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Fatigue Life Prediction of Components Using Multiaxial Criteria

REFERENCE Vidal, E., Kenmeugne, B., Robert, J. L. and Bahuaud, J., **Fatigue life prediction of components using multiaixal criteria**, *Multiaxial Fatigue and Design*, ESIS 21 (Edited by A. Pineau, G. Cailletaud, and T. C. Lindley) 1996, Mechanical Engineering Publications, London, pp. 365–378.

ABSTRACT The present paper deals with fatigue life prediction for periodic multiaxial loading. An implicit method is developed; it is an application of the use of multiaxial criteria which are extended from long fatigue life to a finite lifetime. Two multiaxial criteria are presented. One (RB criterion) uses the concept of the most critical plane and the other (FB criterion) has a global approach as it intends to describe the damage all over the possible planes through a point. Biaxial fatigue tests give the experimental support of the practical application. This implicit method shows a good correlation between predicted lifetimes and experimental ones. The extension to variable-amplitude multiaxial loading is then presented.

1 Introduction

The fatigue behaviour of materials depends on the interaction between the loading sequence, material characteristics and component geometry. Nowadays designers have to take into account the fatigue phenomenon which is responsible for the majority of mechanical failures. The most general case of loading applied to structures is a multiaxial loading which can be either of constant or of variable amplitude.

The aim of this paper is to present a method of fatigue lifetime prediction for cyclic loading using two different criteria. These criteria are available to check the behaviour for long fatigue life as well as for finite fatigue life. Required material fatigue characteristics are also mentioned. Fatigue life predictions are compared to experimental test results and show a good correlation. The reliability of the results is strongly dependent on the accuracy of the fatigue data. Finally, the extension of the method to multiaxial variable amplitude loading is presented.

2 Finite Fatigue Life Criteria for Multiaxial Stress Cycles

2.1 *General form of a fatigue criterion*

Let us consider a state of cyclic stress $[\sigma_{ij}(t)]$, with period $T(\sigma_{ij}(t+T) = \sigma_{ij}(t))$.

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The number N of cycles, defined by state of stress $[\sigma_{ij}(t)]$ and corresponding to crack initiation, is obtained by using a multiaxial criterion developed initially for long fatigue life and periodic states of stress. The general functional form of such a criterion can be written as

$$E([\sigma_{ij}(t)], \sigma_{-1}, \sigma_0, \tau_{-1}, \ldots) = 1 \tag{1}$$

where: σ_{-1}, σ_0 and τ_{-1} are fatigue limits of the material for, respectively, completely reversed axial loading ($R = -1$), zero-to-maximum axial loading ($R = 0$) and completely reversed torsion ($R = -1$) tests. Here these values will be assigned to correspond to fatigue strengths for $N = 10^7$, 2×10^6, 10^6 or 10^5 cycles rather than conventional fatigue limits, which are denoted $\sigma_{-1}(N)$, $\sigma_0(N)$ and $\tau_{-1}(N)$. Doing so, equation (1) has the new form

$$E([\sigma_{ij}(t)], \sigma_{-1}(N), \sigma_0(N), \tau_{-1}(N)) = 1 \tag{2}$$

and can be interpreted as a finite fatigue life criterion suitable for periodic states of stress (**1**).

2.2 *Conditions of validity*

Any criterion which can be derived from equation (2) has validity conditions which need to be checked.

(1) The formulation of the criterion must remain independent under coordinates transformation.
(2) We have assumed, equation (2), that the application of the criterion requires the knowledge of three material fatigue strengths $\sigma_{-1}(N)$, $\sigma_0(N)$ and $\tau_{-1}(N)$ found from experimental tests. The aim of the criterion is to predict the result of any other periodic possible test. Of course these theoretical predictions must be in good agreement with experimental results.

For example, let us consider the effect of mean stress: the amplitude $S_A(N)$ of the fatigue strength (for the number N of cycles) depends generally on the mean stress $S_M(N)$. This relation may be represented by a Haigh diagram (or constant-life diagram) $S_A(N) = f(S_M(N))$. The shape of the theoretical diagram derived from the criterion, equation (2), must be similar to the experimental one.

