J. Lu, J. F. Flavenot,* A. Diboine,† S. Lasserre,‡ C. Froustey,‡ M. Bennebach,‡ and T. Palin-Luc‡*

Development of a General Multiaxial Fatigue Criterion for High Cycles of Fatigue Behaviour Prediction

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ABSTRACT The aim of our study is to propose a global approach for calculating the fatigue strength in the presence of multiaxial stress field and stress gradient. In order to develop a general criterion, we have tried to take into account the different important effects involved, based on the results obtained for different materials. These include the notch effect and the scale effect (that is to say the gradient effect) in the case of notched components, the effect of a combined in-phase or out-of-phase stress and the effect of residual stresses. To complete the study and validate the criterion on a material in the presence of all the parameters, tests were carried out on FGS 700–2 cast iron, strengthened using roller burnishing.

1 Introduction

The most frequent and most serious incident likely to occur to a mechanical structure is its failure as a result of fatigue.

Although it is now possible to calculate the fatigue strength of a component or structure subjected to uniaxial stresses, a solution has still not been found for the more general case in which the stresses are multiaxial and the component is subjected to complex static stresses resulting from either loading or residual stress produced during manufacture.

There is no recognized general calculation method which can be used when the applied cyclic stresses are multiaxial and when a stress gradient is present (due to either the loading or to the geometry of the component). In reality, stresses are very rarely simple, but rather a combination of normal alternating stresses and shear alternating stresses superposed on mean stresses. It is practically impossible to test each material, under all the possible combinations of alternating stress and static stress; however, it is essential to make an estimation of the fatigue strength or the lifetime of a mechanical part based on the usual laboratory tests.

*Cetim Senlis, France.
†Renault, Research Department, Boulogne Billancourt, France.
‡Ensam Bordeaux, France.

2 Definition of the Criterion

2.1 *Basic criterion*

The aim of our study is not to propose a new criterion to calculate the fatigue strength for high cycle fatigue. In the text which follows, we are going to adapt existing criteria to reduce calculation errors according to the type of loading. For all the cases analysed, conventional critera, involving the amplitude of octahedral shear ($\tau_{oct,a}$) or maximum shear ($\tau_{max,a}$) and the maximum hydrostatic pressure (P_{max}) already produce satisfactory results. We have simply modified the formula for the case of out-of-phase combined stresses, and cases in which there is a stress gradient in the critical zones caused by stress concentration.

The basic criterion can therefore be expressed as indicated below

$$\tau_{max,a} \text{ or } \tau_{oct,a} = f(\mathrm{A}, \mathrm{B}P_m, CP_a)$$

An example can be given using the formula below

$$\tau_{max,a} \text{ or } \tau_{oct,a} = \mathrm{A} + \mathrm{B}P_m^{\mathrm{D}} + CP_a^{\mathrm{E}}$$

A, B, C, D and E are material constants. The coefficients B and C enable the effect of the mean hydrostatic pressure (P_m) to be separated from the alternate hydrostatic pressure (P_a). The exponents D and E offer the possibility of formulating the hydrostatic pressure effect nonlinearly.

If τ_a is taken in the maximum shear plane, the D = E = 1 and B = C, we get the Dang Van criterion (**1**).

If τ_a is taken on the octahedral shear plane ($\tau_{oct,a}$), when D = E = 1 and B = C, we get the Crossland criterion (**2**); when D = E = 1 and C = 0, we get the Sines criterion (**3**) and when D = E = 1 and B is different from C, we get the Kakuno criterion (**4**).

This type of development can be continued to invent new criteria but it only makes matters more complicated, because more and more parameters have to be determined, since even with a Dang Van linear relationship, two Wöhler curves have to be determined to obtain at least the two points needed to construct the diagram. If additional stresses are introduced, the test plane will be even greater which means that the criterion cannot be used in industry, with the necessity to simplify the criterion as much as possible. In our case, the Dang Van criterion is used as a basic criterion.

In this criterion, it is indicated that the calculations must be carried out with the material in the elastic shakedown state. This enables the effect of residual stresses to be taken into account directly. But, at present, it is difficult to calculate stabilized residual multiaxial stresses, especially for an out-of-phase stress. This makes a complete calculation difficult, for the simple reason that very few studies have been carried out in the past in which taking the residual stress effect into account in the calculation of complex stresses is analysed in detail. In our study, the stabilized residual stresses are calculated for simple stresses (tension and compression) and measured for other types of stresses.

