MIXED MODES OF FRACTURE UNDER BIAXIAL CYCLIC TENSION: METHODS AND RESULTS

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ABSTRACT

The investigation of characteristics of the aluminium alloys cyclic crack-stability under biaxial load of arbitrary direction was carried out. The dependence of parameters to be determined on the properties of material, the stressed state form and the initial crack orientation was obtained. For the assumed elasto-plastic interpretation of the experimental data one has obtained the plastic stress fields in the angled crack vertex for the different strain hardening factors.

METHODS OF INVESTIGATION

The methods for investigation of the characteristics of cyclic crack-stability for the mixed modes of the crack propagation under biaxial load of arbitrary direction has been elaborated. The following parameters are determined: the crack growth rate; the strain energy density factor; the angle, determining the further direction of crack propagation as a function of its initial orientation; the trajectory of crack growth and the introduced T-parameter characterizing the cyclic crack-stability. The processing of the test results was carried out in accordance with the (Shlyannikov and Dolgorukov, 1987) procedure. The experimental data interpretation is based on the parameter of the strain energy density S (Sih, 1974) and on the introduced concept of an equivalent straight crack which allows to obtain the computational-experimental and theoretical trajectories of the crack growth under a complex stressed state as well as the fatigue fracture diagrams (Shlyannikov and Dolgorukov, 1987; Slyannikov and Ivanyshin, 1983). In this work we shall distinguish following three ways to determine the crack growth trajectory: experimental, computational-experimental and theoretical ones. The experimental crack growth trajectories (ECGT) were obtained as a result of the cyclic analysis on aluminium alloy specimens under the both uniaxial $\eta = \sigma_x^\infty / \sigma_y^\infty = 0$ and biaxial tension with the nominal stresses relation $\eta = 0.5$. Then the experimental crack growth trajectory has been transfered into the clear film having the dividing net-work with a pitch equal to 0.5 mm. In the work (Shlyannikov and Dolgorukov, 1987) one has introduced the determination of computational-experimental crack growth trajectory (CECGT) being the integral part of the method to research the cyclic crack-stability characteristics for the crack propagation mixed modes. Interpretation of experimental results in limits of this method is based on substitution of the real curvilinear trajectory by an equivalent rectilinear one (fig. 1). Such a crack is the geometrical sum of the rectilinear crack preceding length a_{i-1} with its increase Δa_i in the direction of its further growth determining by the angle θ^*. In experiment only the crack increase Δa_i has been measured on every measuring pitch and then with its help by the known θ^* (determining α) the new length of rectilinear crack and its angle of inclination have been calculated by the following equations

$$\begin{cases} a_i = \left[a_{i-1}^2 + \Delta a_i - 2a_{i-1}\Delta a_i \cos(\pi - \theta_{i-1}^*)\right]^{1/2} \\ \alpha_i = \alpha_{i-1} + \arcsin \dfrac{\Delta a_i \sin(\pi - \theta_{i-1}^*)}{a_i} \end{cases} \qquad (1)$$

Every pair of values a_i and α_i coordinates i-point of the crack growth trajectory. Since these equations include the both experimental and computational values, then the crack growth trajectory determining by the pair a_i and α_i is called the computational-experimental one. At the experimental data treatment successive transition from one point to other one allows to construct the CECGT under the cyclic load with a help of the pitching method. Theoretical crack growth trajectory (TCGT) has been also determined with a help of the pitching method. The pitching method was first proposed (Parton and Morozov, 1974), while in (Shlyannikov and Ivanishin, 1983) the equation was proposed from other positions connecting the coordinates of the preceding and the next crack vertex location on its growth trajectory

$$a_i = a_{i-1}\left[\cos(\alpha_i-\alpha_{i-1})+\sin(\alpha_i-\alpha_{i-1})tg\left(\frac{\pi}{2}-\Theta^*_{i-1}+\alpha_i-\alpha_{i-1}\right)\right] \quad (2)$$

Thus, the trajectory is theoretical, if it is obtained only by the calculation proceeding from initial length and the angle of crack orientation as well as from direction of its propagation taking into account the type of the stressed state $\Theta^* = f(\alpha,\eta)$. The above relations (1-2) are invariant regarding to the form of criterion $\Theta^* = f(\alpha,\eta)$ Fig2.Comparison of CECGT and TCGT with ECGT has allowed to ground accordingly the way to interpret the characteristics of cyclic crack-stability for the mixed modes and also to determine the correctness of engineering formulae for the trajectory calculation predicting a longevity. The principal peculiarity of the tests of materials with an initial angled crack is that the crack does not propagate at initial direction and that it has the curvilinear trajectory (fig. 1). The purpose of investigations of the cyclic crack-stability characteristics is to obtain the dependence of the crack growth rate on the range of the stress intensity factor. In order to obtain such a dependence one must put the values of the stress intensity factor (SIF) into the conformity with each crack length (under an accumulated quantity of load cycles). In the angled crack vertex under the plane stressed state there are realized two forms of fracture described by the corresponding SIF of the normal separation and the pure shear (Panasyik, 1968)

$$K_I = \frac{\sigma\sqrt{\pi a}}{2}\left[(1+\eta)-(1-\eta)\cos 2\alpha\right]Y_I$$
$$K_{II} = \frac{\sigma\sqrt{\pi a}}{2}\left[(1-\eta)\sin 2\alpha\right]Y_{II} \quad (/)$$

in which σ is the nominal normal stress in the axis OY direction; a is the crack length; α is the angle of the crack orientation about the axis OY; $\eta = \sigma_x^\infty/\sigma_y^\infty$ is the relationship of nominal biaxial stresses; Y_I and Y_{II} are the K-taring functions. Thus, in the case of mixed modes of load one must use the equivalent value of SIP which is the function of K_I and K_{II}. As such an equivalent we propose to use the parameter Sih

$$S = b_{11}K_I^2 + 2b_{12}K_IK_{II} + b_{22}K_{II}^2 \quad (4)$$

$$B_{11} = \frac{1}{16G}(\text{æ} - \cos\theta^*)(1+\cos\theta^*)$$

$$B_{12} = \frac{2}{16G}(\cos\theta^* - \text{æ}+1)\sin\theta^*$$

$$B_{22} = \frac{1}{16G}[(\text{æ}+1)(1-\cos\theta^*)+(3\cos\theta^*-1)(1+\cos\theta^*)] \quad (5)$$

Here θ^* is the angle determining the crack propagation direction as a function of its orientation α taking into account the stressed state type; G is a shear modulus; $\text{æ} = (3-\nu)/(1+\nu)$; ν is a Poisson factor. In order to describe dependences θ^* on α the different criteria have been used in the series of works. However, we have been showed (Shlyannikov and Dolgorukov, 1988) that the existing criteria of the crack growth direction do not spread to all the range of change of the experimental data for materials having the different properties under biaxial tension. Therefore in the procedure of experimental data interpretation we have directly used with a help of the Lagrange polynoms the one-dimensional interpolation of dependences $\theta^*-\alpha$ for each material and stressed state type under the calculation of the current values of B_{12}, B_{11}, B_{22} in (5) as well as the two-dimensional interpolation over tha crack length and its inclination angle under the calculation of the current values of K_I and K_{II} (3) and as a result S (3). These interpolation calculations have been carried out for each position of the equivalent straight crack on the real curvilinear trajectory of its propagation. K-taring functions Y_I and Y_{II} from (3) for the rectangular and eight-petal specimens have been determined by the finite element method (FEM) taking into account the singularity.