For a tension–compression test a low compressive mean stress σ_M has a favourable effect on the fatigue behaviour: the allowable stress amplitude σ_A increases.

For a torsion test (with smooth specimens) the mean shear stress τ_M affects only slightly the shear stress amplitude τ_A. In addition a graphical representation of the relationship between τ_M and τ_A gives a curve symmetrical with respect to the τ_A-axis. An horizontal tangency is generally found at the intersection point of the curve with the τ_A-axis.

From these points of view, Haigh diagrams are considered as physical properties of the material fatigue behaviour.

(3) A further condition involves the effect of multiaxial stress. The ability of the criterion to predict fatigue behaviour of the material under a multiaxial periodic state of stress must be checked with the corresponding experimental tests results.

2.3 *Choice of the damage indicator*

The fatigue phenomenon is related to the stresses induced by the external loading. Experimentally it has been pointed out that the origin of the crack initiation on one material plane is directly dependent on the stresses which exist on this plane. The effect of the normal stress and the shear stress are clearly distinguishable (2). As a result, the damage indicator to be chosen must be related to these characteristic quantities. That led us to use the normal stress and the shear stress as loading parameters to define the fatigue indicator.

According to the experimental observations, two formulations have been established. The first one uses the critical plane approach, the second one is based on a global approach.

2.4 *First proposed criterion*

Formulation

In order to calculate the damage indicator E_h, let us consider the loading path, that is the locus of the tip of the shear stress vector $\vec{\tau}_h$ acting on a surface element with unit normal vector $\vec{h}$. Let us determine the centre of the smallest circle surrounding the loading path. This centre is required to define the shear stress amplitude $\vec{\tau}_{ha}(t)$ at time t (see Fig. 1); the damage indicator is formulated as

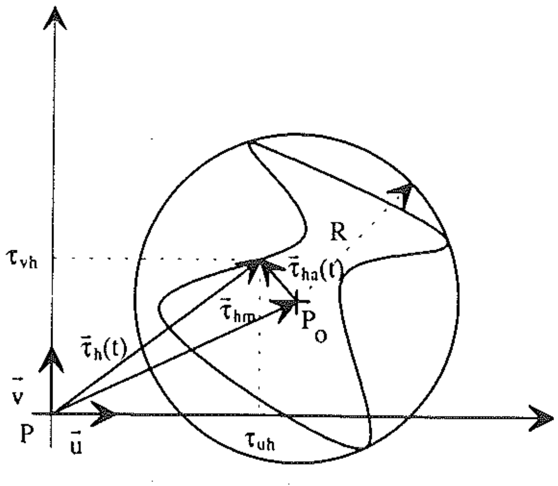

Fig 1 Locus of the tip of the shear stress vector during a cycle (loading path).

$$E_h(t) = \| \vec{\tau}_{ha}(t) \| + \alpha(N)\sigma_{hha}(t) + \beta(N)\sigma_{hhm} \quad (3)$$

where $\alpha(N)$ and $\beta(N)$ are criterion parameters depending on the number N of cycles at crack initiation and σ_{hhm}, $\sigma_{hha}(t)$ are, respectively, the mean normal stress and the normal stress amplitude at time t.

Introducing a third parameter $\theta(N)$, the criterion is defined by

$$E_{RB} = \frac{\max_{h,t} E_h}{\theta(N)} = 1 \quad (4)$$

Therefore this first proposition (RB criterion) (3,4) describes the fatigue phenomenon by seeking the most critical plane, that is to say the one where the damage indicator is the highest. From a physical point of view, this criterion is especially suitable in the case where principal stress directions are fixed during the whole cycle. The most activated plane always remains the same.

Identification of criterion parameters

The formulation of RB criterion uses three parameters $\alpha(N)$, $\beta(N)$ and $\theta(N)$. They are determined from three independent relations using three basic fatigue values of the material. Two types of tests provide these values.

