STRAIN PARAMETERS FOR FATIGUE LIFE PREDICTION UNDER OUT-OF-PHASE BIAXIAL FATIGUE

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ABSTRACT

Two new strain parameters are proposed for correlating fatigue failure under out-of-phase biaxial loading conditions. The parameters are based on the maximum shear strain and the normal strain acting on the plane of maximum shear strain. It is shown that fatigue life prediction approaches based on maximum strain amplitudes, which may be applicable under proportional fatigue loading, are not suitable for out-of-phase multiaxial fatigue. The new parameters depend on the phase difference between controlled strains on the surface of the material. It is further shown that under out-of-phase biaxial loading conditions, three crack growth planes may be operative within a cycle but only one or two of the planes dominates and causes failure. The dominating crack growth plane depends on the phase angle. The final fracture plane is directed along the intersection of the material's free surface and the dominant crack plane. Tests conducted on a low alloy carbon steel A516 Grade 70 used extensively in the fabrication of pressure vessels are analysed and plotted on a constant endurance curve using the new parameters.

INTRODUCTION

A number of fatigue studies has been undertaken in recent years to aid engineers in their attempt to derive, from simple laboratory test data, theories which will permit an adequate assessment of the fatigue behaviour of materials under various loading conditions. These studies have been carried out in order to generate valid experimental data and to propose adequate criteria for correlating the test results [1-4]. Most of the experimental work in the literature has been carried out under proportional multiaxial loading conditions. In real life situations, however, the loading may be complex, that is, the principal surface stresses or strains may be applied out-of-phase with each other resulting in a rotation of principal stress or strain axes during each cycle of loading. Reviews by many investigators [1, 5] have shown that attempts to predict out-of-phase fatigue life from in-phase results are highly unconservative.

In this paper, a new approach using strain-based parameters is proposed for predicting fatigue life under out-of-phase multiaxial loading conditions. The new approach is based on the observation of the process of crack initiation and propagation under complex strain states. Experimental results obtained from out-of-phase loading of thin-walled tubes subjected to combined axial load and internal-external pressure are correlated using the new parameters.

REVIEW OF MULTIAXIAL FATIGUE

In-phase multiaxial fatigue

A general review of fatigue failure theories can be found in the literature [1-4] and as such no exhaustive review of the subject will be given in this paper. We shall only review those aspects relevant to the present study. Different experimental methods have been used to study the fatigue failure of materials. The most common is the in-phase loading of thin-walled tubes subjected to combined cyclic tension and torsion. For this type of loading, the controlled surface strain ratios are proportional throughout a cycle and limited to between -1 and $-\nu$ where ν is Poisson's ratio. Experimental results for positive ratios of the principal surface strains, using thin-walled tubes, are very limited since this requires the development of complicated testing systems involving a simultaneous application of axial load and internal-external pressure [4, 6, 7]. Another method used in the study of both the negative and the positive strain ratio values consists in applying two mutually perpendicular axial loads on cruciform specimens [8]. This method also requires the development of complicated testing systems besides the fact that it is difficult to determine the strains imposed on the gauge section of the cruciform specimen [9].

Two different approaches have been used in the literature in the development of stress or strain based fatigue criteria. The first is based on a direct adaptation of plastic yield criteria of Tresca and von Mises where the multiaxial stress or strain state is reduced to an equivalent uniaxial stress or strain state. Reviews by many investigators [10, 11] have shown that these phenomenological criteria are very limited in their predictive capability and are not able to account for variations in life observed under different multiaxial loading conditions.

The second approach considers the physical mechanisms of fatigue failure, which are crack initiation and growth, in identifying the salient macroscopic variables controlling fatigue failure. Fatigue cracks initiate at surface defects resulting from the accumulation of slip bands into slip zones called persistent slip bands in single crystals and these slip bands occur on planes of maximum resolved shear stress. For polycrystals, fatigue cracks also initiate from microscopic shear movements along slip planes and crack propagation is in general along the plane of maximum shear stress [12, 13]. The rate at which the cracks propagate is influenced by the normal stress on the plane of maximum shear stress and by other microstructural factors such as intercrystalline decohesion, inclusions and the presence of more than one phase in the material. Crack propagation still occurs by a process of shear decohesion at the crack tip and the direction of the crack is in general normal to the greatest principal stress [14].

The need to use a directly measurable parameter in correlating low-cycle fatigue led Brown and Miller [10] to propose that crack initiation and growth is controlled by two parameters: the maximum amplitude of shear strain in the plane of maximum shear stress $\gamma_{max}/2 = (\varepsilon_1-\varepsilon_3)/2$ and the amplitude of the normal strain $\varepsilon_n = (\varepsilon_1+\varepsilon_3)/2$ acting on the plane of maximum shear strain over a cycle of loading. Brown and Miller [10] then proposed that under proportional loading, constant endurance fatigue curves can be described using the following relations:

$$f(\gamma_{max}/2) + g(\varepsilon_n) = 1 \quad \text{for Case A}$$

and (1)

$$\gamma_{max}/2 = \text{constant} \quad \text{for Case B}$$

where $\varepsilon_1 > \varepsilon_2 > \varepsilon_3$ are the principal strain amplitudes over a cycle. Case A corresponds to combined axial and torsional loadings, while Case B corresponds to principal strain ratios between $-\nu$ and 1. The relation proposed for Case B does not allow to account for the influence of hydrostatic pressure on fatigue life [15]. To account for this, Lefebvre [16] proposed a relation of the form:

$$\frac{\gamma_{max}}{2} + k\,\varepsilon_s = \text{constant} \qquad (2)$$

where $\varepsilon_s = \varepsilon_n$ if $-1 \leqslant \rho < -\nu$ (Case A), $\varepsilon_s = \varepsilon_m = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$ if $-\nu \leqslant \rho \leqslant 1$ and k is a constant.