RESULTS AND DISCUSSIONS

On the aluminium alloys having the different properties (whose main mechanical characteristics are presented in table 1) the experimental investigation and analysis of the crack growth has performed taking into account their orientation under uniaxial η=0 and biaxial η =1 and η=0.5 tension on the electrohydraulical stand with antisymmetry factor R = 0.05 at a frequency 3.5 1/s on the rectangular (80 x 320) mm and eight-petal specimens by the thickness 3 ÷ 5 mm (fig. 3). The fatigue fracture diagrams and the crack growth trajectory have been obtained for all the materials at the following stress relationships: η = 0;0.5;1 and at the angles of initial orientation α = 0;25;45;65;90° of crack with its length a = 10 mm. In fig. 4a the crack growth trajectories for their initial orientation under uniaxial tension in Al.alloy 1 and Al.alloy 6 are shown, corresponding to limits of the range of relation $\sigma_B / \sigma_{0.2}$ change. One can see that the crack trajectories for both materials trend to come out during the process of their growth into the normal direction to the nominal stress. Difference of properties is greater for the small values of α . One can observe the good agreement between ECGT, CECGT and TCGT for given loading conditions. As it follows from fig. 4,b, the type of the stressed state has essential influence to the prosecc of crack propagation. First of all the shape of trajectory is changed under biaxial tension So, on example of Al.alloy 1 one can see, that the trajectory curvature changes into the opposite side regarding to uniaxial tension. Moreover, the crack having the different initial orientation do not come out during their growth into the normal direction to a greater nominal stress. One can observe the good agreement between ECGT and CECGT for the complex stressed state. It should be considered that correspondence of the theoretical and the experimental ways (calculated only on criterion Sih) is only satisfactory. This fact regards to Al.alloy 1 as well as to other tested materials. Essentially greater is the range of the crack growth trajectory change under biaxial tension than it there was under uniaxial one (for the same angles of initial orientation) depending on the material properties (fig. 5,b). Comparison of fig. 4 and fig. 5 shows that properties of observed materials have the same quantitative influence as a type of the stressed state on process of crack propagation. Cyclic trajectories under biaxial tension of most brittle material Al.alloy 5 are close to the normal direction to maximum normal stress, that corresponds to a physical sense of σ_θ -criterion. However, we have established (Shlyannikov and Dolgorukov, 1988) that this criterion describes badly experimental diagrams $\theta^* = f(\alpha, \eta)$ (Fig.2) for Al.alloy 5 under biaxial tension. We consider that in a present time the universal criterion $\theta^* = f(\eta, \alpha)$ suitable for all the range of the materials observed is absent. This circumstance defines the necessity to elaborate more adaptable criterion of view $\theta^* = f(\alpha, \eta)$ regarding their properties. Plastic materials have more high crack growth trajectories. In this connection one is interested in results of the cyclic experiments (fig. 5,a) under biaxial tension of specimens with initial crack orientation along the greatest tensile stress (i.e. α =0 for η =0.5). This situation is the state of unstable equilibrium. In motion of experimental curves one can see some regularity: for the plastic material Al.alloy 1 the crack trajectory first deviates from an initial orientation direction, while for the brittle material Al.alloy 6 the perceptible increase in this direction can be observed, and then the crack turning occurs. A good agreement between ECGT and CECGT allows to recommend the proposed method (Shlyannikov and Dolgorukov, 1987) of interpretation of the cyclic crack-stability characteristics for the mixed modes of the crack growth. For the mixed modes of fatigue loading we have proposed criterion (Shlyannikov and shkanov, 1982) as follows

$$\frac{da}{dN} = \left(\frac{da}{dN}\right)^{*}\left(\frac{S_{max}}{S^{*}}\right)^{m} \quad (6)$$

in which da/dN = 10^{-7} m/cycle, S^{*} and m being the experimentally determined constants. In fig. 6 the diagrams of fatigue fracture for some materials is shown. The cases (a) and (b) relate to the symmetrical uni- and biaxial tension (α= 90°). It was established that for all the materials observed tha crack growth rate is greater at uniaxial symmetrical tension than at biaxial one. For the plastic materials the smallest crack growth rate corresponds to biaxial tension η = 0.5 while for the more brittle materials η = 1. Moreover some difference in shapes of their fatigue fracture diagrams is observed. The cases (c) and (d) in fig. 6 correspond to mixed modes of the crack propagation at η = 0 and η = 0.5. At uniaxial cyclic tension the influence of the crack initial orientation angle is not very great. However, as a matter of fact, the greatest crack growth rate corresponds to α = 45° at which the value of K is maximum. At biaxial tension the crack initial orientation exerts a greater influence than at uniaxial one. From the common series of diagrams one can distinguish the diagram of fatigue fracture, when the initial crack is directed along the line of the maximum nominal stresses, i.e. α = 0°; η = 0.5. For the generalized evaluation of characteristics of the cyclic crack-stability at mixed modes of loading the undimensional parameter has been proposed (Shlyannikov and Dolgorukov, 1988), in which the arbitrary values of empirically determined constants of material m and S (in equation 5) were divided on their values in the case of equally-biaxial tension ($\eta = 1$) m_1 and S_1^*

$$T = (m_1/m)/(S^*/S_1^*) \qquad (7)$$

The case of equally-biaxial tension ($\eta = 1$) is invariant to the crack orientation angle and hence it makes sense to use it as a basic experiment. In fig. 7 the character of change of the parameter T as a function of α is shown for the different relationships of nominal stresses η. Here one can see that the crack-stability under biaxial load is greater than under uniaxial one at all the regions of the α change. Character of the T-parameter change corresponds to the mutual disposition of the fatigue fracture diagrams. The most dangerous from the point of view of the fracture under biaxial tension are the cases of $\alpha = 0°$ and $\alpha = 90°$. In the other cases the deviation of the plane of initial crack orientation from the specimen axis of symmetry leads to decrease of the crack growth rate. It may be seen that the plastic materials have the greater cyclic crack-stability than the brittle ones. The character of dependence of the crack growth rate determined at $S_{max} = 0.8$ MPa m on the T-parameter value (fig. 8) is notable. The indivisible character of this curve for all the materials the types of SSS and the angles of the initial crack orientation (which can be approximated by the m-degree polynom) opens the possibility to predict characteristics of the cyclic crack-stability under mixed modes of fracture. It is obvious that the case of the equally-biaxial tension with $\eta = 1$ has to appear as a base experiment.

