MEAN STRESS EFFECTS IN BIAXIAL FATIGUE WHERE THE STRESSES ARE OUT-OF-PHASE AND AT DIFFERENT FREQUENCIES.

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ABSTRACT

The effect of mean stress on biaxial fatigue tests on thin wall steel cylinders covering a range of principal stress amplitude ratio, out-of-phase angle and frequency ratio are reported and discussed. Three different types of crack growth behaviour have been identified in these tests, depending on the state of multiaxial stress existing. Mean stress parallel to the plane of stage II crack growth had little effect on fatigue strength. Mean stress normal to the plane of stage II crack growth reduced fatigue strength in accordance with the modified Goodman relationship. Uniaxial push-pull fatigue test data cannot be used as being generally representative of multiaxial fatigue stress conditions. Each particular type of crack growth behaviour can be correlated by the Tresca shear stress criterion of failure.

KEYWORDS

Biaxial and multiaxial fatigue; mean stress; biaxial crack propagation; life assessment; principal stress ratio; out-of-phase cyclic loading; frequency ratio; cumulative damage; anistropy.

NOMENCLATURE

α	Frequency ratio = frequency of σ_{2a}/frequency of σ_{1a}
λ	Principal stress amplitude ratio = σ_{2a}/σ_{1a}
ϕ	Out-of-phase angle, where σ_2 leads σ_1; related to σ_2 where 1 cycle of σ_2 is 360°
$\sigma_1, \sigma_2, \sigma_3$	Principal stresses ($\sigma_1 > \sigma_2 > \sigma_3$)
$\sigma_{1a}, \sigma_{2a}, \sigma_{3a}$	Principal stress amplitudes
σ_a	Stress amplitude
σ_A	Uniaxial reversed fatigue strength
σ_m	Mean stress

σ_n	Normal stress amplitude on the plane of maximum range of shear stress
σ_{nm}	Mean normal stress on the plane of maximum range of shear stress
σ_T	Tensile strength
$\sigma_{n12}, \sigma_{n23}, \sigma_{n31}$	Normal stress amplitudes in the 12, 13, 31 planes of maximum range of shear stress
τ	Shear stress amplitude
$\tau_{12}, \tau_{23}, \tau_{31}$	Shear stress amplitudes on the 12, 23, 31 planes of maximum range of shear stress
τ_a	Shear stress amplitude on the plane of maximum range of shear stress
τ_m	Mean shear stress on the plane of maximum range of shear stress
12, 23, 31	Planes of maximum range of shear stress associated with the 1, 2, 3 principal stress directions.
1, 2, 3	Longitudinal, transverse and radial directions respectively.

INTRODUCTION

It has long been known that fatigue is the main cause of many mechanical failures. Despite over one hundred years of intensive research activity in this field it is still not possible to prevent the failure of components in service due to fatigue. This is partly due to the fact that many components in service are subjected to complex multiaxial fatigue stress conditions which make it very difficult to assess how fatigue damage occurs and how this damage accumulates during the life of the components. Commonly occurring structural examples are aircraft, nuclear reactors, pressure vessels and gas turbines along with components such as axles, crank shafts and propellor shafts which are subjected to combined bending and twisting which can be out-of-phase and at different frequencies. Many attempts have been made to derive theories which can cope with these complex multiaxial fatigue stress situations based on simple laboratory test data such as the uniaxial reversed stress fatigue test. A large number of theories of multiaxial fatigue have been proposed and many of these are reviewed in references (1-3).

The author has, among others, proposed a critical plane approach (4, 5) which has the advantage of a physical interpretation of fatigue damage accumulation. This approach proposed that the important parameters for long life fatigue in the unnotched situation are the alternating and mean stresses, both normal and shear, occurring on the plane of maximum range of shear stress. This theory has been extended for the case of out-of-phase biaxial stresses (6, 7) where it is shown that the out-of-phase stresses produce shorter fatigue lives than equal in-phase stresses. A further extension of this work (8) has been carried out for the case where the biaxial stresses are not only out-of-phase, but also at different frequencies.

Further tests (9) have been conducted under similar conditions to those in (8) to extend the work to include the effect of mean stress. The results of these tests led to the realisation that a number of different cracking systems operate, depending on the particular multiaxial fatigue stress system being applied. These cracking systems are discussed in (10).

EXPERIMENTAL PROGRAM

A series of fatigue tests were carried out on thin-wall tubular specimens to investigate the effect of mean stress while the specimens were subjected simultaneously to constant amplitude alternating longitudinal load and alternating differential pressure across the wall thickness. These tests covered the range of principal stress ratio, out-of-phase angle, frequency ratio and mean stress shown in Table 1.