(i) *Tension–Compression tests.* Let us use a reference space with coordinate axes fixed with respect to the body and denoted $\vec{1}$, $\vec{2}$, $\vec{3}$. In the case of a tensile–compressive test along axis $\vec{1}$, the normal stress $\sigma_{11}(t)$ is the only component of the stress tensor. Its evolution versus time t is described by

$$\sigma_{11}(t) = \sigma_{11m} + \sigma_{11a} \sin \omega t \quad (5)$$

where σ_{11m} and σ_{11a} are respectively the mean normal stress and the normal stress amplitude. Substituting equation (5) into equation (4) gives the relationship between the components of the fatigue strength $\sigma_{11a} = \sigma_A$ and $\sigma_{11m} = \sigma_M$. It is the equation of the constant-life diagram in tension–compression according to the criterion. Two particular points belong to this curve: one corresponds to the fatigue strength for tension–compression ($R = -1$) with coordinates ($\sigma_M = 0$, $\sigma_A = \sigma_{-1}(N)$) and the other corresponds to the fatigue strength for pulsating tension ($R = 0$) with coordinates ($\sigma_M = \sigma_A = \sigma_0(N)/2$). The third equation is obtained from a torsion test.

(ii) *Torsion tests.* For this kind of test, the shear stress $\sigma_{12}(t)$ (and of course $\sigma_{21}(t)$) is the only component of the stress tensor. $\sigma_{12}(t)$ is expressed versus time t by

$$\sigma_{12}(t) = \sigma_{12m} + \sigma_{12a} \sin \omega t \quad (6)$$

where σ_{12m} and σ_{12a} are respectively the mean shear stress and the shear stress amplitude. Substituting equation (6) into equation (4) gives the relationship between the components of the fatigue strength $\sigma_{12a} = \tau_A$ and $\sigma_{12m} = \tau_M$. It is the equation of the constant-life diagram in torsion according to the criterion. This curve contains the point corresponding to the fatigue strength for reversed torsion ($R = -1$) with coordinates $\tau_M = 0$ and $\tau_A = \tau_{-1}(N)$.

The identification of the three criterion parameters $\alpha(N)$, $\beta(N)$ and $\theta(N)$ by using the quoted tests gives the following equations

$$\begin{cases} \dfrac{\sigma_{-1}(N)}{2}\left(\alpha(N) + \sqrt{(\alpha(N))^2 + 1}\right) = \theta(N) \\[2ex] \dfrac{\sigma_0(N)}{4}\left(\alpha(N) + \beta(N) + \sqrt{(\alpha(N) + \beta(N))^2 + 1}\right) = \theta(N) \\[2ex] \tau_{-1}(N)\sqrt{(\alpha(N))^2 + 1} = \theta(N) \end{cases} \tag{7}$$

The system (7) is solved and gives

$$\frac{1}{2} < \frac{\tau_{-1}(N)}{\sigma_{-1}(N)} < 1 \tag{8}$$

This double inequality defines the validity domain of the proposed criterion.

$$\begin{cases} \alpha(N) = \dfrac{\dfrac{2\tau_{-1}(N)}{\sigma_{-1}(N)} - 1}{\sqrt{\dfrac{2\tau_{-1}(N)}{\sigma_{-1}(N)}\left(2 - \dfrac{2\tau_{-1}(N)}{\sigma_{-1}(N)}\right)}} \\[3ex] \theta(N) = \tau_{-1}(N)\sqrt{(\alpha(N))^2 + 1} \\[3ex] \beta(N) = \dfrac{\left(\dfrac{4\theta(N)}{\sigma_0(N)}\right)^2 - 1}{2\left(\dfrac{4\theta(N)}{\sigma_0(N)}\right)} + \alpha(N) \end{cases}$$

2.5 *Second proposed criterion*

Formulation

In many cases the principal stress directions rotate during a cycle. In such circumstances various slip planes are activated (**2**). The criterion must predict correctly that physical multiplicity. Therefore, the second proposed criterion (FB criterion) (**4**, **5**) takes into account the damage indicator of each plane and is based on the root mean square of the damage indicator on all possible physical planes through a point.