Out-of-phase multiaxial fatigue

Under out-of-phase loading conditions, the stress-strain history is complex, that is, corresponding principal stresses or strains are non-proportional and the principal directions of load rotate during each cycle. The maximum range of shear strain and the normal strain on the plane of maximum shear strain are no longer in-phase and relations such as those described in the preceding section are no more applicable. Besides, the idea that only a single plane experiences the maximum shear strain is no longer valid; more than one plane may experience the same value of maximum shear strain in a cycle of loading. Because of this, unusual fracture modes may be present and fatigue life may be reduced depending on the value of the phase difference between applied strains [2, 17].

In the high-cycle regime, McDiarmid [18] has reanalysed the results of several researchers and concluded that the important parameters controlling fatigue failure are the maximum range of shear stress and the stress acting normal to the plane of maximum shear stress. A very limited amount of work has been done in low-cycle fatigue under out-of-phase multiaxial loading conditions. Results of low-cycle fatigue under non-proportional loading has been reviewed by Jordan et al. [19] who concluded that fatigue assessment techniques based on strain amplitudes cannot accomodate complex loading paths because strain path influences endurance. They also suggested that the accumulation of damage, that is crack extension, should be assessed throughout the cycle because a gradual build-up of damage during a cycle is physically more realistic than a sudden discontinuous crack extension on reaching the peak strain values. Jordan et al. [19] also noted that integral theories (see Taira et al. [20] and Garud [21]) yield a much closer representation of experimental results than strain amplitude based theories.

STRAIN ANALYSIS

Tests were conducted under strain control on thin-walled tubes subjected to combined axial load and internal-external pressure. The controlled cyclic strains were:

$$\begin{aligned} \varepsilon_a &= \varepsilon_{amax} \sin(\omega t) + \varepsilon_{am} \\ \varepsilon_t &= \rho\, \varepsilon_{amax} \sin(\omega t + \phi) + \varepsilon_{tm} \end{aligned} \qquad (3)$$

where the subscripts a and t refer to the axial and tangential directions of the tube, respectively, $\rho = \varepsilon_{tmax}/\varepsilon_{amax}$, ε_{am} and ε_{tm} are mean strains and ϕ is the phase angle. The third uncontrolled strain ε_r in the radial direction of the tube is calculated from the following relation:

$$\varepsilon_r = \frac{-\nu^e}{1-\nu^e}(\varepsilon_a^e + \varepsilon_t^e) - \frac{-\nu^p}{1-\nu^p}(\varepsilon_a + \varepsilon_t - \varepsilon_a^e - \varepsilon_t^e) \qquad (4)$$

where ν^e , ν^p are elastic and plastic Poisson's ratios, respectively.

At each instant of loading during a cycle, the instantaneous values of the maximum shear strain γ_{max} and the normal strain ε_n on the plane of maximum shear are given by:

$$\frac{1}{2}\gamma_{max} = \frac{\varepsilon_1 - \varepsilon_3}{2}$$

$$\varepsilon_n = \frac{\varepsilon_1 + \varepsilon_3}{2}$$

where ε_1, ε_2 and ε_3 are the instantaneous values of the three principal strains and are such that $\varepsilon_1 \geqslant \varepsilon_2 \geqslant \varepsilon_3$.

The evolutions of the two strain parameters $\gamma_{max}/2$ and ε_n over a cycle are shown in Figs. 1(a) and (b) for different phase angles. The figures have been traced for $\rho = 1$ and $\varepsilon_{amax} = \varepsilon_{tmax} = 1\%$. From these two figures, it would be seen that under in-phase loading ($\phi = 0°$ and $180°$), variations of the two parameters are sinusoidal and the maximum amplitudes of $\gamma_{max}/2$ and ε_n occur at the same time. However, under out-of-phase loading conditions, $0° < \phi < 180°$, maximum amplitudes of $\gamma_{max}/2$ and ε_n do not occur at the same time, the variations are of any shape and depend on the phase angle. Of importance again in Fig. 1(b) is the fact that the variation of ε_n attains its maximum value when $\phi = 90°$ and the minimum variation occurs when $\phi = 0°$ (except for $\phi = 180°$ where $\varepsilon_n = 0$ throughout the cycle). Consequently, it may be possible to use the amplitudes of $\gamma_{max}/2$ and ε_n as fatigue correlating