ELASTOPLASTIC ANALYSIS

Further we go to give an elastoplastic interpretation of above experiments, as the influence of loading biaxiality is displayed over the range of a plastic strain in the crack vertex. The first stage of these works was the computational analysis of stress fields at the region of angled crack vertex in physically nonlinear formulation. The well known results (Shih, 1974) were the first and almost the only attempt to solve the above mentioned kind of problems. In (Shlyannikov and Dolgorukov, 1988) authors have proposed the new method for solution of the strain compatibility equation in a case of angled cracks taking into account the following works (Hutchinson, 1968; Rice and Rosengren, 1968; Machutov, 1973). Using these approaches we have introduced the new model concept of boundary conditions employing for solution of nonlinear differential fourth-order equation. The boundary condition problem consists of two subproblems: quantitative and qualitative. As a quantitative side of the problem one has to imply an integral evaluation of the stressed state by means of the stress intensity parameter in an elastoplastic region of the crack vertex. Conditional theoretical factors of the stress and strain concentration being obtained from the elastic problem one can obtain the maximum acting stress intensity in accordance with a work (Machutov, 1973). In this case all the boundary conditions of problem: biaxial stress relation η and crack orientation angle α, as well as the loading level and the material properties are naturally taken into account. Qualitative side of the boundary conditions problem may be reduced as follows. The boundary conditions for solution of the differential equation in terms of the stress function of Airy and its derivatives were obtained after the minimum analysis of the series stressed-strained state parameters of the crack vertex region (Dolgorukov and Shlyannikov, 1988). These conditions give the fracture initiation direction θ^*, which in turn is determined either from an elastic analysis, or from an experiment. The above approach is true for both the plane stressed state and the plane strain (Dolgorukov, 1988). In such a problem formulation the most complex stage is to obtain undimensional components of the stress tensor. These results for the plane strain are known (Shih, 1974), while the plane stressed state still is little studied. Therefore we present some results of our investigations without detailed solution. So in fig. 9,10 undimensional plastic stress fields are shown for the series angle values of fracture initiation $\theta^* \doteq 15°, 35°, 70°, 90°$ under asymmetrical loading of the angled crack in materials having the strain hardening degree n = 3;13. It should be noted that all the obtained solutions correspond to different combinations of α and η, since the straight line $\theta^* = \mathrm{Const}$ crosses the graphs of criterion equation $\theta^* = f(\alpha, \eta)$ in a few points.

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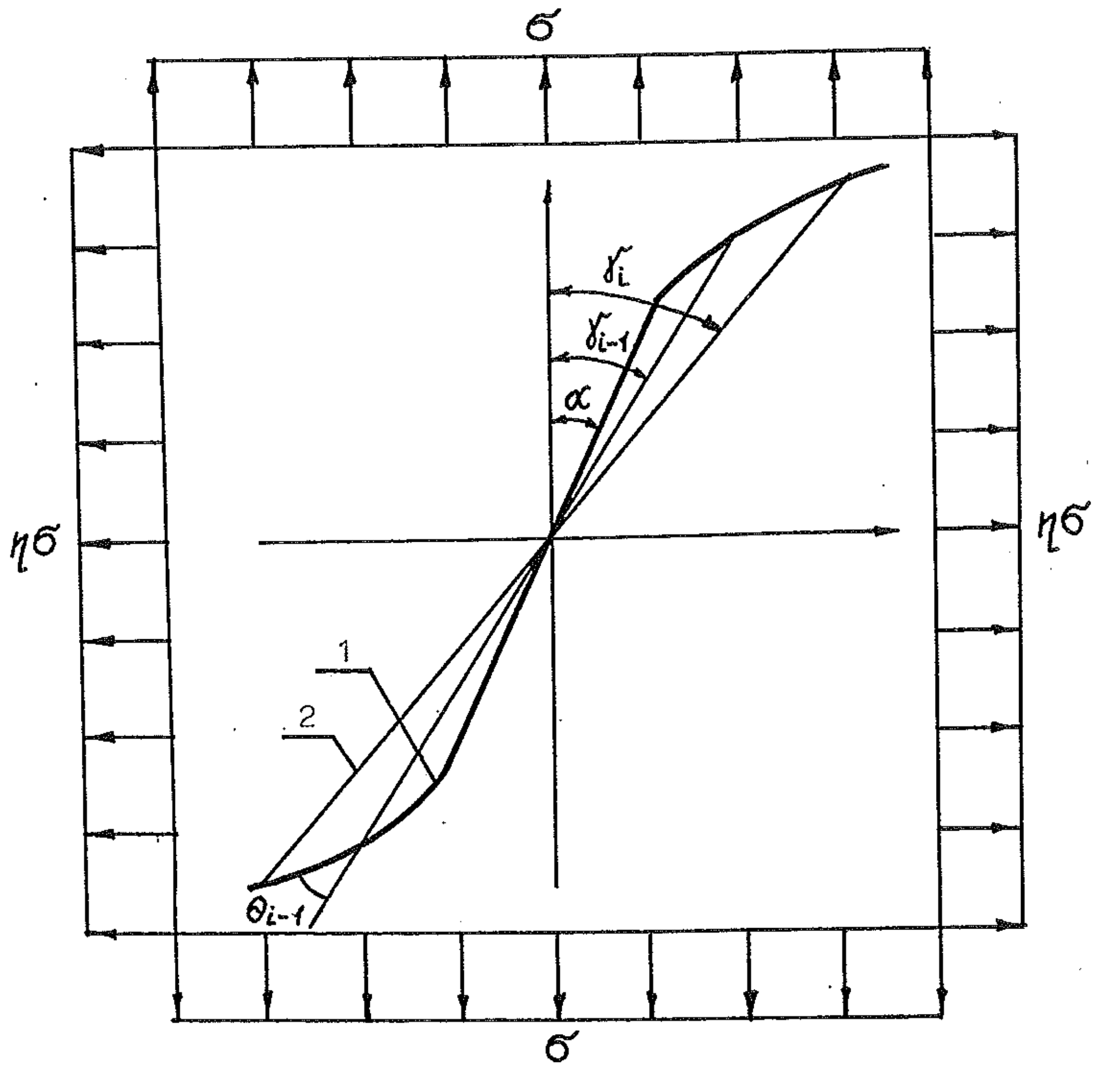

Fig. 1. The plate under biaxial loading

σ_θ-CRITERION

$$\frac{\sigma_\theta}{\sigma} = \left(\frac{a}{32\mu}\right)^{1/2}\left\{\left[(1+\eta)-(1-\eta)\cos 2\alpha\right]\times\right.$$
$$\times\left(3\cos\frac{\theta}{2}+\cos\frac{3\theta}{2}\right) - 3\left[(1-\eta)\sin 2\alpha\right]\times$$
$$\times\left(\sin\frac{\theta}{2}+\sin\frac{3\theta}{2}\right)\Big] + (1+\eta)\cos 2\alpha\,\sin^2\theta$$

S-CRITERION

$$S = a_{11}K_I^2 + 2a_{12}K_I K_{II} + a_{22}K_{II}^2$$
$$K_I = \frac{\sigma\sqrt{\pi a}}{2}\left[(1+\eta)-(1-\eta)\cos 2\alpha\right]$$
$$K_{II} = \frac{\sigma\sqrt{\pi a}}{2}\left[(1-\eta)\sin 2\alpha\right]$$

σ_i-CRITERION

$$\sigma_i^2 = \frac{K_I^2}{32\pi\mu}\left(f_\mu^2 + f_\theta^2 - f_\mu f_\theta + 3f_{\mu\theta}^2\right) +$$
$$+\frac{K_I\varphi}{4\sqrt{2\pi\mu}}\left[(2f_\mu - f_\theta)\cos^2\theta + (2f_\theta - f_\mu)\times\right.$$
$$\left.\times\sin^2\theta - 3f_{\mu\theta}\sin 2\theta\right] + \varphi^2$$

MECHANICAL CHARACTERISTICS OF MATERIALS

marker	material	σ_B MPA	$\sigma_{0,2}$ MPA	δ %	$\sigma_B/\sigma_{0,2}$
○	Al.alloy 1	320	160	20	2.00
+	Al.alloy 2	390	225	14	1.74
△	Al.alloy 3	440	285	20	1.54
◇	Al.alloy 4	430	335	13	1.28
▽	Al.alloy 5	345	300	9	1.15
□	Al.alloy 6	570	510	11	1.12