The damage indicator is formulated as

$$E_h = \frac{1}{\sigma_{-1}(N)}\left[a(N)\max_t \|\vec{\tau}_{ha}(t)\| + b(N)\max_t \sigma_{hha}(t) + c(N)\|\vec{\tau}_{hm}\| + d(N)\sigma_{hhm}\right] \tag{10}$$

where $a(N)$, $b(N)$, $c(N)$ and $d(N)$ are the criterion parameters and $\vec{\tau}_{hm}$ is the mean shear stress acting on the surface element during the cycle (see Fig. 1).

The criterion is then defined by

$$E_{FB} = \sqrt{\frac{1}{S}\int_S E_h^2 \, dS} = 1 \tag{11}$$

where S is the area of a sphere with a radius equal to unit ($S = 4\pi$).

Identification of criterion parameters

The four parameters $a(N)$, $b(N)$, $c(N)$ and $d(N)$ are obtained from the same three basic fatigue test results as for the first criterion, and also with the condition of symmetry of the constant-life diagram in torsion. The identification of the four parameters leads to the following system

$$\begin{cases} c(N) = 0 \\ 2(a(N))^2 + 3(b(N))^2 + 4a(N).b(N) = 15 \\ 15 + 3(d(N))^2 + 6b(N).d(N) + 4a(N).d(N) = 60\,\dfrac{(\sigma_{-1}(N))^2}{(\sigma_0(N))^2} \\ 6(a(N))^2 + 4(b(N))^2 + 7a(N).b(N) = 15\,\dfrac{(\sigma_{-1}(N))^2}{(\tau_{-1}(N))^2} \end{cases} \tag{12}$$

The system (12) is solved and gives, on the one hand, the validity domain of the criterion

$$\frac{1}{\sqrt{3}} < \frac{\tau_{-1}(N)}{\sigma_{-1}(N)} < \frac{\sqrt{3}}{2} \tag{13}$$

and, on the other hand, the values of parameters

$$b(N) = \sqrt{\frac{15 - \sqrt{\Delta(N)}}{2}} \text{ with } \Delta(N) = 225\left[1 - \frac{8\left[\left(\frac{\sigma_{-1}(N)}{\tau_{-1}(N)}\right)^2 - 3\right]^2}{25}\right]$$

$$a(N) = \sqrt{\frac{12\left(\frac{\sigma_{-1}(N)}{\tau_{-1}(N)}\right)^2 - 21 + (b(N))^2}{2}} \tag{14}$$

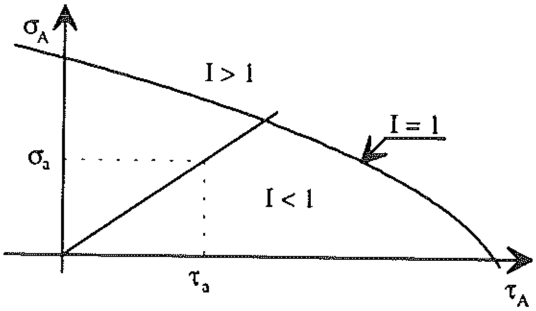

Fig 2 Strength curve $\sigma_A = f(\tau_A)$.

$$d(N) = \frac{-(3b(N) + 2a(N)) + \sqrt{(3b(N) + 2a(N))^2 + 3\left(60\left(\frac{\sigma_{-1}(N)}{\sigma_0(N)}\right)^2 - 15\right)}}{3}$$

2.6 *Validity of the proposed models*

Because all the possible slip planes are reviewed, the intrinsic aspect of the RB criterion is verified; and as the integration of the FB criterion is carried out through the whole area of a sphere, the intrinsic aspect of the second criterion is also checked.