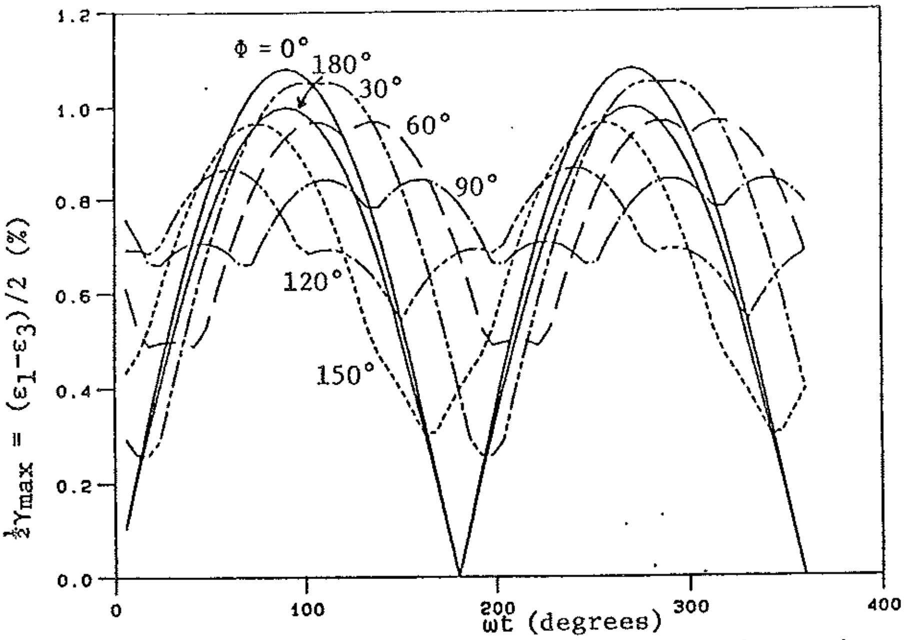


Fig. 1(a) - Variation of the values of the maximum shear strain $\frac{1}{2}\gamma_{max} = (\varepsilon_1-\varepsilon_3)/2$ over a cycle of loading at different phase angles and for $\varepsilon_{amax} = \varepsilon_{tmax}$ = 1% and ν = 0.33.

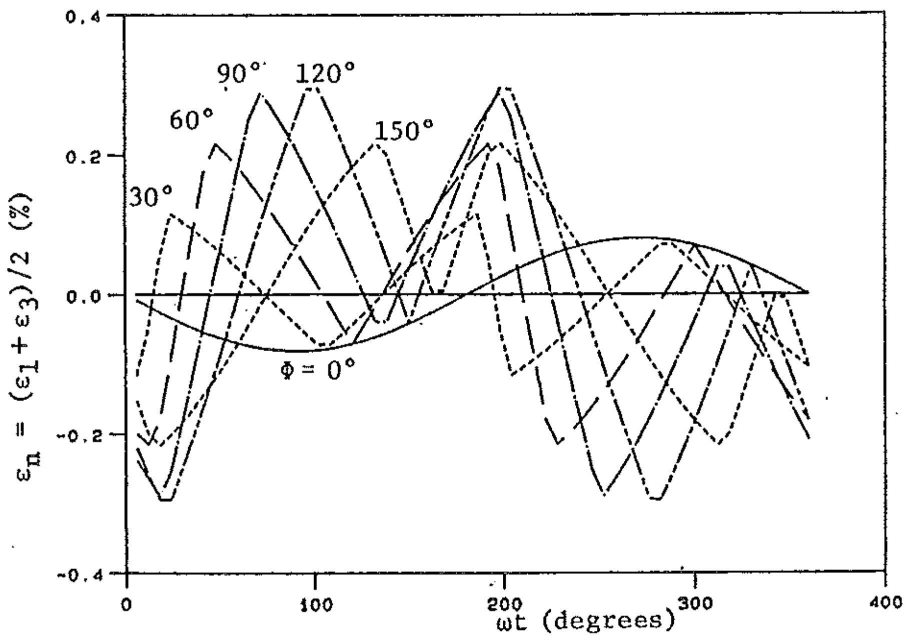


Fig. 1(b) - Variation of the values of the normal strain $\varepsilon_n = (\varepsilon_1+\varepsilon_3)/2$ acting on the plane of maximum shear strain over a cycle of loading at different phase angles and for $\varepsilon_{amax} = \varepsilon_{tmax}$ = 1% and ν = 0.33. For ϕ = 180°, ε_n = 0 throughout the cycle.

parameters under proportional loading, but under non-proportional loading it would be necessary to consider not only the amplitudes of $\gamma_{max}/2$ and ε_n but also the way these parameters vary over a cycle.

CRACK GROWTH PLANES

An examination of the rotation of the principal strain parameters ε_1, ε_2, ε_3, $\gamma_{max}/2$ and ε_n over a cycle shows that three distinct cracking systems can be operative within any single cycle [2]. Each system forms a different crack growth plane and is defined in relation to material axes as follows:

System 1: $\varepsilon_1 = \varepsilon_a$, $\varepsilon_2 = \varepsilon_r$ and $\varepsilon_3 = \varepsilon_t$ or $\varepsilon_1 = \varepsilon_t$, $\varepsilon_2 = \varepsilon_r$ and $\varepsilon_3 = \varepsilon_a$

$$\frac{1}{2}\gamma_{max} = \frac{1}{2}\left|\varepsilon_a - \varepsilon_t\right| \quad \text{and} \quad \varepsilon_n = \frac{1}{2}(\varepsilon_a + \varepsilon_t)$$

System 2: $\varepsilon_1 = \varepsilon_a$, $\varepsilon_2 = \varepsilon_t$ and $\varepsilon_3 = \varepsilon_r$ or $\varepsilon_1 = \varepsilon_r$, $\varepsilon_2 = \varepsilon_t$ and $\varepsilon_3 = \varepsilon_a$

$$\frac{1}{2}\gamma_{max} = \frac{1}{2}\left|\varepsilon_a - \varepsilon_r\right| \quad \text{and} \quad \varepsilon_n = \frac{1}{2}(\varepsilon_a + \varepsilon_r)$$

System 3: $\varepsilon_1 = \varepsilon_t$, $\varepsilon_2 = \varepsilon_a$, $\varepsilon_3 = \varepsilon_r$ or $\varepsilon_1 = \varepsilon_r$, $\varepsilon_2 = \varepsilon_a$ and $\varepsilon_3 = \varepsilon_t$

$$\frac{1}{2}\gamma_{max} = \frac{1}{2}\left|\varepsilon_t - \varepsilon_r\right| \quad \text{and} \quad \varepsilon_n = \frac{1}{2}(\varepsilon_t + \varepsilon_r)$$

The fracture planes corresponding to each system of crack is schematized in Fig. 2. System 1 corresponds to cases where the planes of maximum shear strain are at ± 45° to the axial direction of the specimen and cracks propagate along this direction on the surface of the specimen. System 2 corresponds to cases where the maximum shear strain plane is in-depth and inclined at 45° to the free surface of the specimen. The fracture plane for this system intersects the free surface and grows along the circumferential direction of the specimen. System 3 is also an in-depth plane similar to System 2 but in this case, the fracture plane intersects the free surface and grows along the axial direction of the tube.

