model of this component is illustrated in Fig. 4.

The model consists mainly of 8-noded HEX elements with a minimum number of wedges. A set of 12 load cases was identified which in linear combination were considered able to describe any possible loading condition on the component. These loads were applied at the strut mount, the lower suspension

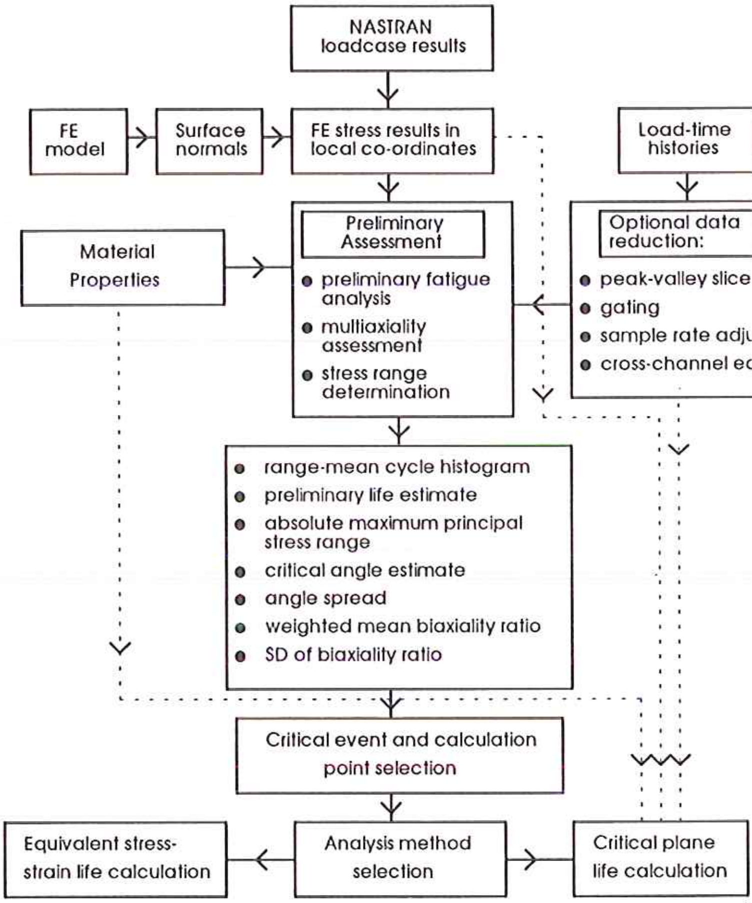

Fig 3 Overview of durability design process.

ball-joint and the steering rod mount, the spindle axis being fixed. Linear static FE stress analyses were carried out for each of these static load cases. Sets of load histories $P_k(t)$ corresponding to each of the 12 load cases were identified for a number of different test track events. These were reduced using a peak–valley slicing technique, which extracts the maxima and minima from multiple channels while retaining the cycle sequence and phase information. While this procedure retains the load peaks, it may conceivably lead to some truncation of local stress maxima and minima. Some of the resulting load histories, for a slalom manoeuvre on a cobblestone surface, are illustrated in Fig. 5.

An initial fatigue life assessment was carried out using a commerical fatigue life estimation program called MSC/FATIGUE. This program computes elastic strain histories $\varepsilon_{ij,e}(t)$ for each surface node according to the following formula.

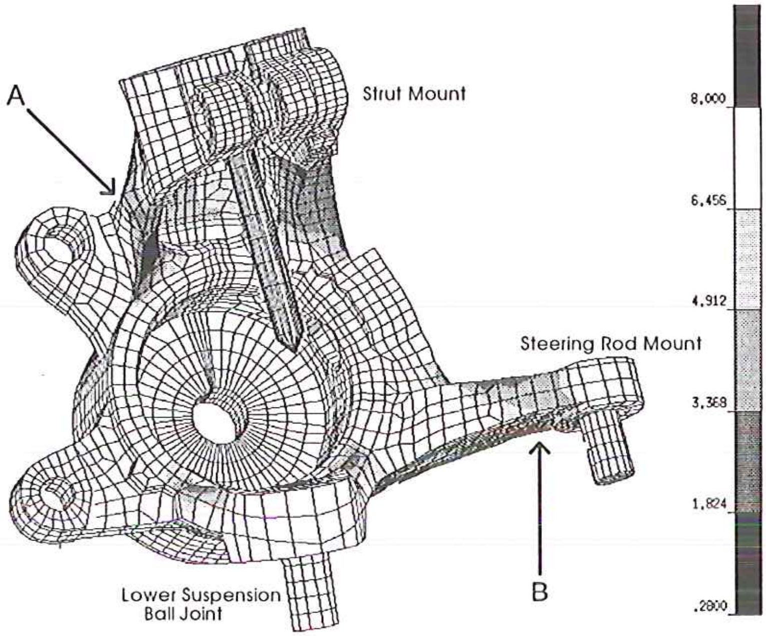

Fig 4 FE model of steering knuckle, with contours of log(life/repeats).