Each criterion contains a constant-life diagram induced by its formulation which predicts a favourable effect of a low compressive mean stress in a tension–compression test, which is in good agreement with usual experimental Haigh diagrams.

For torsion test the diagrams given by both criteria show a symmetry with respect to the τ_A-axis with a low effect of the mean shear stress.

In order to check the validity of the models, we define a validity parameter I. At crack initiation, the stress state $[\sigma_{ij}(t)]$ is such that $E = 1$. For example let us consider a biaxial state of stress $[\sigma_{ij}(t)]$ with the shear stress amplitude $\sigma_{12a} = \tau_A$ and the normal stress amplitude $\sigma_{11a} = \sigma_A$. When the stresses σ_A and τ_A are taken as coordinates axes, the curve $E = 1$ (so-called strength curve for the biaxial state of stress (σ_A, τ_A) can be plotted in a stress plane (see Fig. 2). If the stress state $[\sigma_{ij}(t)]$ does not verify the criterion, i.e. $E([\sigma_{ij}(t)], \sigma_{-1}(N), \sigma_0(N), \tau_{-1}(N)) \neq 1$, it means that crack initiation occurs after a number of cycles different from N. Let us denote I the stress ratio between the current values $[\sigma_{ij}(t)]$ and the ones for which the criterion is verified, that is to say

$$E([\sigma_{ij}(t)]/I, \sigma_{-1}(N), \sigma_0(N), \tau_{-1}(N)) = 1 \qquad (15)$$

Considering the linearity of the damage indicator E_h of both proposed criteria with respect to stresses, this equation gives

$$I = E([\sigma_{ij}(t)], \sigma_{-1}(N), \sigma_0(N), \tau_{-1}(N)) \qquad (16)$$

If $I < 1$ ($E < 1$) the criterion is non conservative or optimistic: crack initiation will occur for stresses smaller than the predicted ones. If $I > 1$ ($E > 1$) the criterion is conservative or pessimistic: crack initiation occurs for stresses larger than the predicted ones (see Fig. 2).

The validation of the proposed model uses experimental data given with courtesy by investigators of the L.B.F. (Laboratorium Für Betriebsfestigkeit, Darmstadt, Germany) (**6**). Proposed criteria are implemented on computer. That allows us to calculate the validity parameters for each test.

The specimens are cylinders or bars. When the axis of specimens is taken as axis $\vec{1}$, the stresses are expressed versus time t as

$$\sigma_{11} = \sigma_{11m} + \sigma_{11a} \sin \omega t$$

$$\sigma_{22}(t) = \sigma_{22m} + \sigma_{22a} \sin(\omega t + \delta_{22})$$

$$\sigma_{12}(t) = \sigma_{12m} + \sigma_{12a} \sin(\omega t + \delta_{12})$$

The studied material is a heat-treated carbon steel Ck45 (or XC48: 0.43% C, 0.25% Si and 0.63% Mn) with ultimate tensile strength $\sigma_u = 850$ MPa, yield strength $\sigma_{0,2} = 810$ MPa, $A = 22\%$; testing is conducted at room temperature.

Specimens are thin-walled cylinders with interior diameter of 32 mm and 1.5 mm wall thickness. Two tests series are performed: tests with combined tension and internal pressure and tests with combined tension and torsion.

For each tests series two stress levels are considered and the number of cycles up to crack initiation are noted. All the results plotted on a probabilistic diagram (probability–logN) may be adequately described by a straight line. This allows the graphical evaluation as the number of cycles up to crack initiation N_{exp} defined with a probability of 50%. Adopting then the assumption of a linear logS/logN dependence, the stress corresponding to a number $N = 10^5$ cycles up to crack initiation can be graphically determined. Fatigue data are summarized in Table 1; all the bold-faced values are exact data and the others are graphically determined.