Because each system of crack acts on its corresponding planes for a certain period of time during a cycle of loading, we shall henceforth refer to them as zones, that is, System 1 becomes Zone 1, etc. Fig. 3 shows the experimental evolution of the strain parameters and the widths of each zone at half-life for the particular case where $\phi = 90°$, $\varepsilon_a = 0.12\%$, $\rho = 1$ and $N_f = 630$ where N_f is the number of cycles to failure. Zone 1 corresponds to Case A of Brown and

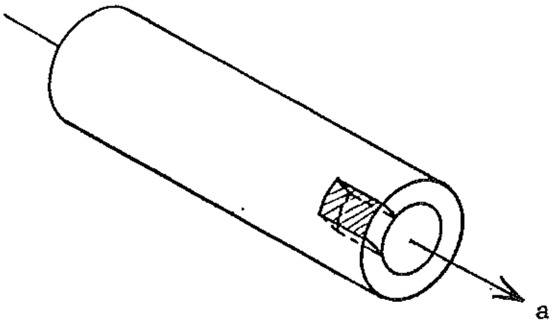
Miller [10] and is such that the instantaneous strain ratios ρ_i^A is between -1 and $-\nu$ where $\rho_i^A = \varepsilon_3/\varepsilon_1$. Zones 2 and 3 correspond to Case B of Brown and Miller [10] and the instantaneous strain ratios ρ_i^B is between $-\nu$ and 1 where $\rho_i^B = \varepsilon_2/\varepsilon_1$.

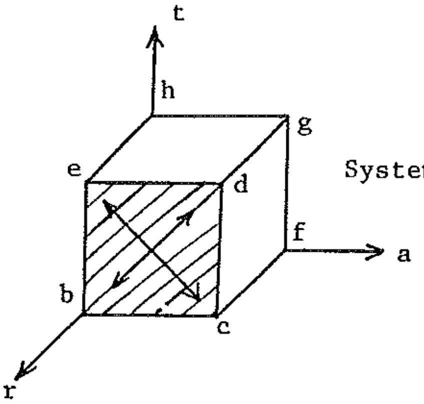
From test results under in-phase biaxial fatigue, it is known that for the same maximum shear strain value, fatigue life is much shorter for Case B than for Case A. One would then expect that when Zones 2 and 3 dominate in a cycle fatigue life would be shorter than when Zone 1 is the dominating plane for the same maximum shear strain value.

As shown in Table 1, the relative width of each zone in a cycle varies depending on the phase angle. When $\phi = 0°$, only Zones 2 and 3 are present, while for $\phi = 90°$, all the zones are present and each fills a third of the complete cycle. As a result of the change in the relative importance of each zone as a function of phase angle, fatigue life and failure mode will be influenced by the width of each zone. Consequently, failure mode corresponding to Zone 1 (Case A tension-torsion) will dominate for phase angles between 150° and 180°. On the other hand, failure modes corresponding to Zones 2 and 3 will dominate for $0° < \phi < 90°$. Finally, since failure planes corresponding to Zones 2 and 3 are through thickness planes, the two zones may interfere and reinforce each other, then one would expect that failure modes corresponding to Zones 2 and 3 be dominant for phase angles up to 120°.

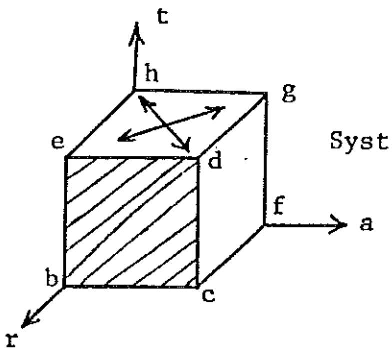
Table 1 - Relative width of each zone for different phase angles and for $\rho = \varepsilon_{tmax}/\varepsilon_{amax} = 1$.

	Zone 1	Zone 2	Zone 3
Uniaxial	$\frac{1}{2}$	$\frac{1}{2}$	0
$\phi = 0°$	0	$\frac{1}{2}$	$\frac{1}{2}$
$\phi = 30°$	$\frac{1}{10}$	$\frac{4.5}{10}$	$\frac{4.5}{10}$
$\phi = 60°$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{2}{5}$
$\phi = 90°$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
$\phi = 120°$	$\frac{2}{5}$	$\frac{1.5}{5}$	$\frac{1.5}{5}$
$\phi = 150°$	$\frac{3}{5}$	$\frac{1}{5}$	$\frac{1}{5}$
$\phi = 180°$	1	0	0

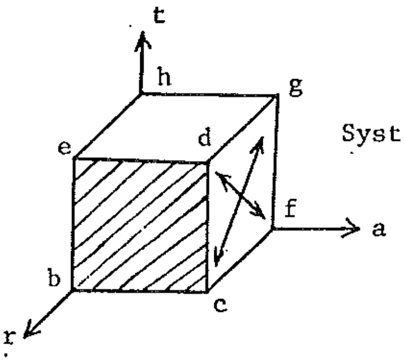




System 1 : crack growth and fracture planes are inclined at ±45° to the axial direction and on the surface of the tube.



System 2 : crack growth plane is across the thickness and inclined at 45° to the free surface of the specimen. Fracture plane is along the circumferential direction of tube.



System 3 : crack growth plane is across the thickness and inclined at 45° to the free surface of the specimen. Fracture plane is parallel to the axial direction of tube.