$$\varepsilon_{ij,e}(t) = \sum_k \left(\frac{P_k(t)}{P_{k,fea}}\right)\varepsilon_{ij,e,k}$$

where k is the static load case i.d., and $P_{k,fea}$ is the FE load case load for load case k. To simplify subsequent analysis, the stress and strain results are presented in local coordinate systems whose z-axes are outward surface normals. From the full strain tensor, an equivalent elastic strain $\varepsilon_{q,e}(t)$ is extracted and cycle counted using the Rainflow method. Here the initial calculation uses the absolute maximum principal strain, that is the principal strain which is largest in magnitude. The resulting cycles are elastoplastic converted using the Neuber approximation according to the formulation

$$E\varepsilon_{q,e}^2 = \varepsilon_q \sigma_q$$

in conjunction with the uniaxial cyclic material properties. Mean stress effects are taken into account using the Smith–Watson–Topper method **(16)**.

The results of this preliminary analysis indicate that the most damage occurs on the inside of the rearmost rib of the strut mount, position A in Fig. 4, node 7977 from the FE model. Two questions need to be asked at this stage.

(1) Is the analysis valid at this point, i.e. is the loading at this point uniaxial?

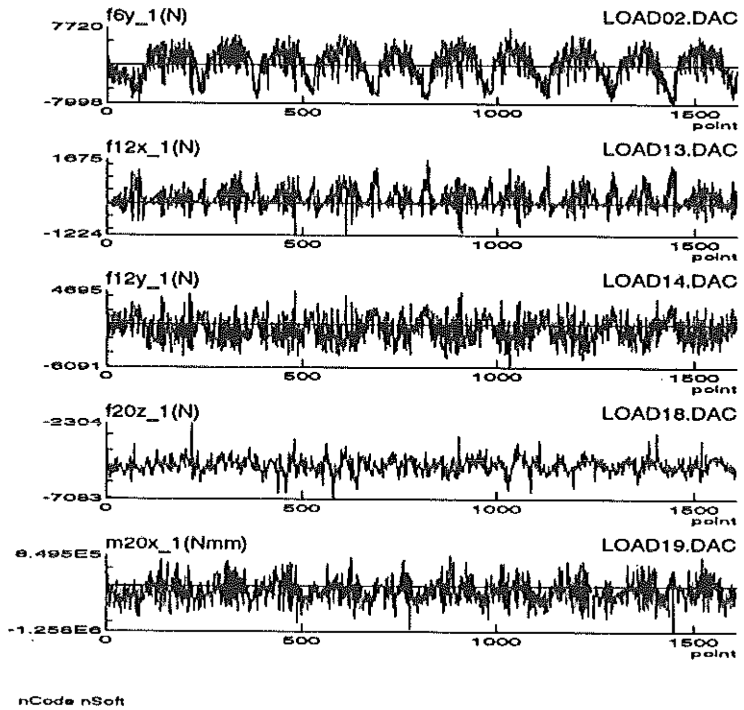

Fig 5 Selected load histories from cobblestone slalom.

(2) Are there other locations on the model which would have more predicted damage if we analysed them by a different, and perhaps more appropriate route?

These issues are addressed in the second stage of the procedure. To illustrate this using the knuckle it was necessary to artificially increase the damage, and this was achieved by scaling up the load histories by a factor of 3.33. Contours of log(life/repeats) for this analysis are plotted in Fig. 4. Two additional parameters were now computed: the angle ϕ_p which the absolute maximum principal stress σ_1 (elastic) makes with the local x-axis, and the ratio of the two in-plane principal stresses $a_e = \sigma_2/\sigma_1$ (elastic), where σ_2 is the lesser in magnitude of the two in-plane principal stresses. The stress state at free surfaces is simple, because one of the principal stresses must be zero and normal to the surface. These two parameters therefore provide a complete description of the surface stress state. Summary statistics of their behaviour can be produced and may be contour plotted to give an indication of the regions in which the validity of the uniaxial analysis must be questioned, those where some other equivalent