Table 1 Testing conditions – experimental number of cycles up to crack initiation

Test No	σ_{11a}	σ_{11m}	σ_{22a}	σ_{22m}	σ_{12a}	σ_{12m}	δ_{22}	δ_{12}	N_{exp}
	491.0								22 627
201	**451.0**								57 003
	419.0								**100 000**
	392.0								175 227
	373.0	**373.0**							33 470
202	359.5	359.5							**100 000**
	344.0	**344.0**							367 156
					294.0				44437
203					286.0				**100 000**
					275.0				314 378

2083	275.3	−275.3	302.8	−302.8			90°		**100 000**
2103	291.9		321.1	321.1			90°		**100 000**
2143	**326.4**				**187.7**			**0°**	**100 000**
2163	288.8				166.1	166.1		90°	**100 000**
2173	291.4				167.5			60°	**100 000**
2183	284.4				163.5	163.5		0°	**100 000**
2193	308.5				177.4			90°	**100 000**
2203	399.4					199.7			**100 000**

The validity parameters according to both criteria are calculated for all the tests corresponding to 10^5 cycles (see Table 2). These validity parameters are reasonably close to the theoretical value ($I = 1$). The proposed criteria are in good agreement with results from testing under multiaxial periodic states of stress. However we can enquire the real signification of a value of 0.909 or 1.030 for the validity parameter I. That information is not really satisfying because it may be insufficient for a fatigue analysis. It is more judicious to predict fatigue lives in order to compare different stress states, since design engineers need to know when crack initiation occurs rather than if it happens or not. In addition, a deviation of the validity parameter can correspond to a very important deviation of the fatigue life. It is the reason why we have proposed a fatigue life prediction method.

Table 2 Validity parameters – experimental and predicted fatigue lives.

Test	I_{RB}	I_{FB}	N_{RB}	N_{FB}	N_{exp}	N_{RB}/N_{exp}	N_{FB}/N_{exp}
2083	1.032	1.090	48 523	29 190	100 000	0.5	0.3
2103		1.030		68 303	100 000		0.7
2143	1.078	1.018	43 636	80 186	100 000	0.4	0.8
2163		0.880		355 159	100 000		3.6
2173		0.890		311 085	100 000		3.1
2183	1.020	0.909	66 824	386 147	100 000	0.7	3.9
2193		0.920		178 279	100 000		1.8
2203	1.057	0.958	29 756	148 546	100 000	0.3	1.5

3 Fatigue Life Prediction Method

3.1 *Principle*

We have seen in Section 2 that any criterion $E([\sigma_{ij}(t)], \sigma_{-1}(N), \sigma_0(N), \tau_{-1}(N)) = 1$ can be interpreted as a limited fatigue life criterion suitable for a periodic state of stress.

The number N of cycles corresponding to the periodic state of stress $[\sigma_{ij}(t)]$ is the solution of this equation and is obtained from an iterative process. This procedure is an implicit method of fatigue life prediction.

The procedure is as follows: for a given number of cycles the identification of the criterion parameters is realized. Then, for the given state of stress $[\sigma_{ij}(t)]$, I is calculated. If $I < 1$ the fatigue strength is not reached for N. The criterion parameters are calculated for $N' > N$, and I is evaluated once again ... If $I > 1$, the fatigue strength is over-estimated for N. The criterion parameters are consequently calculated for $N' < N$; I is evaluated, and so on ...

The calculations are carried out for a large number of cycles. Therefore for each value of N, we have to know $\sigma_{-1}(N)$, $\sigma_0(N)$ and $\tau_{-1}(N)$ in order to identify the criterion parameters. In other words, we must know the three corresponding S-N curves.

3.2 *Validity of the proposed method*

Experimental data of Table 1 represent material fatigue characteristics with a crack initiation probability of 50%. The ten upper lines give the values used to define the three required S/N curves $\sigma_{-1}(N)$, $\sigma_0(N)$, and $\tau_{-1}(N)$. The relationship between $\log S$ and $\log N$ is assumed to be linear. All the numbers of cycles up to crack initiation (see Table 2) are calculated with that assumption. N_{RB} and N_{FB} denote the fatigue lifetimes obtained respectively with RB and FB criteria. The experimental life is 10^5 cycles for all the tests. The comparison is made by the use of ratios N_{RB}/N_{exp} and N_{FB}/N_{exp}.