Note that in the testing $\sigma_1 = \sigma_{LONGITUDINAL}$ and $\sigma_2 = \sigma_{TRANSVERSE\ (HOOP)}$, and the frequency of σ_{2a} was the greater and equalled 30 Hz in the tests conducted.

MATERIAL AND SPECIMEN

The material used was EN 24 T steel. The test specimens were thin wall tubes of internal diameter 25.4 mm and wall thickness 0.635 mm. Full details of material and test specimen are given in (8, 9).

TEST EQUIPMENT

The thin wall tubular specimen was mounted in a pressure test cell device, this assembly then being used in a standard fatigue test machine. While the test machine applied fluctuating longitudinal stress, the pressure cell applied fluctuating transverse stress via differential pressure across the specimen wall thickness. Full details of the test system are given in (8, 9).

CRITICAL SHEAR PLANES AND CRACK GROWTH PLANES AND DIRECTIONS

The critical shear planes and crack growth planes and directions for all cases are shown in Table 2, the detailed information being obtained from (10).

Table 1 gives the values of the shear and normal stresses, amplitude and mean values, acting on the critical shear planes in terms of the applied transverse stress amplitude in case 2 where only transverse stress is applied.

It has been shown in (8) that when biaxial principal stresses of the same amplitude, but of different frequency are applied, the frequency difference causes the critical shear stress and associated normal stress on the same plane to have varying amplitudes. For each cycle of σ_{1a} we obtain a number of cycles of τ_{12} and τ_{n12} of different amplitude depending on the value of the frequency ratio.

TEST RESULTS AND DISCUSSION

In (9), attempts were made to correlate biaxial fatigue test results, including the effect of mean stress, using a shear stress criterion of failure where the allowable shear stress amplitude was reduced due to the effects of σ_n, τ_m and τ_{nm}. Cases 4A, 4D and 1A and 4B in (9) were used to determine the separate effects of τ_a, σ_{nm}, σ_n and τ_m respectively. It is now realised that this approach is unlikely to be successful as different crack systems operate in Cases 1 and 4.

Case 1, (Longitudinal stress only):

Test results are shown in figure 1 plotted on a shear stress amplitude basis. Table 1 shows that there are two possible critical shear planes, 12 and 31 and also shows the stress conditions on the critical shear planes.

Table 2 indicates that the type of crack growth in this case is transverse A/B with a circular crack front growing into the surface through thickness. Test results for specimens tested within the pressurised test cell are slightly higher than those produced when the specimen is tested without being in the pressure cell.

Longitudinal mean stress effects fall close to a Goodman line relationship as shown in figure 9.

Mean transverse stress appears to have little effect on longitudinal fatigue life. Tests at higher mean transverse stress levels than those shown in figure 1 buckled before fatigue failure.

Case 2, (Transverse stress only):

Test results are shown in figure 2 where the transverse fatigue strength is seen to be about 75% of the longitudinal fatigue strength. Table 1 shows the 23 shear plane to be critical. Table 2 indicates the crack growth behaviour to be longitudinal type B with the crack front growing into the surface through thickness. In the extruded bar material used the 23 shear plane can contain extruded grains which would act as starter cracks. In the past, lower fatigue strength in the transverse direction, relative to the longitudinal fatigue strength, has been adjudged due to material anisotropy, whereas it is probably due at least in part to the different types of crack growth occuring in the two cases. In support of this argument, metallurgical examination of transverse and longitudinal sections of the material show little difference in material structure. Tensile tests of small specimens showed the material to be about 8% weaker in the transverse direction compared to the longitudinal direction.

Transverse mean stress effects again fall close to a Goodman line relationship, using transverse fatigue strength, as shown in figure 9.

Longitudinal mean stress appears to have very little effect on transverse fatigue strength.

Case 3, (λ=1, ϕ=0^o, α=1):

Test results are shown in figure 3 and are seen to be virtually identical to Case 2. This was to be expected as seen from Tables 1 and 2 which show that the 23 plane is again critical, crack behaviour is longitudinal type B into the surface and also that stress conditions on the critical 23 shear planes are the same in both cases. Again, longitudinal mean stress has no effect.

Case 4, (λ=1, ϕ=180^o, α=1):

Test results are shown in figure 4 where the fatigue strength is seen to be greater than for Cases 1, 2 or 3. Again, this is to be expected as Tables 1 and 2 indicate that a third type of crack growth behaviour, longitudinal Type A along the surface occurs in this case. Type A cracks growing along the surface are less severe than Type B cracks growing into the surface.