It has already been demonstrated in the past that a large number of the criteria proposed and the Dang Van criterion in particular do not satisfactorily deal with loadings or geometries which produce steep stress gradients in components (**5**) or with the phase lag effects of combined stresses (**6-8**).

2.2 *Taking the scale effect and notch (or the stress gradient) effect into account*

The scale effect under fatigue stress is a result of the following observation: for the same nominal stress, the greater the dimensions of the part, the lower the fatigue strength.

The scale effect can have two main causes.

- a mechanical cause (the stress gradient effect: the greater the stress gradient, the greater the fatigue strength);
- a statical origin (the larger the component, the greater the chance of finding a defect which will initiate a fatigue crack).

It is well established that it is mainly the gradient effect which causes the scale effect.

In the case of the notch effect, we have to take into account both a local increase in the stress and the presence of a stress gradient which increases with the stress concentration in the notch.

To take the gradient effect and therefore the notch effect into account, we propose to use the critical layer criterion (**5**).

From a physical point of view, the critical layer criterion is based on the following reasoning: the fatigue behaviour of a material cannot be explained by simply observing the stresses which exist at a particular point in the material. The heterogeneous nature of metals and the breakdown of stresses below the surface where the fatigue crack is initiated are necessarily involved in any damage process caused by fatigue. It would therefore seem judicious to apply the calculation to a sufficiently large volume in order to represent the real behaviour of the material, whence the idea of a volume or distance which is characteristic of the material. In (**9**) and (**10**), it has been demonstrated that, for the same material, the larger the grain, the thicker the critical layer.

The tension–compression results obtained by (**11**) on a quenched and tempered 37Gr4 grade steel part are given in Table 1. It can be seen that for $K_t = 1$, there is no scale effect, as indicated in the bibliography for the other tests. On the other hand, for $K_t = 2$, there is a fairly large discrepancy in terms of the fatigue strength; the greater the diameter, the lower the fatigue strength.

Figure 1 shows the experimental points located in the Dang Van diagram of the material using the K_t coefficients of the test specimens (theoretical stress concentration factor). A fairly large difference is therefore observed for the same $K_t = 2$ according to the diameter of the test specimen. Figure 2 shows the results

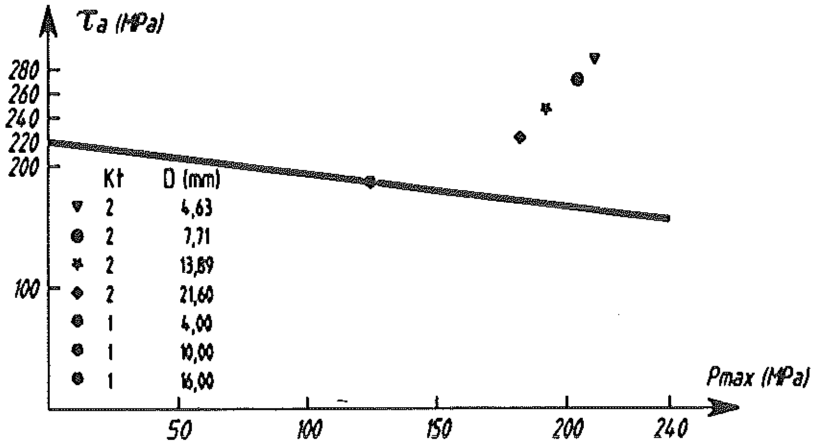

Fig 1 Dang Van diagram showing the experimental points obtained for different tension–compression test specimens.

obtained for a critical layer of 150 μm using the finite-element calculation method. It can be seen that, in this case, there is much less scatter. It can therefore be concluded that, in the case of tension–compression, the critical layer approach takes into account both the scale effect due to the volume effect and the stress gradient.