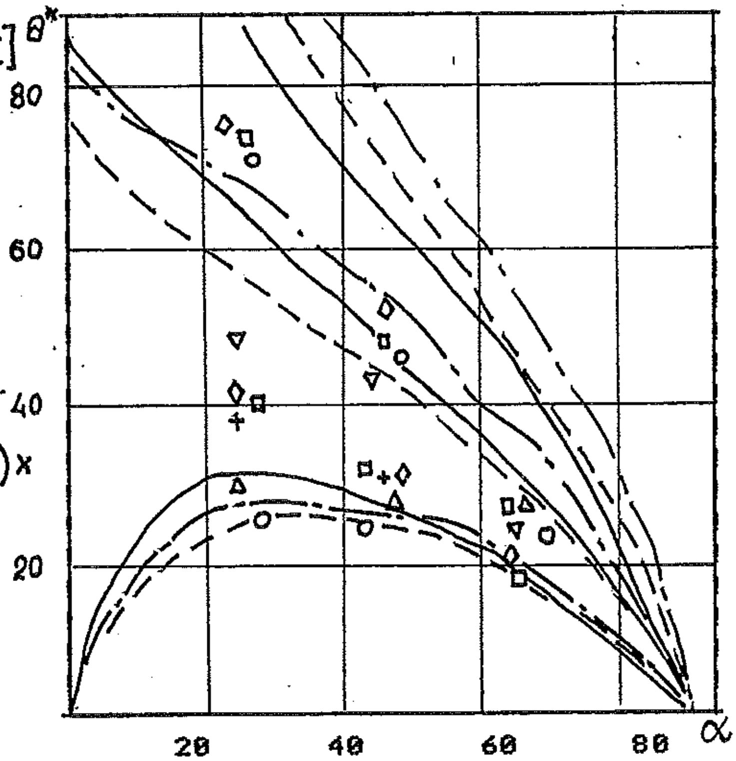

Fig. 2. Dependence $\theta^*-\alpha$ in comparison with tha crack growth criteria