Many observations can be made:

- For four tests, fatigue life predictions are not possible with RB criterion because fatigue life is out of the validity domain of the criterion.
- The number of cycles up to crack initiation has a greater significance than the validity parameter I. A little deviation of I corresponds to a greater deviation of the fatigue life. For instance I is equal to 0.958 and 0.909 (and N_{FB}/N_{exp} is equal to 1.5 and 3.9) respectively for tests 2203 and 2183.
- N_{FB}/N_{exp} is generally greater than N_{RB}/N_{exp}. RB criterion is less optimistic than FB criterion, except for test 2083 for which the stress components are out-of-phase.
- The predictions of FB criterion are closer to experimental values (0.3 to 3.9) than RB predictions when they are possible (0.3 to 4.9).

The ratios between predicted and experimental fatigue lifetimes are really different and more explicit than the corresponding validity parameters. In fact, fatigue life predictions are strongly dependent on the models chosen for S/N curves. The assumption of a linear dependence of $\log S$ versus $\log N$ may be a rough estimate. It would be interesting on the one hand to study the influence of S/N curves models and on the other hand to take into account the statistical likelihood of material data to assess properly fatigue lives. We have already done this for the validity parameters (**5**, **12**) by the use of the Monte-Carlo simulation method.

4 Extension of the Proposed Method

An extension to variable amplitude multiaxial stress history can be developed by using a counting method and a damage cumulation law (7–9). The counting variable divides the loading sequence into cycles denoted $[\sigma_{ij}(t)]_k$. A damage is associated to the fatigue life N_k assigned by the criterion to each cycle. The damage cumulation law gives the total damage corresponding to the whole sequence. A fatigue life prediction suitable for multiaxial random loading is given by this method.

The first step of the fatigue life prediction procedure is the application of the Rainflow counting method. In the case of a multiaxial sequence, the counting is not so easy to apply as for a traditional uniaxial loading history. It is possible to apply a Rainflow counting method to each component $\sigma_{ij}(t)$ of the stress tensor but generally peaks and valleys of one component do not correspond simultaneously to peaks and valleys of another component or do not have the same occurence frequency. It is therefore difficult to apply the Rainflow counting in order to obtain cycles at the same moment on each component.

This problem may be overcome by the definition of a variable $V(t)$ which describes properly the stress states and allows a Rainflow counting. Cycles of $V(t)$ are Rainflow counted. Let us denote ΔT_i the time period corresponding to the i^{th} cycle of $V(t)$. When ΔT_i is determined, the values of the components $(\sigma_{ij}(t), t \in \Delta T_i)$ of the stress tensor are pointed out. It is assumed that these values may be considered as the ones of a single stress cycle and allow a fatigue life calculation by the implicit method previously developed. In order to present the practical application of this method, we specify the choice of the counting variable $V(t)$.

In the case of random stresses, the principal stress axes generally rotate versus time t with respect to the body. Hence the maximum shear plane rotates also and may activate several crystallographic slip planes. From this point of view we could choose as counting variable $V(t)$ the root mean square of shear stress on all planes through a point. Let us denote the shear stress vector on a surface element with unit normal vector. The following relation can be proved for any time.

$$\sqrt{\frac{1}{S}\int_S \| \vec{\tau}_h \|^2 \, dS} = \frac{1}{\sqrt{15}} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2}$$

$$= \sqrt{\frac{3}{5}} \| \vec{\tau}_{oct} \| \tag{17}$$

where S is the area of a sphere of radius equal to unit ($S = 4\pi$); σ_1, σ_2, σ_3, are the principal stresses of the stress tensor and, $\| \vec{\tau}_{oct} \|$ is the magnitude of the octahedral shear stress vector.