Fig. 2 - Schema of crack growth planes corresponding to the three crack systems which may be operative within any single cycle under out-of-phase biaxial loading in relation to material axes.

STRAIN PARAMETER FOR CORRELATING FATIGUE

The preceding observations show that to study endurance under out-of-phase biaxial fatigue, it is necessary to consider the following points: a) the zones over which each system of crack operates during a cycle of loading, b) the maximum intensities of the strain parameters $\gamma_{max}/2$ and ε_n for each zone, and c) the variation of these parameters within each zone. In order to take into account the relative importance of each parameter on a crack system, we define the intensity of any strain parameter represented by ε as follows:

$$I(\varepsilon) = \int_{\omega t_1}^{\omega t_2} \varepsilon \, d(\omega t) \tag{5}$$

where ω is the frequency of loading and t_1 and t_2 represent the beginning and the end of any particular zone.

Since the strain parameters controlling crack initiation and propagation are the maximum shear strain $\gamma_{max}/2$, and the normal strain on the plane of maximum shear strain ε_n, we propose that fatigue failure under out-of-phase loading conditions occurs at a given number of cycles to failure if the following relation is satisfied:

$$I(\gamma_{max}/2) + f\,[I(\varepsilon_n)] = C \tag{6}$$

where f is a function of the intensity of ε_n and C is a constant, defined when the phase angle is either equal to 0° or 180°. Because of the fact that only two fracture modes can be present, that is, either Case A or Case B, depending on the phase angle, we have two separate relations for the two cases such that

$$I(\gamma_{max}/2)_1 + f_1[I(\varepsilon_n)_1] = C_1$$

for $150° < \phi < 180°$, and (7)

$$I(\gamma_{max}/2)_{23} + f_{23}\,[I(\varepsilon_n)_{23}] = C_{23}$$

for $0° < \phi < 120°$ where the subscripts 1 and 23 are the intensities in Zone 1 and Zones 2 and 3 combined, respectively.

It has been observed by Jordan et al. [19] that only the positive value of the normal strain ε_n acting on the plane of maximum shear strain influences fatigue failure. Therefore, only the positive values of ε_n are integrated for any zone, that is,

$$I(\varepsilon_n) = \int_{\omega t_1}^{\omega t_2} \varepsilon_n \cdot \langle \varepsilon_n \rangle \, d(\omega t)$$

where

$$\langle \varepsilon_n \rangle = \begin{cases} 1 & \text{if} \quad \varepsilon_n > 0 \\ 0 & \text{if} \quad \varepsilon_n < 0 \end{cases} \qquad (8)$$

EXPERIMENTAL PROGRAM

The experiments were conducted on thin-walled tubes of low-alloy carbon steel A516 Grade 70 used extensively in the fabrication of pressure vessels. The specimens have an external diameter of 27.18 mm, a gauge length of 10 mm and a wall thickness of 0.89 mm. Finite element analysis of the specimen configuration shows that the stress-strain field is uniform over the gauge length of the specimen [22]. The chemical composition, static properties and the uniaxial fatigue curves for the steel have been reported in [23]. Biaxial test results on the same steel under proportional fully reversed constant amplitude strains are reported in [16] at different surface strain ratios. The fatigue life varied between 180 and 82 000 cycles.

About 40 tests have been carried out under out-of-phase loading with the tubes subjected to combined cyclic axial load and internal-external pressure. Axial strain ε_a and tangential strain ε_t were controlled with the strain ratio $\varepsilon_{tmax}/\varepsilon_{amax}$ maintained equal to +1 during the tests (see Equation (30) with $\varepsilon_{am} = \varepsilon_{tm} = 0$).

The sinusoidal strain signals were applied out-of-phase with phase angles $\phi = 0°, 60°, 90°, 120°, 150°$ and $180°$. The strain amplitudes were chosen to give fatigue lives between 300 and 10^6 cycles. During each test, axial and tangential strains ε_a and ε_t , axial force F , internal and external pressures P_i and P_e were recorded at sixty different times over a cycle by a data acquisition system [7]. Hysteresis loops along the axial and tangential directions were plotted and the variations of the following parameters ε_1, ε_2, ε_3, $\gamma_{max}/2$ and ε_n were examined over a complete cycle and at different cycles during the tests. The specimen was considered to have failed after any of the loads has dropped by more than 10% of the stable hysteresis loop values.

EXPERIMENTAL RESULTS

Results of the experiments under out-of-phase biaxial fatigue are presented in Fig. 4 in terms of the imposed maximum axial strain amplitude ε_{amax} as a function of fatigue life N_f for different phase angles. It would be observed that all the curves tend towards the same fatigue strain limit approximately equal to 0.05%. Also, the pure shear strain state loading (ϕ = 180°) is the least unfavourable while the curves at ϕ = 0°, 60° and 90° are almost identical.

The same results are presented in Figs. 5 and 6 in terms of the intensity of maximum shear strain $I(\gamma_{max}/2)$ and the intensity of the normal strain $I(\varepsilon_n)$ as a function of N_f, respectively. For ϕ = 150° and 180° only the integrals of the strain parameters in Zone 1 have been taken into consideration, while for 0° < ϕ < 120°, the integrals have been evaluated in both Zones 2 and 3 and added together. From Fig. 5, the integrals of $\gamma_{max}/2$ is practically the same for 0° < ϕ < 90°. In general, for the same intensity of $\gamma_{max}/2$, Case A (150° < ϕ < 180°) leads to longer fatigue lives than Case B(0° < ϕ < 120°). Also, fatigue life is shorter when ϕ = 150° as compared to ϕ = 180° because there is an effect of $I(\varepsilon_n)$ on the fracture process at γ = 150° whereas for ϕ = 180°, $I(\varepsilon_n) = 0$. The influence of $I(\varepsilon_n)$ on fatigue life can also be shown by examining Figs. 5 and 6 for 0° < ϕ < 90°. For the same value of $I(\gamma_{max}/2)$ in Fig. 5, fatigue life is shortest for ϕ = 90°. This is again due to the differences in the values of $I(\varepsilon_n)$ at ϕ = 0°, 60° and 90° shown in Fig. 6 and this confirms the result of Kanazawa et al. [17] who demonstrated that at constant shear strain range, failure time is influenced by the maximum value of ε_n.