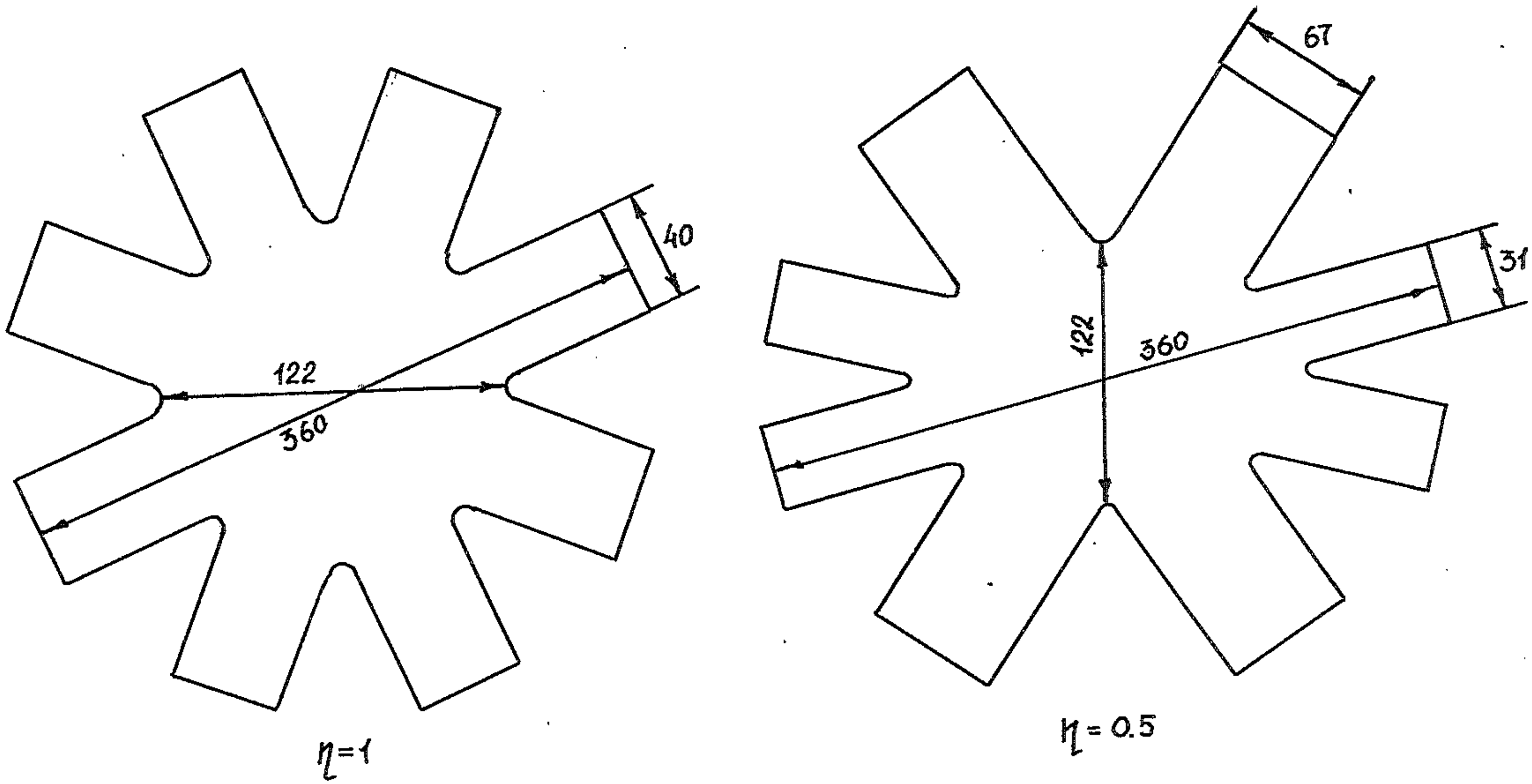

Fig. 3. Experimental specimens

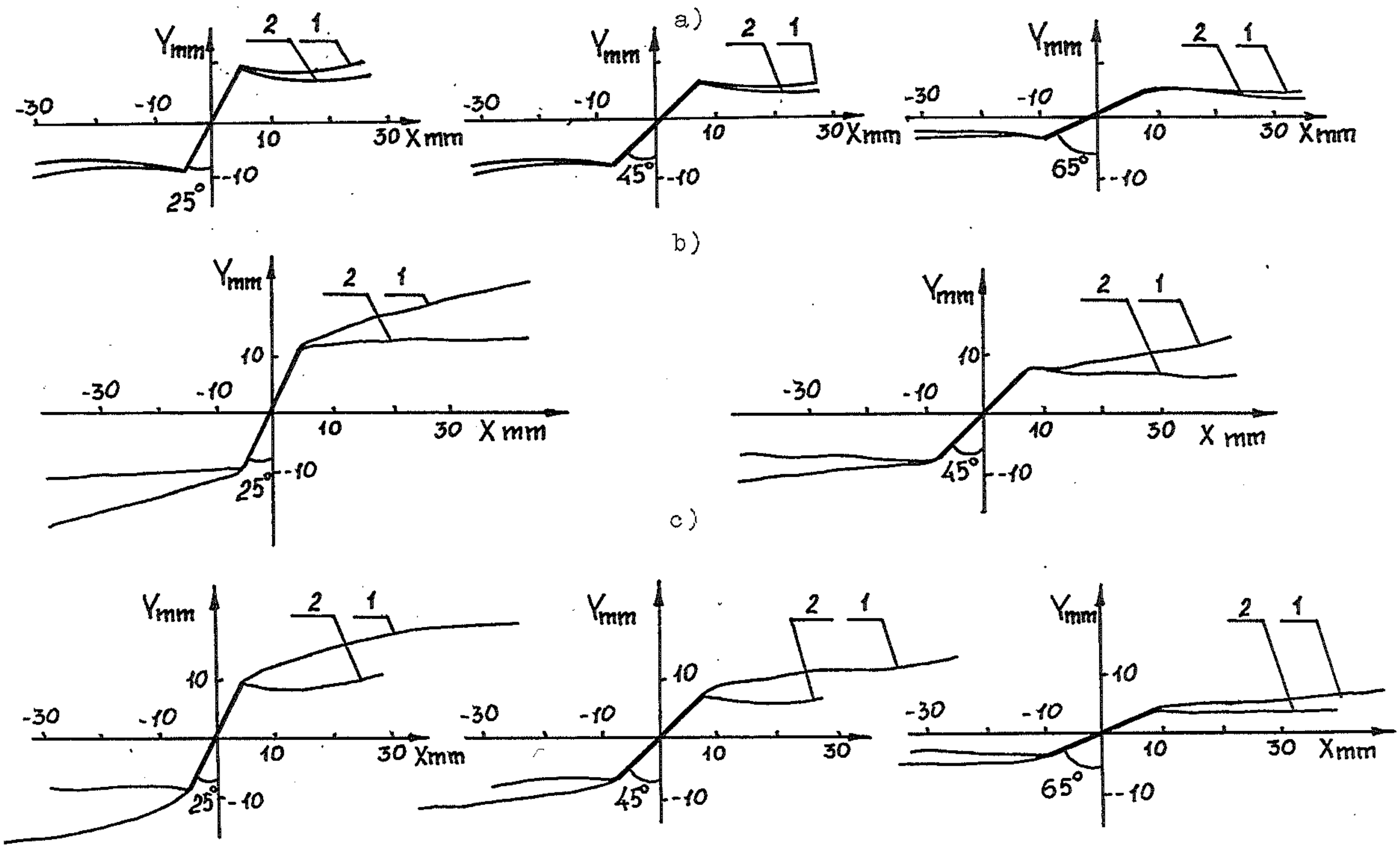

Fig. 4. The crack growth trajectories

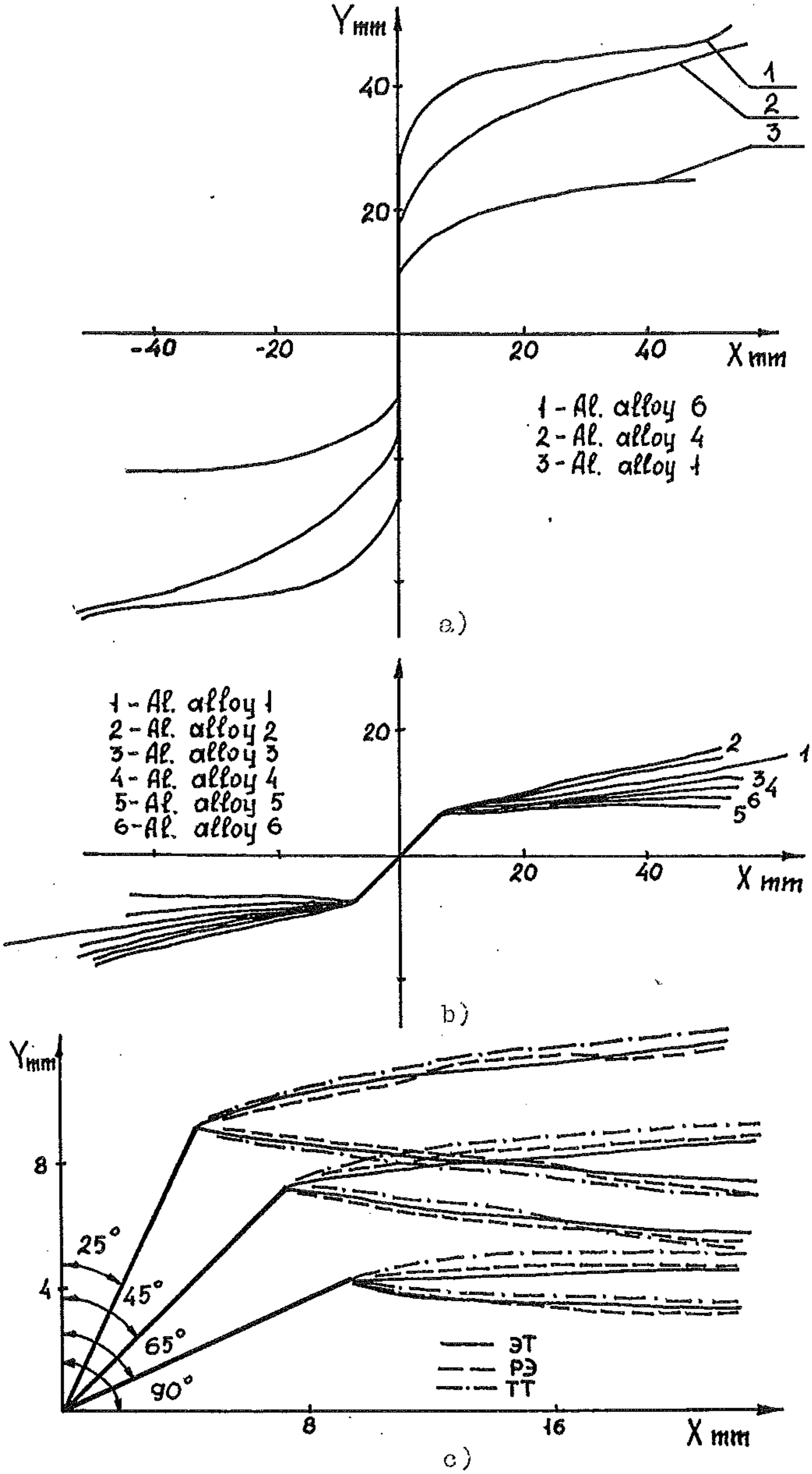