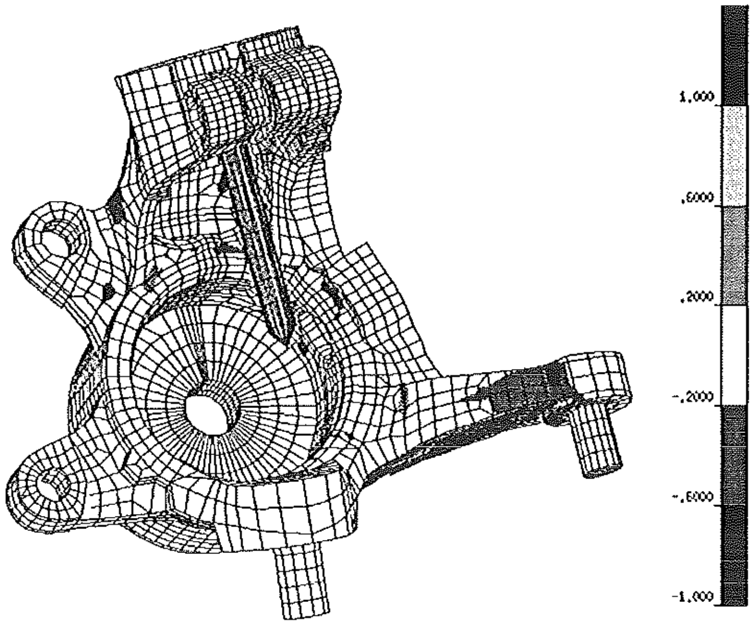

Fig 6 Steering knuckle with contours of mean biaxiality ratio.

strain method would be preferable, and those where the loading is clearly nonproportional, and some more complex method may be required to obtain a result of reasonable reliability.

The directional mobility of the principal stress axes is characterized by the spread of angles ϕ_p for which the magnitude of σ_1 exceeds a gate value, here set to 160 MPa. The biaxiality is characterized by the mean and standard deviation of the values of the biaxiality ratio a_e for which the magnitude of σ_1 exceeds the gate value. The mean biaxiality ratio is contour plotted in Fig. 6. A study of the statistics indicates that the loading at node 7977 is very close to being uniaxial, since the directional mobility of the in-plane principal stresses is small and the biaxiality ratio exhibits little variation and is close to zero. This suggests that the method selected for the preliminary analysis should yield reasonable results, within the limitations of the local strain approach and the FE model. However contour plots of these parameters indicate that there are other areas of the knuckle where the behaviour of the stress tensor is such that an equivalent strain approach could not be used with much confidence. For instance the statistics for node 1045 in region B in Fig. 4 indicate that the absolute maximum principal stress moves through an angle of around 90° and that the biaxiality ratio not only tends to be large and negative, indicating shear loadings, but also exhibits considerable variation. The loading here is clearly

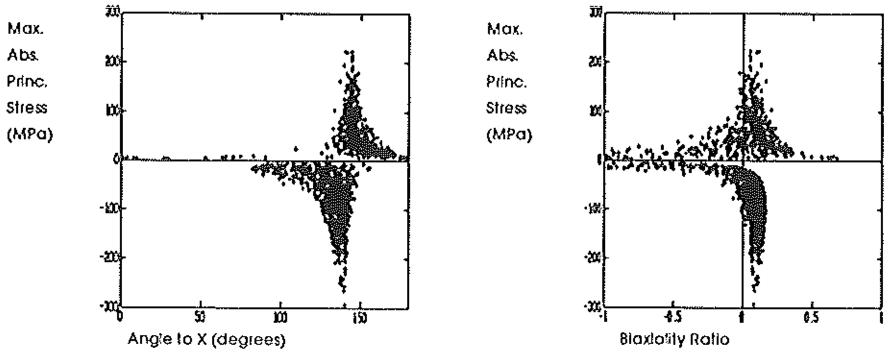

Fig 7 Biaxiality assessment plots for node 7977, cobblestone slalom.

nonproportional, and if this were a critical location on the component, a more complex procedure would be required. However, the stress levels are low (never above the 160 MPa gate with the unscaled loads) so that in practice the simplest method of damage assessment is sufficient.

3.2 *Detailed assessment*

For illustrative purposes, the behaviours of the stresses at nodes 7977 and 1045 were next examined in more detail using a prototype multiaxial fatigue analysis program PTRACK, developed for the purpose. This allows more detailed analysis of the stress tensor behaviour than is available in MSC/FATIGUE and also implements some different equivalent strain and critical plane damage assessment options. In each case two plots are produced, scatter plots of absolute maximum principal stress plotted against the angle to the local x-axis and the biaxiality ratio (see Figs 7 and 8).