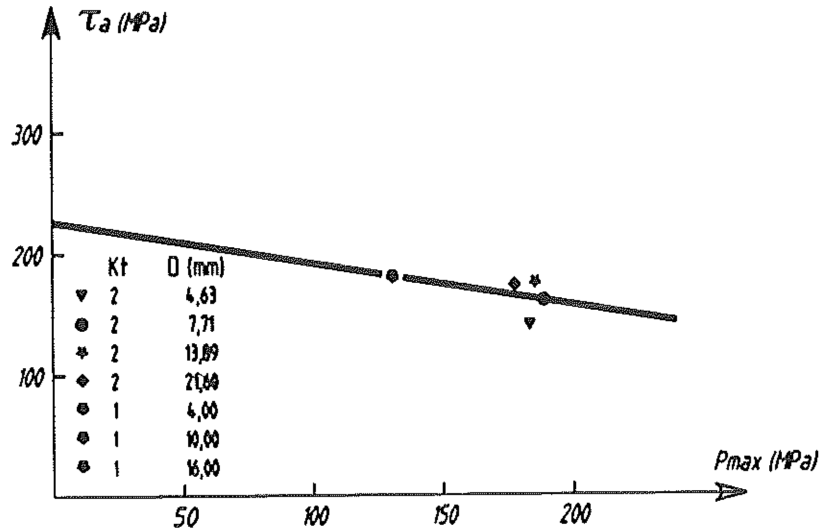

Fig 2 Dang Van diagram obtained for stresses calculated by averaging over a critical layer thickness of 150 μm for tension–compression test specimens with different K_t coefficients and dimensions.

Table 1 Size effect for a 37Cr4 grade steel (11)

K_t	1	1	1	2	2	2	2
Diameter (mm)	4	10	16	4.63	7.71	13.89	21.6
Fatigue strength σ_D (MPa)	370	370	370	255	240	220	200

2.3 *Taking residual stresses into account*

A detailed discussion of the problem has already been developed in (**12–15**). We use the same type of approach by making calculations based on the stabilized residual stress distributions measured using the X-ray diffraction method.

2.4 *Taking in account the combined in-phase and out-of-phase stress effect*

The different results show that the original formula for the Dang Van criterion does not take out-of-phase combined stresses into account. Several criteria have been proposed to deal with this phenomenon (**6–8**). But the results obtained in our study show that these criteria do not enable either the residual stresses or the stress gradient effects to be satisfactorily integrated. We therefore propose a global approach.

The basis of this approach is to find a criterion based on the shear amplitude and the hydrostatic pressure which takes into account the effect of out-of-phase combined stresses.

In the literature, the stress which has most often been studied is combined bending and torsion.

The Dang Van criterion can be written in the following form

$$\frac{1}{2}\sqrt{\sigma_a \sin(\omega\tau))^2 + 4(T_a \sin(\omega\tau + \Psi))^2} = A + \frac{B}{3}(\sigma_m + \sigma_a \sin(\omega\tau))$$

where σ_a = stress amplitude during bending, σ_m mean stress during bending; T_a = shear stress amplitude during torsion; Ψ = phase lag angle.

Since this formula does not take the results during out-of-phase loading stress into account, we suggest to modify it as follows. Shear amplitude according to: Tresca or Mises (τ_a, maximized on all the component axes and for all the terms of the loading sequence) = A + BP_{max} where P_{max} = maximum hydrostatic pressure; A, B = material constants.

This gives, for a combined torsion–bending out-of-phase loading, using Tresca criterion

$$\frac{1}{2}\sqrt{(\sigma_{a\,max}) + 4K(T_{a\,max})^2} = A + \frac{B}{3}(\sigma_m + \sigma_{a\,max})$$