Fig. 5. The crack growth trajectories

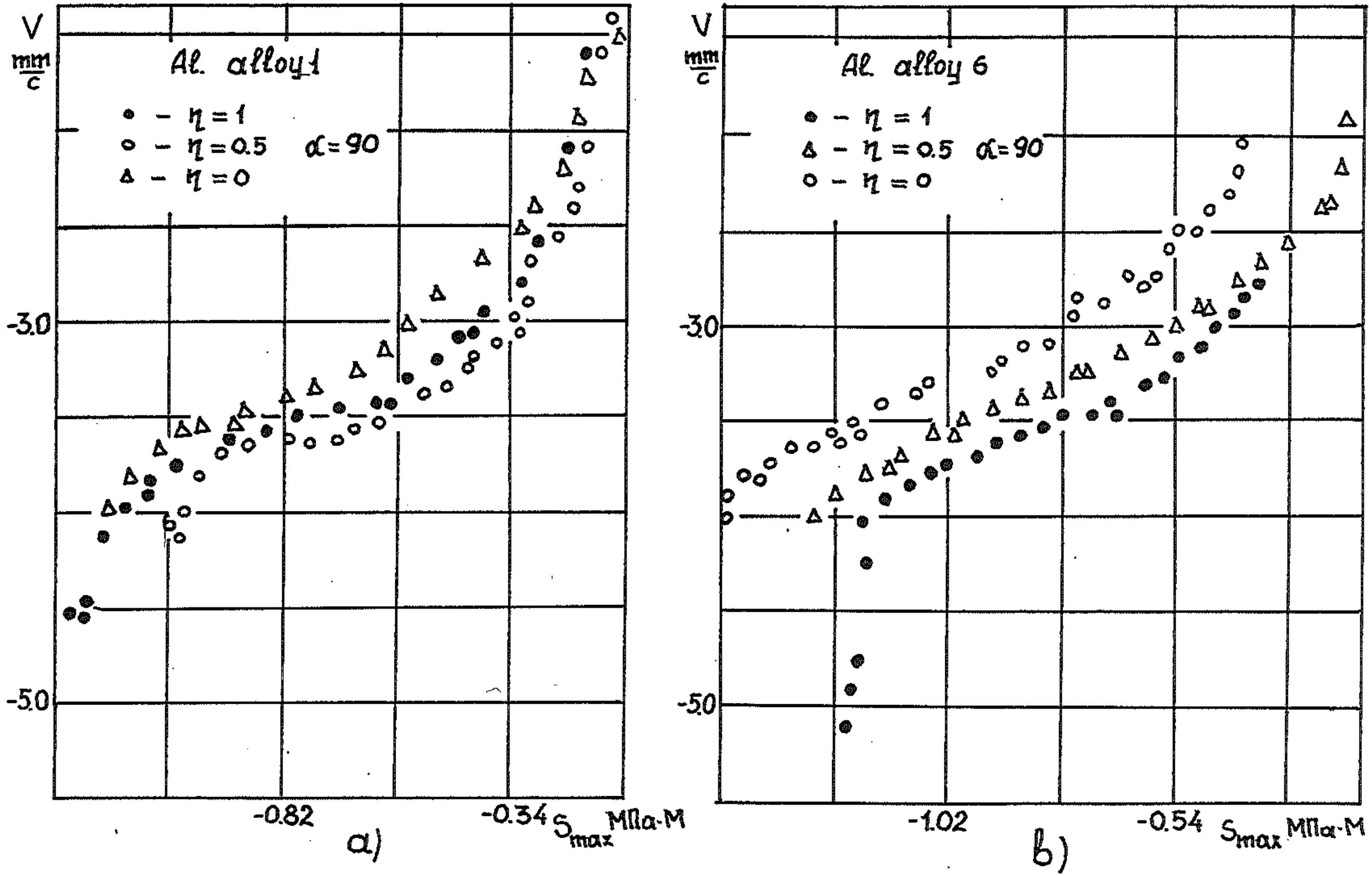

Fig. 6. Fatigue fracture diagrams

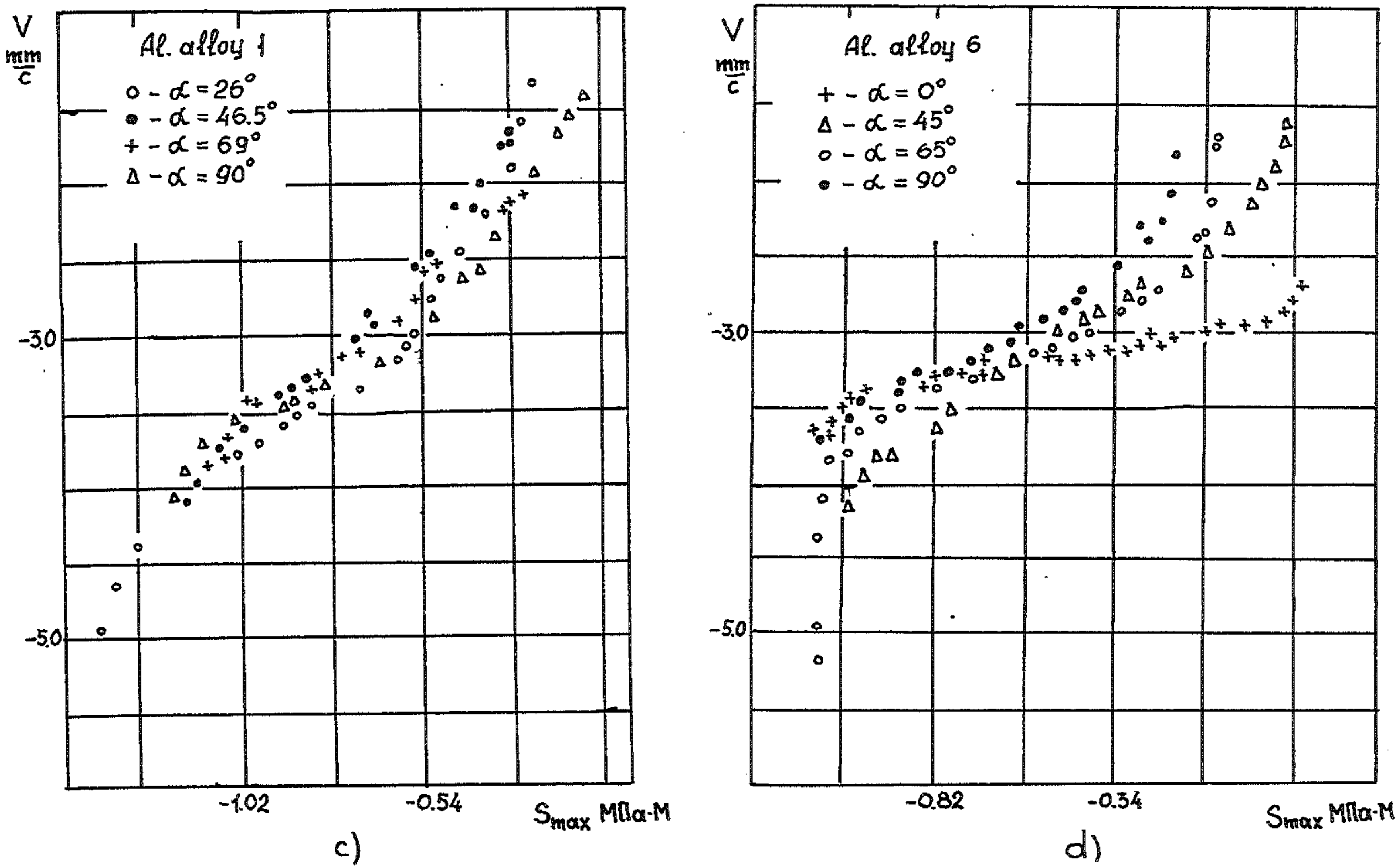