Figure 7 confirms that although there appears to be some variation in the orientation of the principal stresses at node 7977, this all occurs at low stresses where it is of little significance. Similarly, there is some variation in the biaxiality ratio, but again it is mostly at low stresses. At the highest stresses, the biaxiality is nearly constant and close to zero. This confirms that the method used in the preliminary analysis might be expected to yield reasonable results, within the limitations imposed by the accuracy of the nodal stress and strain results.

Node 1045 presents a quite different picture. The plots (Fig. 8) indicate that the absolute maximum principal stress 'flips' through approximately 90°, due to the fact that as the biaxiality approaches -1 (pure shear loading), the absolute maximum principal switches between the conventional maximum and minimum principal. The biaxiality plot indicates that the loading is genuinely nonproportional.

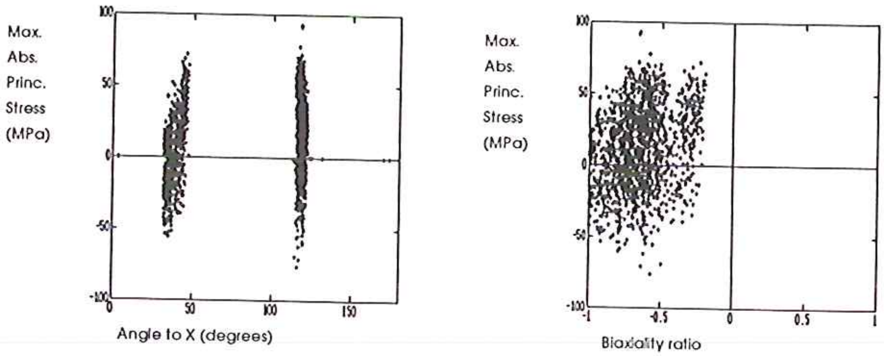

Fig 8 Biaxiality assessment plots for node 1045, cobblestone slalom.

Conventional wisdom dictates that prediction of the elastoplastic stresses and strains at this location should be carried out using a suitable kinematic hardening model, and damage by a critical plane method. Taking a purely pragmatic view, these methods are only beneficial if they lead to a significant difference in the predicted lifetime. If only small variations in life are predicted, they may be insignificant. in relation to the experimental scatter of fatigue lives and the inherent deficiencies of the local strain approach. A detailed study of the sensitivity of life predictions to the modelling of stress–strain behaviour is beyond the scope of the current work. However, it was possible to compare critical plane and equivalent stress–strain methods. For the purposes of comparison, the following calculations were carried out on the basis of the elastic FE stresses and strains, with no correction for plasticity, using the program PTRACK. This program implements four critical plane damage models:

1. *Normal strain only*

$$\varepsilon_n = \frac{\sigma_f'}{E}(2N_f)^b + \varepsilon_f'(2N_f)^c$$

2. *Bannantine's* (2) *'tensile' model (analogous to the Smith–Topper–Watson model)*

$$\varepsilon_n \sigma_{n,\max} = \frac{(\sigma_f')^2}{E}(2N_f)^{2b} + \varepsilon_f \sigma_f'(2N_f)^{b+c}$$

3. *Shear strain only*

$$\gamma = \frac{(1 + \nu_e)}{E}\sigma_f'(2N_f)^b + (1 + \nu_p)\varepsilon_f'(2N_f)^c$$

4. *Combined shear and normal model due to Fatemi and Socie* (**17**)

$$\gamma\left(1 + n\frac{\sigma_{n,\max}}{\sigma_y}\right) = \frac{(1+\nu_e)}{E}\sigma_f'(2N_f)^b + \frac{n(1+\nu_e)(\sigma_f')^2}{2E\sigma_y}(2N_f)^{2b}\ldots$$

$$+ (1+\nu_p)\varepsilon_f'(2N_f)^c + \frac{n(1+\nu_p)\varepsilon_f'\sigma_f'}{2\sigma_y}(2N_f)^{b+c}$$

where ε_n and σ_n are normal stress and strain, σ_y denotes yield stress, $n = 0.6$ and all other values take their normal definitions. Each of these models can be applied to planes intersecting the free surface at 90° and 45° (suitable for shear-based models and positive biaxiality ratios). Equivalent stress–strain versions of these models were also implemented using the absolute maximum principal stress and strain in place of the direct stress and strain on the critical plane, and signed maximum shear strain (γ_{max} with the sign of the absolute maximum principal) instead of the shear strain. Note that $\sigma_{n,max}$ is the maximum normal stress that acts on the critical plane during any complete strain (Rainflow) cycle.