Figure 7 shows the results presented in the plane of $I(\gamma_{max}/2)$ against $I(\varepsilon_n)$ at constant fatigue life. Observation of Figs. 5-7 shows that the constant life contours for A516 Grade 70 steel can be divided into three parts:

a) When 0° < ϕ < 90°, fracture occurs at a given number of cycles to failure for a constant intensity of maximum shear strain. The most unfavourable case occurs at ϕ = 90° for which $I(\varepsilon_n)$ is maximum. The correlating parameter is then at constant fatigue life:

$$I(\gamma_{max}/2)_{23} = \text{constant}$$

where $I(\gamma_{max})/2$ is the integral of the $\gamma_{max}/2$ in Zones 2 and 3 over a cycle. The constant is evaluated at ϕ = 0° and $\rho = 1$. For this

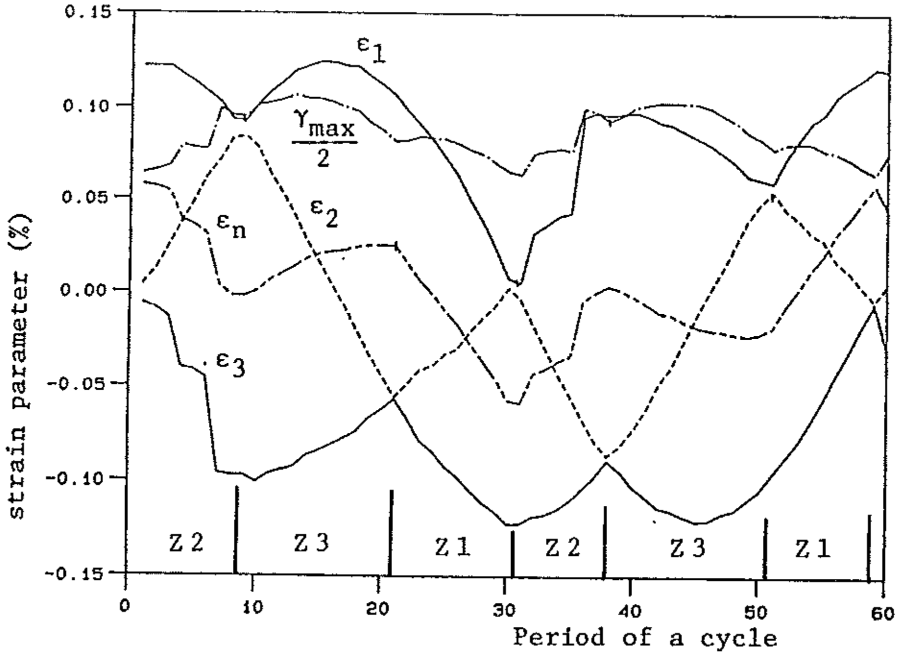


Fig. 3 - Experimental variation of the strain parameters ε_1, ε_2, ε_3, $\gamma_{max}/2 = (\varepsilon_1 - \varepsilon_3)/2$ and $\varepsilon_n = (\varepsilon_1 + \varepsilon_3)/2$ over a cycle of loading for a phase angle of 90°.

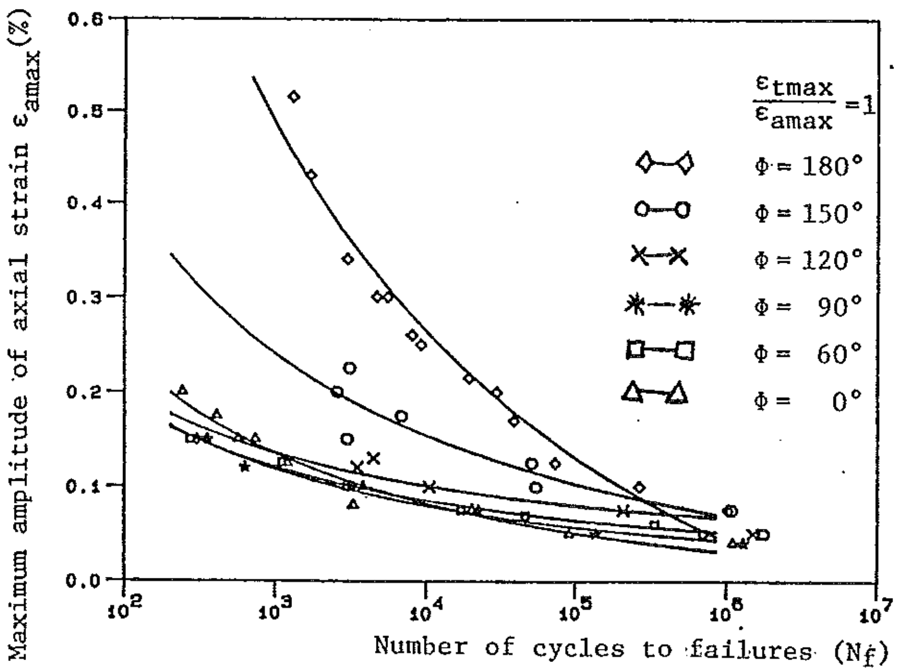


Fig. 4 - Fatigue life as a function of the maximum amplitude of axial strain.