Stress conditions on the 12 critical shear plane are shown in Table 1 and indicate that case 4B, with longitudinal mean stress, should be weaker than case 4A which is confirmed by the test results. However, we might expect cases 4B and 4C to give the same test results which is not supported by the test results. Tests show that cases 4C and 4D give the same results. There is an unexplained discrepancy between the results of cases 4B and 4C.

Transverse mean stress effects fall close to a Goodman line relationship using the case 4A fatigue strength, as shown in figure 9.

Cases 5 (λ=1, ϕ=0, α=2), 6 (λ=1, ϕ=90°, α=2), 7 (λ=1, ϕ=0, α=3) and 8 (λ=1, ϕ=180°, α=3):

Test results are shown in figures 5, 6, 7 and 8 respectively. The 12 shear planes are critical and the crack growth behaviour is longitudinal Type A along the surface in all these cases. Mean stress behaviour is similar to that found in case 4, except for case 8 where the longitudinal mean stress has little effect.

Stress conditions on the 12 critical shear plane are shown in Table 1. It has been shown in (8, 10) that when biaxial principal stresses of the same amplitude, but of different frequency are applied, the frequency difference causes the critical shear stress and associated normal stress on the same plane to have varying amplitudes and thus, there is a cumulative damage problem. It is also clear that these stresses are out-of-phase. For each cycle of σ_{1a} we obtain a number of cycles of τ_{12} and σ_{n12} of different amplitude depending on the value of the frequency ratio. Values of τ_{12} and σ_{n12} for cases 5 to 8, taken from (8) are shown in Table 1.

For the zero mean stress cases (i.e., cases 5A, 6A, etc.) and assuming that only the cycles of greatest shear stress amplitude are damaging and also neglecting any effect of the normal stress acting on the plane of greatest shear stress amplitude, we can compare on a shear stress criterion of failure, the experimental results for cases 5A to 8A with those of case 4A which have the same longitudinal Type A cracks growing along the surface. This comparison is shown in figure 10, where we see that agreement is reasonable for cases 4A, 5A and 7A where the lesser shear stress amplitudes are less than 25% of the greater. In cases 6A and 8A agreement is not good and some allowance has to be made for the damaging effects of the lesser shear stress amplitudes which are 35% and 50% of the greater shear stress amplitudes in cases 6A and 8A respectively.

Transverse mean stress effects fall close to a Goodman line relationship using the appropriate case A fatigue strength, as shown in figure 9.

All Cases 1 to 8:

Test results for cases 1A to 8A, that is without mean stress, are shown in figure 10. This indicates three bands of results, each consisting of the same type of crack growth behaviour.

The highest fatigue strengths are found for longitudinal Type A cracks growing along the surface (cases 4A, 5A and 7A) and also cases 6A and 8A which should be higher as discussed earlier. The next strongest is case 1A with transverse Type A/B cracks growing into the surface. The lowest strength cases are 2A and 3A with longitudinal Type A cracks growing along the surface.

It is clear that the longitudinal stress only case test results can not be used for general multiaxial fatigue strength predictions as this investigation shows fatigue strength differences of ±25%, using a shear stress criterion of failure, depending on the particular type of crack growth behaviour occurring.

With regard to the effect of mean stress on fatigue strength, in the cases of longitudinal or transverse fatigue only (case 1 and 2) and equibiaxial in phase fatigue (case 3), only the mean stress normal to the crack plane has an effect in reducing fatigue strength. This reduction agrees with a Goodman line relationship.

For cases 4 to 8, where mean stresses are equal to stress amplitude, longitudinal mean stress parallel to the crack plane causes a small decrease in fatigue strength of the order of 5 to 10%. Transverse mean stresses, normal to the Stage II crack growth plane cause decreases in fatigue strength in agreement with the Goodman line relationship.

CONCLUSIONS

1. In multiaxial fatigue it is essential to relate the three dimensional cyclic stress state to the characteristics of crack growth, as well as to note any material anistropy existing.
2. Three different types of crack growth behaviour have been identified in these investigations, dependent on the states of multiaxial stress existing.
3. Uniaxial push-pull fatigue test data can not be used as being representative of multiaxial fatigue stress conditions.
4. The Tresca shear stress criterion of failure is appropriate for correlating each particular type of crack growth behaviour.
5. In the tests conducted mean strss parallel to the plane of Stage II crack

growth had little effect on fatigue strength.