The results of these calculations, together with lifetime predictions from MSC/FATIGUE using various damage models are reported in Tables 1 to 4. Note that MSC/FATIGUE corrects for plasticity using the Neuber method, and also works on the basis of range–mean histograms.

Table 1 Lifetimes from PTRACK at node 7977 for cobblestone slalom

Model	*Critical plane*	*Equivalent*
Normal strain	1.547×10^5	1.504×10^5
Shear strain	1.170×10^5	1.150×10^5
Bannantine normal	3.450×10^5	3.371×10^5
Fatemi and Socie	1.296×10^5	1.272×10^5

Table 2 Lifetimes from MSC3/FATIGUE at node 7977 for cobblestone slalom

Analysis method	*Life in repeats*
Absolute maximum principal – strain life	1.49×10^5
Absolute maximum principal – SWT	3.94×10^5
Absolute maximum principal – Morrow	3.03×10^5
Signed Tresca criterion	9.66×10^5
Signed von Mises criterion	1.80×10^5

Table 3 Lifetimes from PTRACK at node 1045 for cobblestone slalom (load × 3.33)

Model	*Critical plane*	*Equivalent*
Normal strain	7236	6794
Shear strain	430	365
Bannantine normal	8769	8089
Fatemi and Socie	742	1428

Table 4 Lifetimes from MSC/FATIGUE at node 1045 for cobblestone slalom (load × 3.33)

Analysis method	*Life in repeats*
Absolute maximum principal – strain life	4364
Absolute maximum principal – SWT	3866
Absolute maximum principal – Morrow	4128
Signed Tresca criterion	172
Signed von Mises criterion	621

4 Discussion

The process of making analytical fatigue life predictions on the basis of FE models involves a number of activities, including obtaining reliable load histories and FE meshes. This work has focused on the actual fatigue analysis, taken to include the process of determining stress–strain histories and calculating damage. A general procedure for the analysis of automotive chassis components has been demonstrated using a steering knuckle as an example. It was shown here that although the component was subject to complex multiaxial load environment, the loadings (that is the stress and strain stage variations) at the most damaged locations on the component were quite simple and amenable to analysis using uniaxial or equivalent strain methods, both for stress–strain history determination and for damage calculation. It is worth noting that such simple loadings have been found at the most damaged locations on several components of widely differing shape. This reinforces the view that the two-stage procedure described here is of considerable practical value.

Where the loading is multiaxial and proportional, procedures such as that suggested by Hoffmann and Seeger (**3**) provide a satisfactory means of correcting elastic strain histories for plasticity. Alternatively, uniaxial cyclic properties, modified according to the biaxiality ratio, could be used. For proportional loadings, equivalent strain parameters such as the absolute maximum principal strain and the signed Tresca parameter, should in principal be able to provide good life predictions; there is nothing to be gained by carrying out more complex material modelling or applying critical plane methods. However, modifications to these methods are needed as $a_e \rightarrow -1$ because the sign becomes difficult to define, and some work remains to be done to define acceptable mean stress corrections.

Where local loadings are more complex, chiefly where parts experience out-of-phase torsion and bending, there may still be surprisingly little difference between fatigue life calculations made using critical plane and appropriate equivalent stress–strain methods. In the example above, this is because although the loading is complex and nonproportional, the predicted damage is dominated by a few of the largest cycles for which there is little variation in the orientation of the principal stress axes. For these cases, the most promising route for determination of local elastoplastic stress–strain histories would seem to be an incremental notch correction procedure (**7**). Having said that, there is a need to explore further the impact that this would have on life predictions. The results described in this paper indicate that in many cases where strains are predominantly elastic, the precise formulation of an elastoplastic correction, or indeed omitting it altogether may not make a significant difference to life predictions; there is always scatter in fatigue lives and the difference made by progressively refining the elastoplastic correction may not be significant in the light of the other inaccuracies and assumptions inherent in the process.

5 Conclusions

Although a component may be subject to a complex nonproportional multiaxial load environment, this does not necessarily generate complex loadings at critical locations.

A pragmatic approach to the fatigue assessment of chassis components has been presented. This involves selecting an analysis route which is the simplest appropriate to the prevailing loading conditions, and uses parameters which are selected on the basis of an assessment of the stress-state variations.

A set of software tools has been developed which forms the basis of a practical and efficient method for the fatigue assessment of components subject to complex multiaxial loadings.

There is a need for further work in incorporating a nonproportional multiaxial notch correction procedure into practical software tools.

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