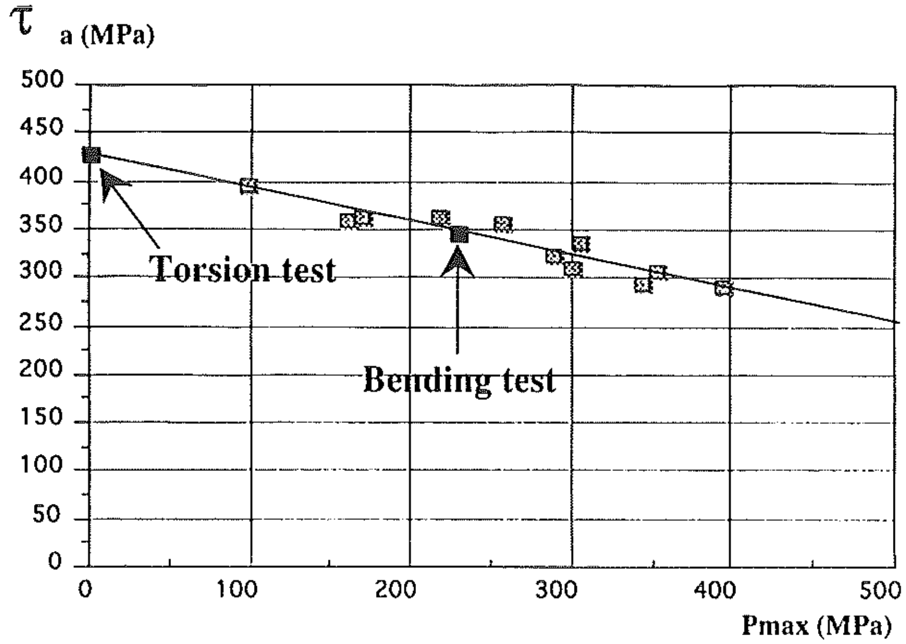

Fig 3 Diagram for a 35NCD16 grade steel (8) for combined in-phase bending–torsion tests.

where $\sigma_{a\,max}$ is the maximum amplitude of the bending stress; $T_{a\,max}$ is the maximum shear amplitude due to the torsion, while σ_m is the average bending stress; K is a so-called phase lag-sensitive coefficient which depends on the nature of the material, the frequency and the level of the stress, the type of stress (biaxial tensile stress, tensile/torsion stress, bending/torsion). The greater the frequency (the lower the loading in general), the closer the value of the K coefficient is to 1. If there is no stress phase lag, then $K = 1$.

The physical significance of this formula is as follows: when there is dephased stress, the time needed by the material to react depends on the nature of the material and the loading level. The material remembers, as it were, the maximum previous stress along another stress axis. Therefore, the stress level to be taken into account is not the maximum stress along one axis and the instantaneous stress along the other axes, but the maximum stress along all the axes. When the stress frequency is high, the time needed for the memory effect to occur is even lower still. To put it more simply, this means 'putting all the stresses back in phase'.

In the studies on fatigue after a large number of cycles (a few million), the frequency used to carry out the tests is usually 20 to 40 Hz. This means that a phase lag of 90° would give a time-lag between the two maximum peaks of

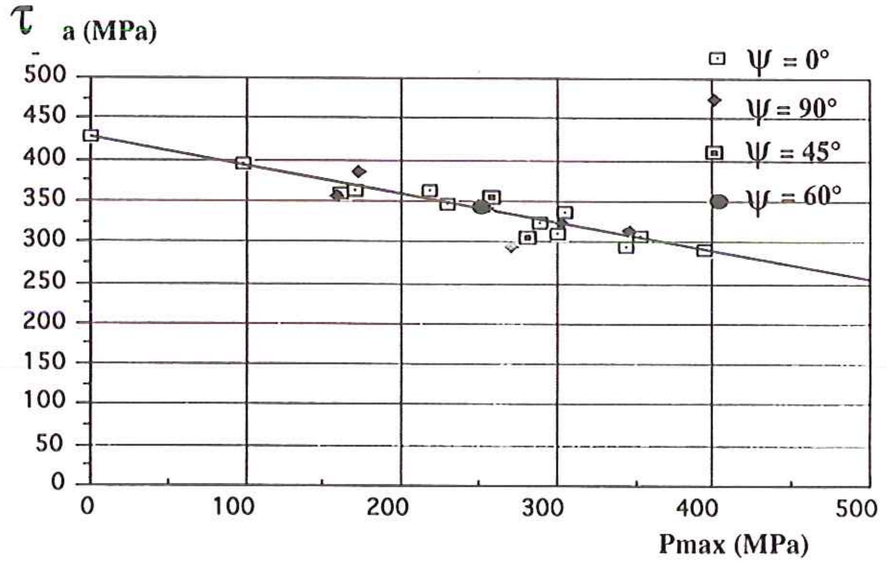

Fig 4 Diagram for a 35NCD16 grade steel (8) for combined out-of-phase bending–torsion tests with different phase lag angles.