6. In the tests conducted mean stress normal to the plane of Stage II crack growth reduced fatigue strength. This reduction is in good agreement with the modified Goodman line relationship using the appropriate reversed stress fatigue strength.

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Case	Longitudinal Stress		Transverse Stress		Critical Shear Plane	$\frac{\tau_a}{\sigma_{1a}}$	$\frac{\sigma_n}{\sigma_{1a}}$	$\frac{\tau_m}{\sigma_{1a}}$	$\frac{\sigma_{nm}}{\sigma_{1a}}$
	At 10^6 cycles, Mn/m^2								
	Amp.	Mean	Amp.	Mean					
1	$\lambda=0$ (Longitudinal Stress only)								
1A	460	–	–	–	12, 31	0.5	0.5	–	–
1B	360	360	–	–	12, 31	0.5	0.5	0.5	0.5
1C					12	0.5	0.5	0.5	0.5
2	$\lambda=$ (Transverse Stress only)								
2A	–	–	350	–	23	0.5	0.5	–	–
2B	–	–	260	260	23	0.5	0.5	0.5	0.5
2C	–	340	340	–	23	0.5	0.5	–	–
3	$\lambda=1$, $\phi=0$, $\alpha=1$								
3A	350	–	350	–	23	0.5	0.5	–	–
3B	350	350	350	–	23	0.5	0.5	–	–
3C	260	–	260	260	23	0.5	0.5	0.5	0.5
3D	260	260	260	260	23	0.5	0.5	0.5	0.5
4	$\lambda=1$, $\phi=180^\circ$, $\alpha=1$								
4A	285	–	285	–	12	1.0	–	–	–
4B	262	262	262	–	12	1.0	–	0.5	0.5
4C	235	–	235	235	12	1.0	–	0.5	0.5
4D	235	235	235	235	12	1.0	–	–	1.0

TABLE 1A. Test Conditions and Stresses on the Critical Shear Planes for Cases 1 to 4.

Case	Longitudinal Stress		Transverse Stress		$\frac{\tau_a}{\sigma_{1a}}$	$\frac{\sigma_n}{\sigma_{1a}}$	$\frac{\tau_m}{\sigma_{1a}}$	$\frac{\sigma_{nm}}{\sigma_{1a}}$
	At 10^6 cycles, Mn/m^2							
	Amp.	Mean	Amp.	Mean				
5	$\lambda=1$, $\phi=0$, $\alpha=2$							
5A	340	–	304	–	0.88	0.88	–	–
					0.18	0.18	–	–
5B	276	276	276	–	0.88	0.88	0.5	0.5
					0.18	0.18	0.5	0.5
5C	247	–	247	247	0.88	0.88	0.5	0.5
					0.18	0.18	0.5	0.5
5D	239	239	239	239	0.88	0.88	0	1.0
					0.18	0.18	0	1.0
6	$\lambda=1$, $\phi=90^o$, $\alpha=2$							
6A	295	–	295	–	0.78	0.78	0.25	−0.25
					0.28	0.28	−0.28	0.28
6B	282	282	282	–	0.78	0.78	0.75	0.25
					0.28	0.28	0.32	0.78
6C	250	–	250	250	0.78	0.78	0.75	0.25
					0.28	0.28	0.32	0.78
6D	250	250	250	250	0.78	0.78	0.25	0.75
					0.28	0.28	−0.28	1.28