Unfortunately if we set $V(t) = \| \vec{\tau}_{oct} \|$, the counting variable is always positive and therefore inappropriate for some stress states (3). For example when we

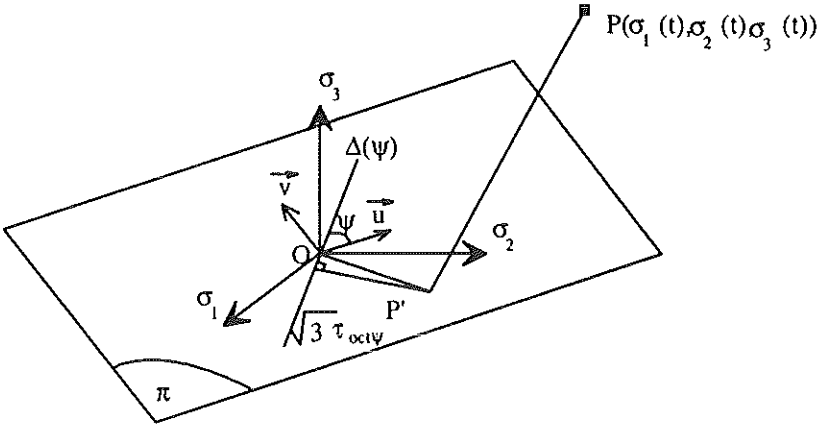

Fig 3 Definition of the counting variable.

consider a reversed torsion test, the Rainflow counting method would give two cycles although there is only one. In order to make a suitable counting, we use a geometrical property (see Fig. 3). Let us consider a given point P of coordinates σ_1, σ_2, σ_3 in the principal stress space. The projection P' of the point P onto the deviatoric plane (π) ($\sigma_1 + \sigma_2 + \sigma_3 = 0$) is such that

$$\mathrm{OP}' = \sqrt{3}\,\|\vec{\tau}_{oct}\| \tag{18}$$

The tip point of the vector $\overrightarrow{\mathrm{OP}'}$ describes a curve, in the deviatoric plane, versus time t. Finally we see that the vector $\overrightarrow{\mathrm{OP}'}/\sqrt{3}$ represents the octhaedral shear stress vector in the space of stresses. Its projection on axis $\Delta(\psi)$ of plane (π), denoted $\tau_{oct\psi}$ is the counting variable $V(t)$. We have

$$V(t) = \tau_{oct\psi} = = \left(\frac{\overrightarrow{\mathrm{OP}'}}{\sqrt{3}}\right).\vec{\Delta}(\psi) \tag{19}$$

The Rainflow counting method used is the AFNOR Recommendation A03 – 406 (**10, 11**).

The implicit method of fatigue life prediction realizes the second step of the procedure. It associates a number of cycles N_k to each extracted cycle. A damage law and its cumulation rule give the damage $D(\psi)$ corresponding to the whole sequence for a given angle ψ. The predicted number n of stress sequences up to crack initiation is obtained with

$$n = \frac{1}{\max\limits_{\psi} D(\psi)} \tag{20}$$

5 Conclusions

A fatigue life prediction method using multiaxial criteria is presented. First two criteria for long and finite fatigue life are proposed. Both of them use a damage indicator which takes into account the shear stress and also the normal stress on the surface element. The first proposition (RB criterion) is based on the research of the critical plane. The second one (FB criterion) uses the root mean square of the damage indicator.

With the tests used, the results given by FB criterion are better than those provided by RB criterion. A comparison with other tests is necessary to give a more exhaustive conclusion.

An implicit method which allows the prediction of fatigue life up to crack initiation is then described. It shows that from an engineering point of view the fatigue life prediction gives a more accurate information about the fatigue behaviour of components than the value of the criterion.

The proposed method is implemented as post-processor of a finite-element code. Its extension to variable amplitude multiaxial loading and experimental validation are in progress.

Acknowledgements

This research was supported by the French Department of Research and Space. The close involvement of industrial partners IRSID, SOLLAC – Florange and Framasoft + CSI– is gratefully acknowledged.

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