0.0125 and 0.00625 seconds respectively. For a phase lag of 30°, the time is divided by three.

Although the stresses should theoretically be transmitted instantaneously with regard to these times, it can be imagined that the time is short enough for the material to have a slight memory. In fact, locally, when there are zones in the material with defects where cracks are initiated, the time needed for these stresses to be propagated can be different. It is for this reason that we cannot obtain sufficiently coherent results if the phase lag phenomenon is considered in an idealized way in the case of fatigue after a very large number of cycles.

However, this type of approach is more successful for combined low-cycle fatigue. In this case, the test frequency is much lower (about 0.1 Hz). The memory time needed is much longer and the loading greater.

By introducing the K coefficient, this memory effect can be expressed gradually according to the frequency and loading level and the nature of the material.

Figures 3 and 4 give Dubar's results (8) treated using the new approach. For the case of in-phase loading (Fig. 3) the results are similar to the classical Dang Van criterion. For the case of out-of-phase loading, the results obtained by the next approach are better than those obtained with the classical approach. Figure 5 shows the errors in a maximum produced by different classical criterion in the case of out-of-phase. The error is a maximum (near 20%) by the classical Dang Van criterion of a phase lag angle 90°. An excellent result for the $K = 1$ value is obtained. For this case, the frequency of the test is about 50 Hz, so the memory effect is very important.

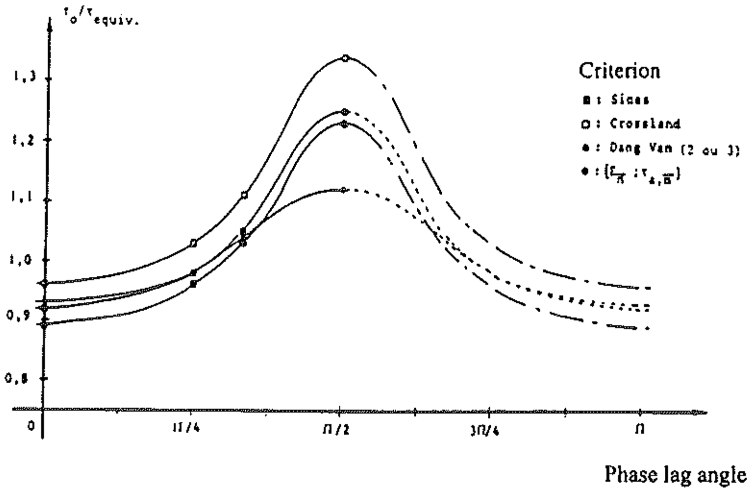

Fig 5 The errors induced as a function of the phase lag angle for out-of-phase bending–torsion tests for a 35NCD16 grade steel (7).

Figure 6 shows the results of (16) treated using the new method. A distinct improvement can be noted in the precision of the prediction with a $K = 1$. In this case, the fitting with the calculation is not perfect. In fact, the frequency of the test is 18 Hz, so the memory effect is smaller than the previous example.

2.5 *Proposal for a global approach*

By synthesizing the different points studied in the paragraphs above, the global

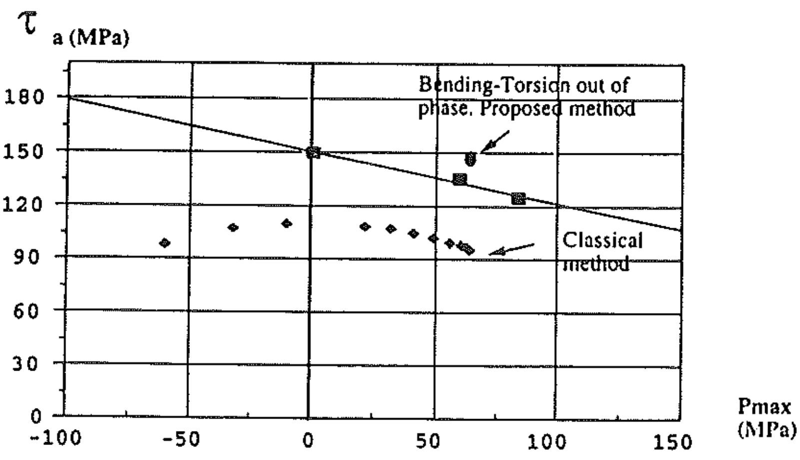

Fig 6 Comparison of results between the classical Dang Van criterion and the new approach of the calculation of the fatigue strength under combined out of phase bending and torsion test for a sintered steel (FE–1,5%Cu) (16).