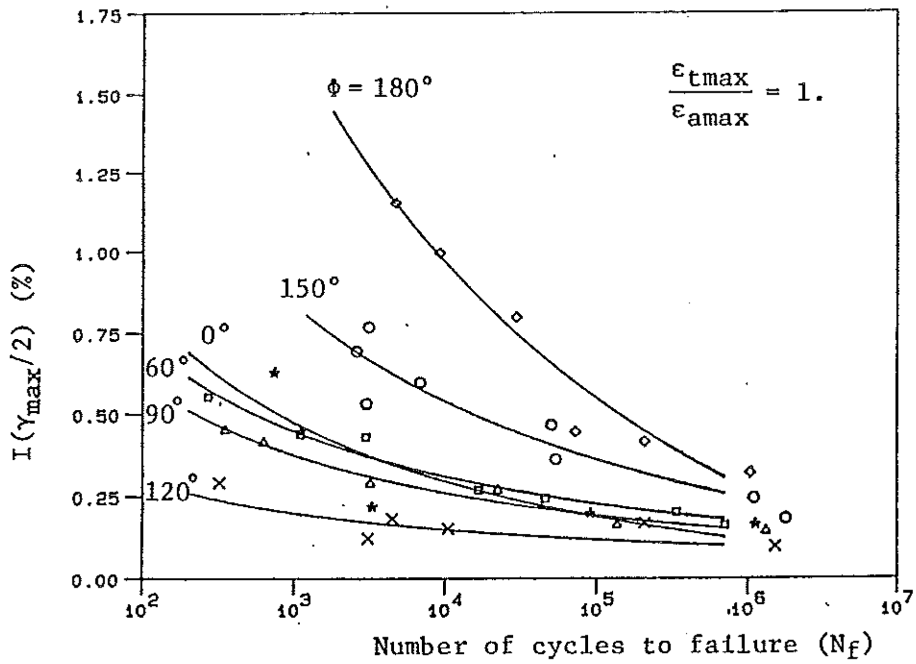


Fig. 5 - Fatigue life correlation by the integral of maximum shear strain $I(\gamma_{max}/2)$ over a cycle of loading for different phase angles.

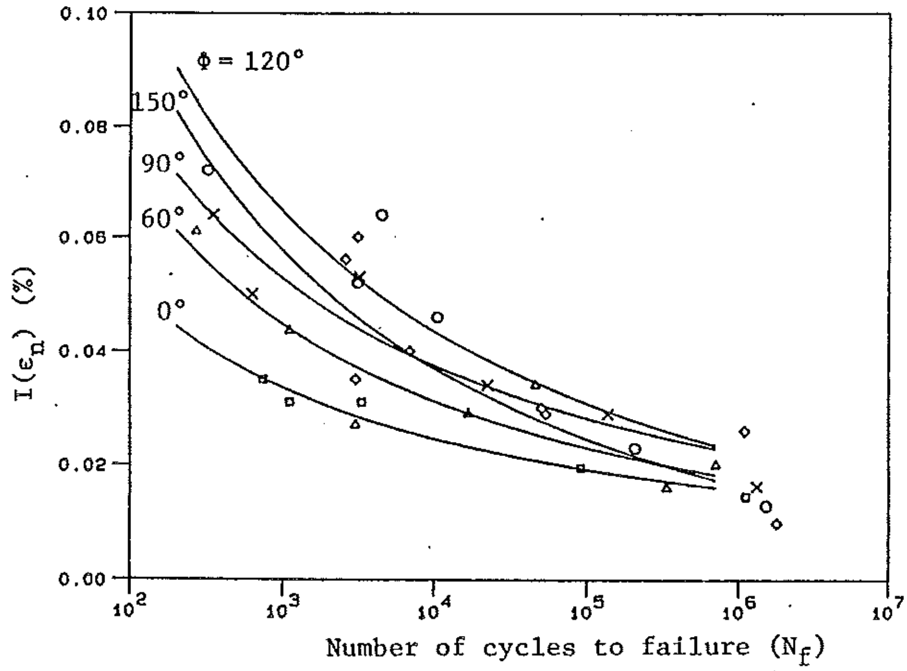


Fig. 6 - Fatigue life correlation by the integral of the normal strain acting on the plane of maximum shear strain $I(\varepsilon_n)$ for different phase angles.

case, cracks propagate across the wall thickness in a direction parallel to the axial direction of the specimen. Intensities of $I(\gamma_{max}/2)$ and $I(\varepsilon_n)$ in Zone 1 are low and the fracture process is controlled by the values of $I(\gamma_{max}/2)$ and $I(\varepsilon_n)$ in Zones 2 and 3.

- b) A transition region with $120° < \phi < 135$ where $I(\gamma_{max}/2)$ and $I(\varepsilon_n)$ in Zone 1 and Zones 2 and 3 have significant values. When $\phi = 120°$, intensities in Zones 2 and 3 are larger than those in Zone 1 but as ϕ tends towards 135°, intensities in Zone 1 become more dominant. Failure modes associated with Zone 1 and Zones 2 and 3 are both present. At $\phi = 120°$, Case B failure (across the wall thickness) mode is dominating marked by maximum intensity of $I(\varepsilon_n)$ and the minimum intensity of $I(\gamma_{max}/2)$. In this transition region, crack propagation seems to be strongly influenced by material microstructural features such as inclusions, and a simple strain parameter cannot be established for life prediction without more experimental results.
- c) When $\phi > 135°$, intensity of $I(\gamma_{max}/2)$ in Zone 1 dominates and failure mode is Case A of Brown and Miller [10]. Fatigue life is influenced by the two strain parameters such that at constant endurance we have:

$$I\ (\gamma_{max}/2)_1 + k\ I(\varepsilon_n)_1 = \text{constant}$$

where the constant is evaluated at $\phi = 180°$, that is, at pure shear strain. Constant k depends on the material and for the results on A516, it is approximately equal to -10.