Fig. 6. Continuation

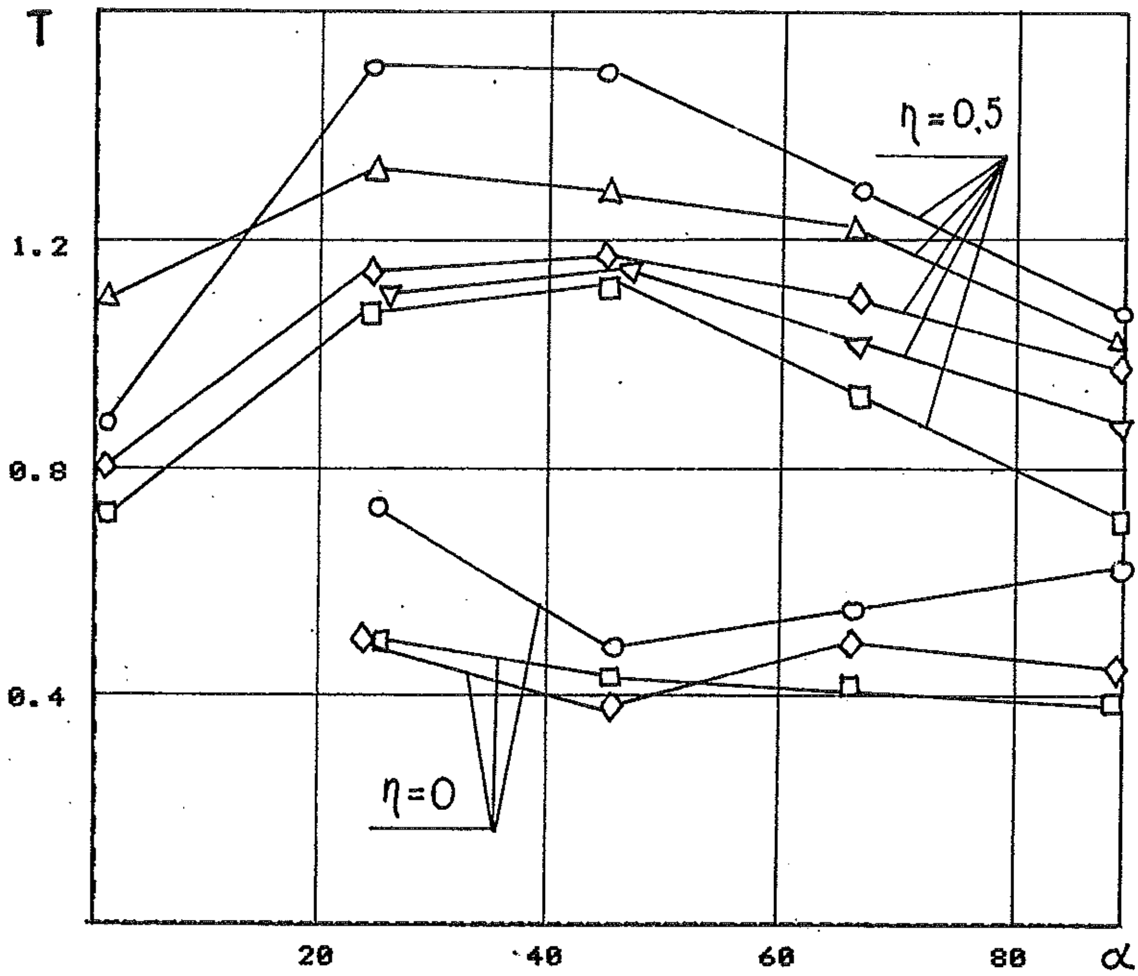

Fig. 7. T-parameter as a function of α

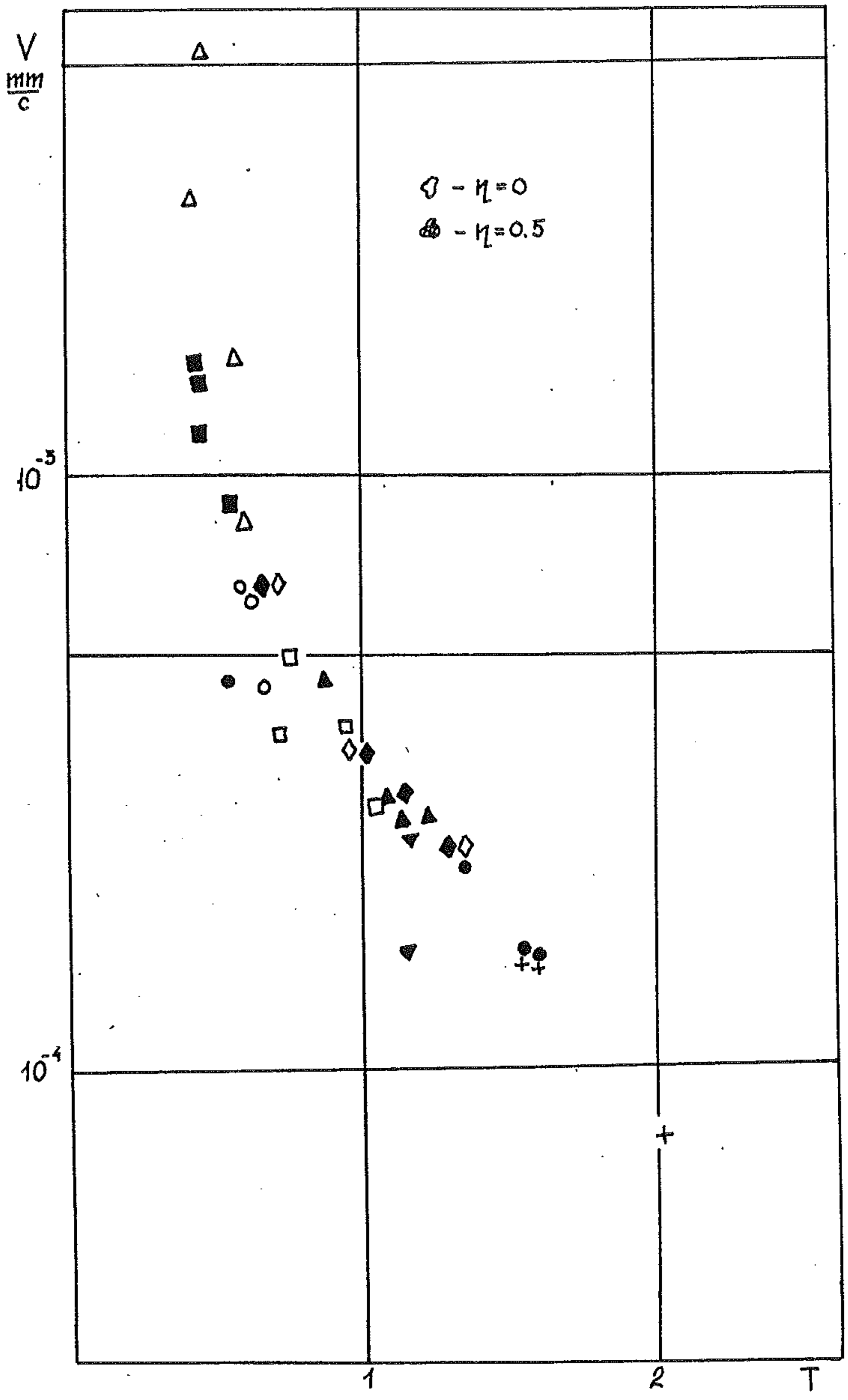

Fig. 8. V as a function of T-parameter