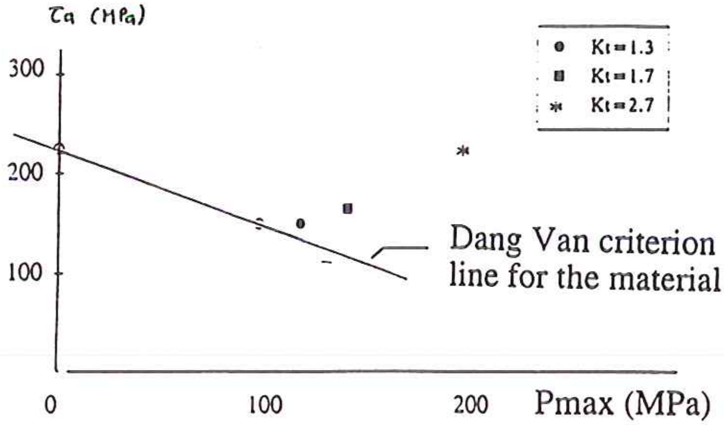

Fig 7 **Dang Van diagram for FGS 700–2 cast iron.**

criterion proposed is therefore a criterion which takes into account the 'phase lag memory effect'. It is used for a stabilized state of residual stresses and of the material, averaged out for a basic damaged volume (thickness of critical layer).

3 Validation of the New Approach on FGS 700–2 Cast Iron

The global criterion was used to calculate the fatigue strength of cast iron (FGS 700–2). The residual stresses were introduced by roller burnishing (17).

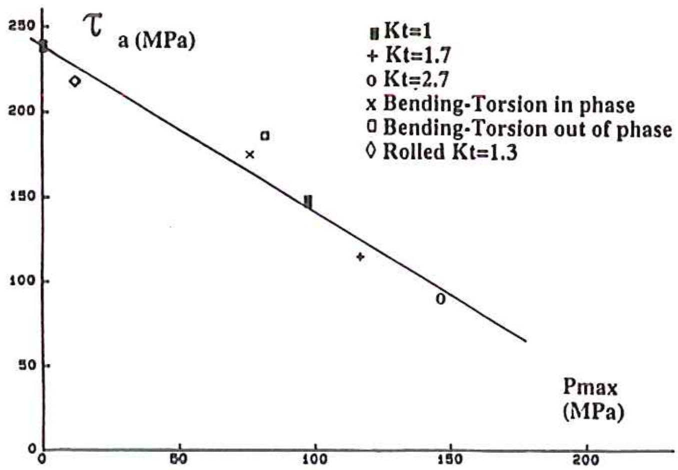

Fig 8 **Global approach diagram in a τ_a vs P_{max} frame of reference.**

Figure 7 shows the results obtained with the Dang Van criterion. Note that the phase lag and notch effects are not taken into account well, even if elastoplastic calculations are carried out when plasticity occurs in the notch.

Figure 8 shows the results obtained with the global approach using adequate "critical layer depth". It can be seen that dispersion of the results is considerably reduced, in the case of bending–torsion loading.

4 Conclusion

A global approach has been proposed so that all the effects which influence the fatigue limit after a large number of cycles can be taken into account. In the new criterion and for the case of bending–torsion loading, we suggest 'putting the combined stresses back into phase' in order to introduce the idea of a sort of 'material memory' of the stresses. The overall results for all the results obtained on a grade of cast iron show that this approach improves the prediction quality with respect to basic criteria of the Crossland or Dang Van type. In the future, developments will be made to introduce this type of approach to research consistencies in the mechanical engineering and car industries. Software for calculating the fatigue strength or life, which takes residual stresses, the effects of stress gradients and phase lag of combined stresses into account will also be developed. The new criterion can be easily connected with a finite-element codes.

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