CONCLUSIONS

Two strain parameters are proposed for correlating fatigue failure under out-of-phase biaxial loading conditions. The parameters are based on the maximum shear strain and the normal strain acting on the plane of maximum shear strain which govern fatigue crack initiation and growth. It is shown that constant life contours can be described by the following relation:

$$I(\frac{\gamma_{max}}{2}) + f\ [I(\varepsilon_n)] = C$$

where $I(\gamma_{max}/2)$, $I(\varepsilon_n)$ are the integrals of the maximum shear strain $\gamma_{max}/2 = (\varepsilon_1-\varepsilon_3)/2$ and the normal strain $\varepsilon_n = (\varepsilon_1+\varepsilon_3)/2$, respectively, f is a function of $I(\varepsilon_n)$ and C is a constant that depends on the material. The

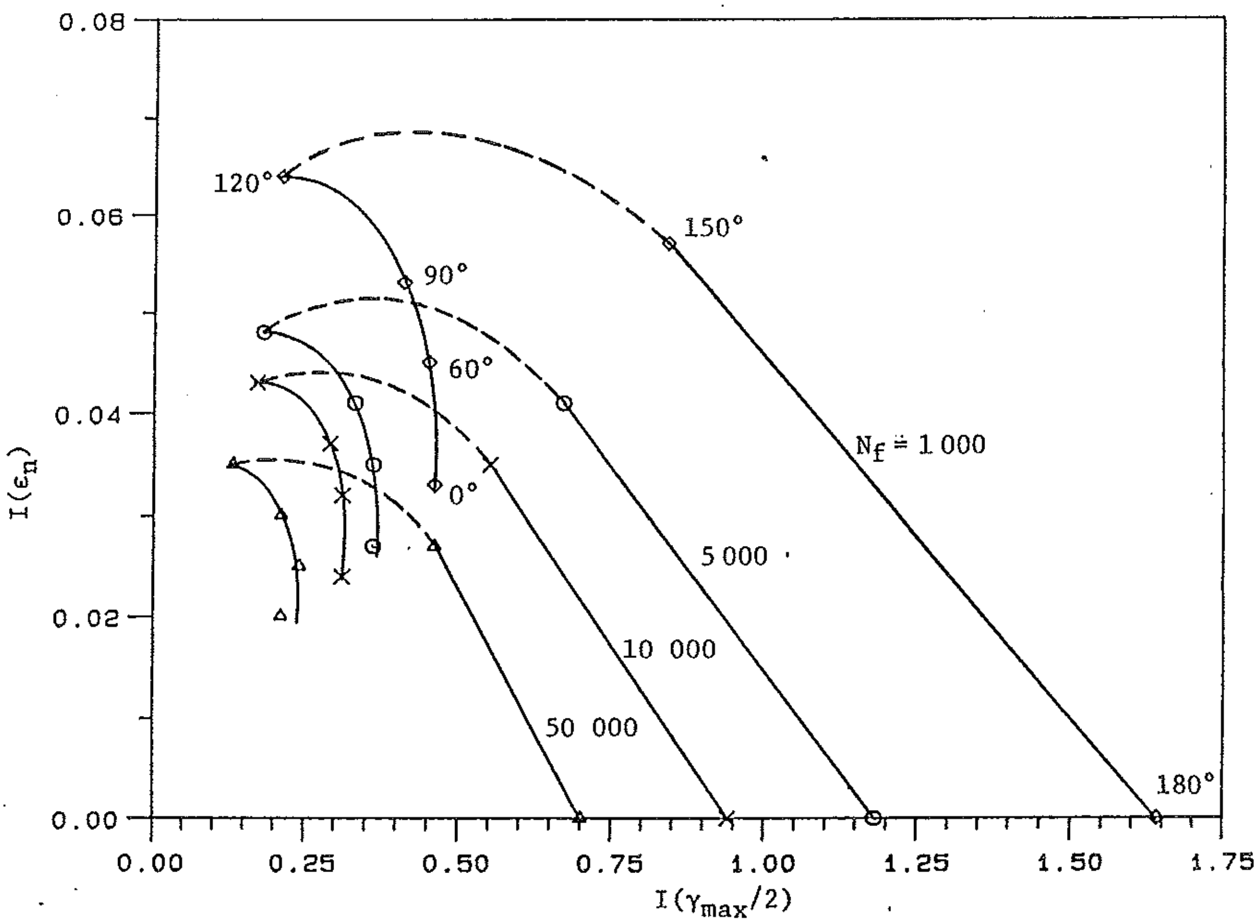


Fig. 7 - Plot of constant fatigue life contours on a graph of $I(\gamma_{max}/2)$ against $I(\varepsilon_n)$.

integrals are evaluated with reference to three crack systems which may be operative within a cycle of loading. Failure of the material is controlled by one or two of the crack systems depending on the phase difference between applied strain-waveforms.

Finally, it is shown that constant life contours can be divided into three regions. In the first region ($0° < \phi < 120°$) crack growth is dominated by two crack systems and is across the thickness of the material. For the second region ($150° < \phi < 180°$) crack growth is controlled by one crack system and is along the surface of the material. Crack growth in the third region, the transition region, is mixed and seems to depend a lot on microstructural details. Further work is necessary to clarify the way the two parameters $I(\gamma_{max}/2)$ and $I(\varepsilon_n)$ vary in this transition region. Also, studies at other maximum strain ratio values $\rho = \varepsilon_{tmax}/\varepsilon_{amax}$, different from $\rho = 1$ studied in this paper, may be necessary to test the general applicability of the criterion proposed in this study.

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