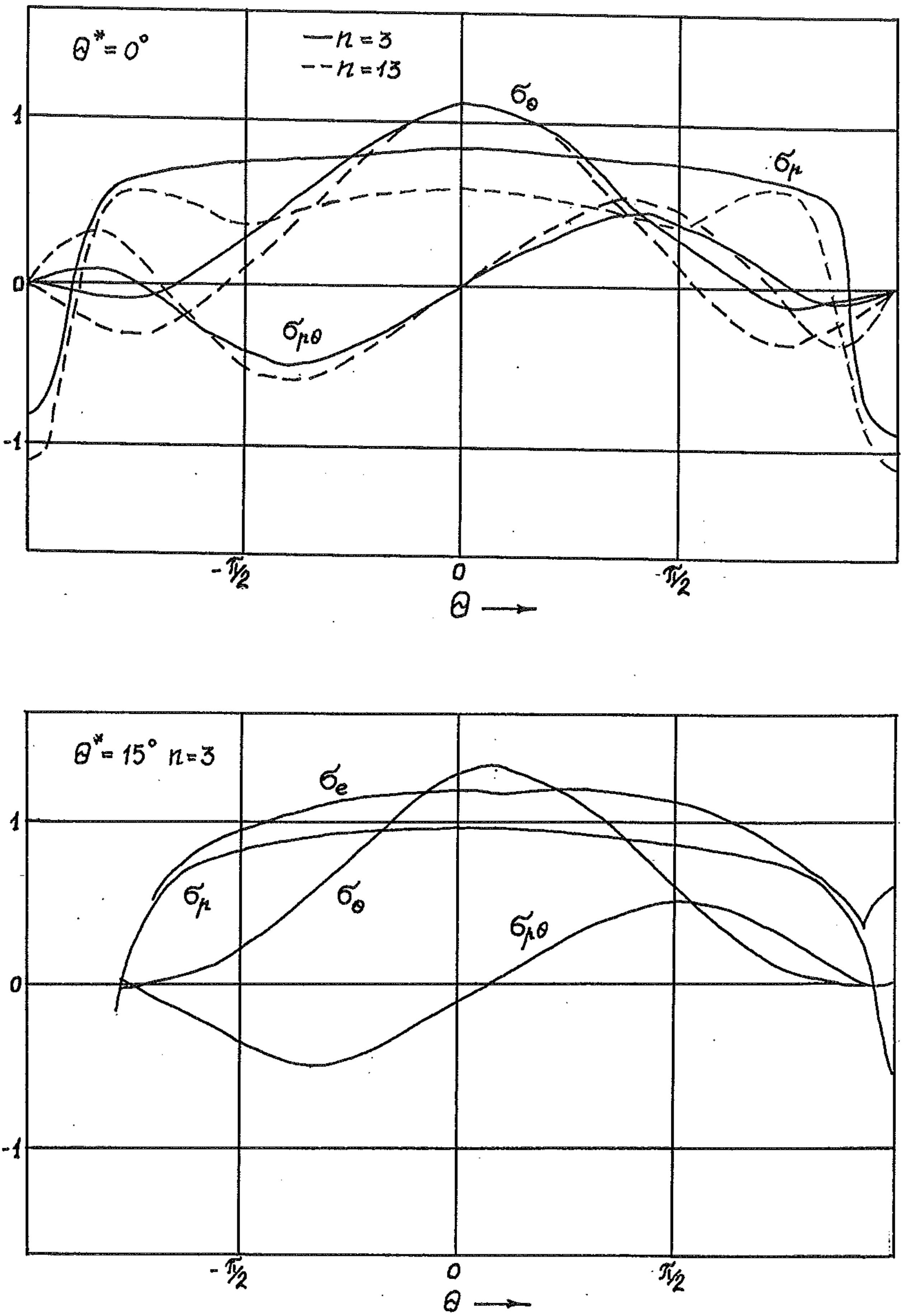

Fig. 9. Undimensional stress θ -distribution

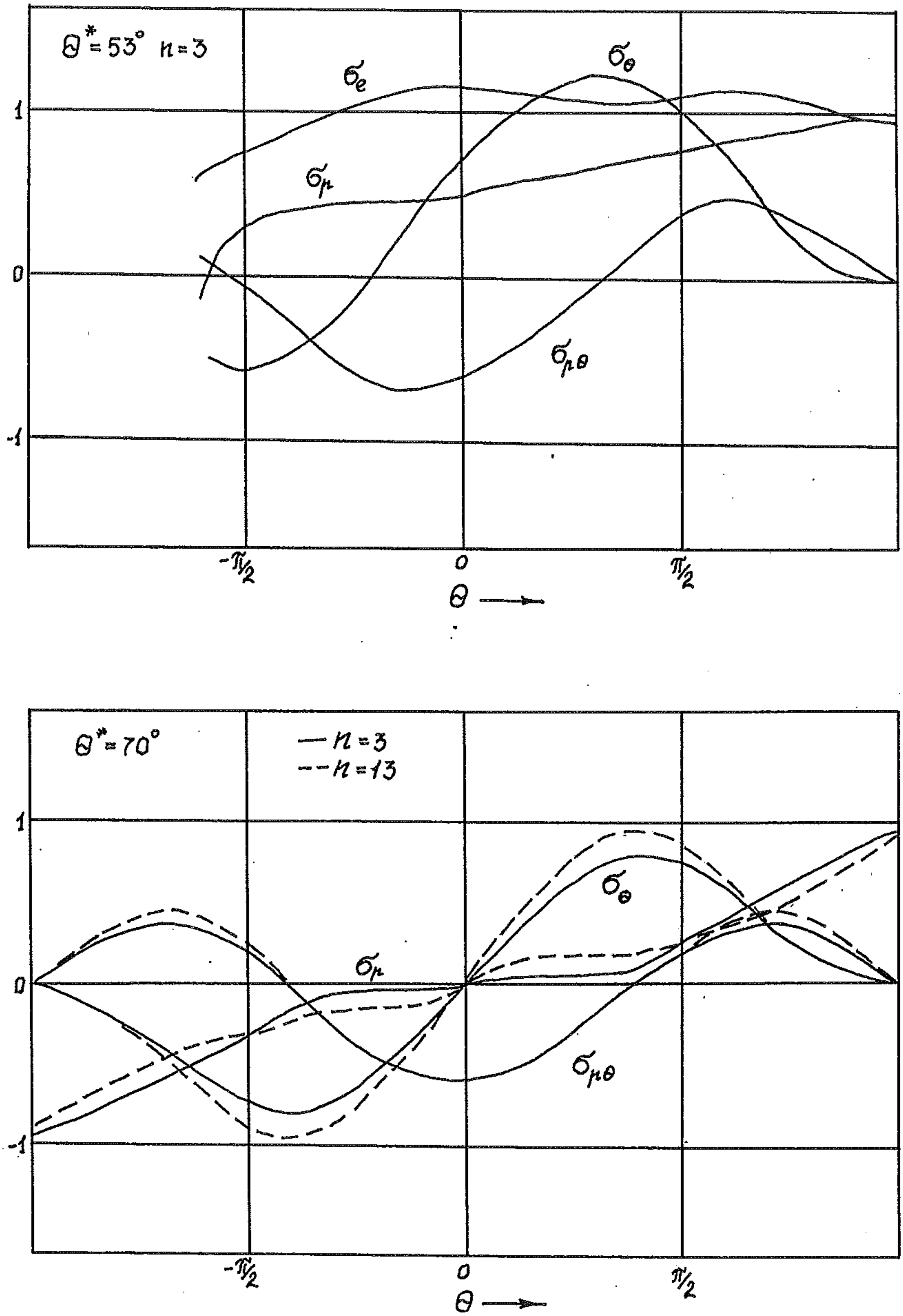

Fig. 10. Undimensional stress θ -distribution