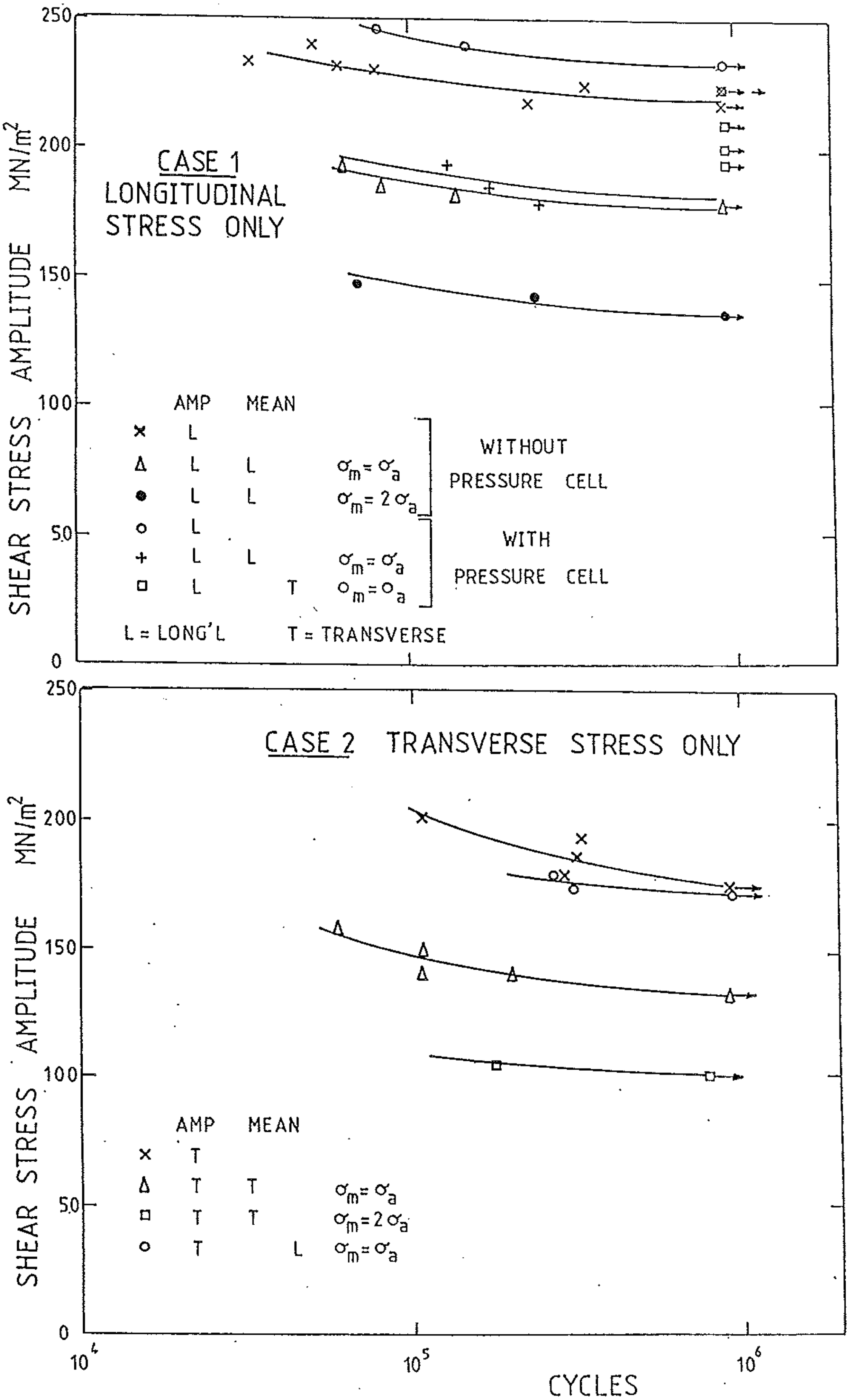
TABLE 1B. Test Conditions and Stresses on the Critical Shear Planes for Cases 5 and 6.

Case	Longitudinal Stress (At 10^6 cycles, Mn/m²) Amp.	Longitudinal Stress Mean	Transverse Stress Amp.	Transverse Stress Mean	$\frac{\tau_a}{\sigma_{1a}}$	$\frac{\sigma_n}{\sigma_{1a}}$	$\frac{\tau_m}{\sigma_{1a}}$	$\frac{\sigma_{nm}}{\sigma_{1a}}$
7	$\lambda=1$, $\phi=0$, $\alpha=3$							
7A	270	–	270	–	1.00	0.78	0	0
					0.28	0.39	0	0.39
					0.28	0.39	0	-0.39
7B	240	240	240	–	1.00	0.78	0.5	0.5
					0.28	0.39	0.5	0.89
					0.28	0.39	0.5	0.11
7C	210	–	210	210	1.00	0.78	0.5	0.5
					0.28	0.39	0.5	0.89
					0.28	0.39	0.5	0.11
7D	215	215	215	215	1.00	0.78	0	1.0
					0.28	0.39	0	1.39
					0.28	0.39	0	0.61
8	$\lambda=1$, $\phi=180^o$, $\alpha=3$							
8A	265	–	265	–	0.76	1.00	0	0
					0.38	0.28	0.38	0
					0.38	0.28	-0.38	0
8B	275	275	275	–	0.76	1.00	0.5	0.5
					0.38	0.28	0.88	0.5
					0.38	0.28	0.12	0.5
8C	205	–	205	205	0.76	1.00	0.5	0.5
					0.38	0.28	0.88	0.5
					0.38	0.28	0.12	0.5
8D	230	230	230	230	0.76	1.00	0	1.0
					0.38	0.28	0.38	1.0
					0.38	0.28	-0.38	1.0

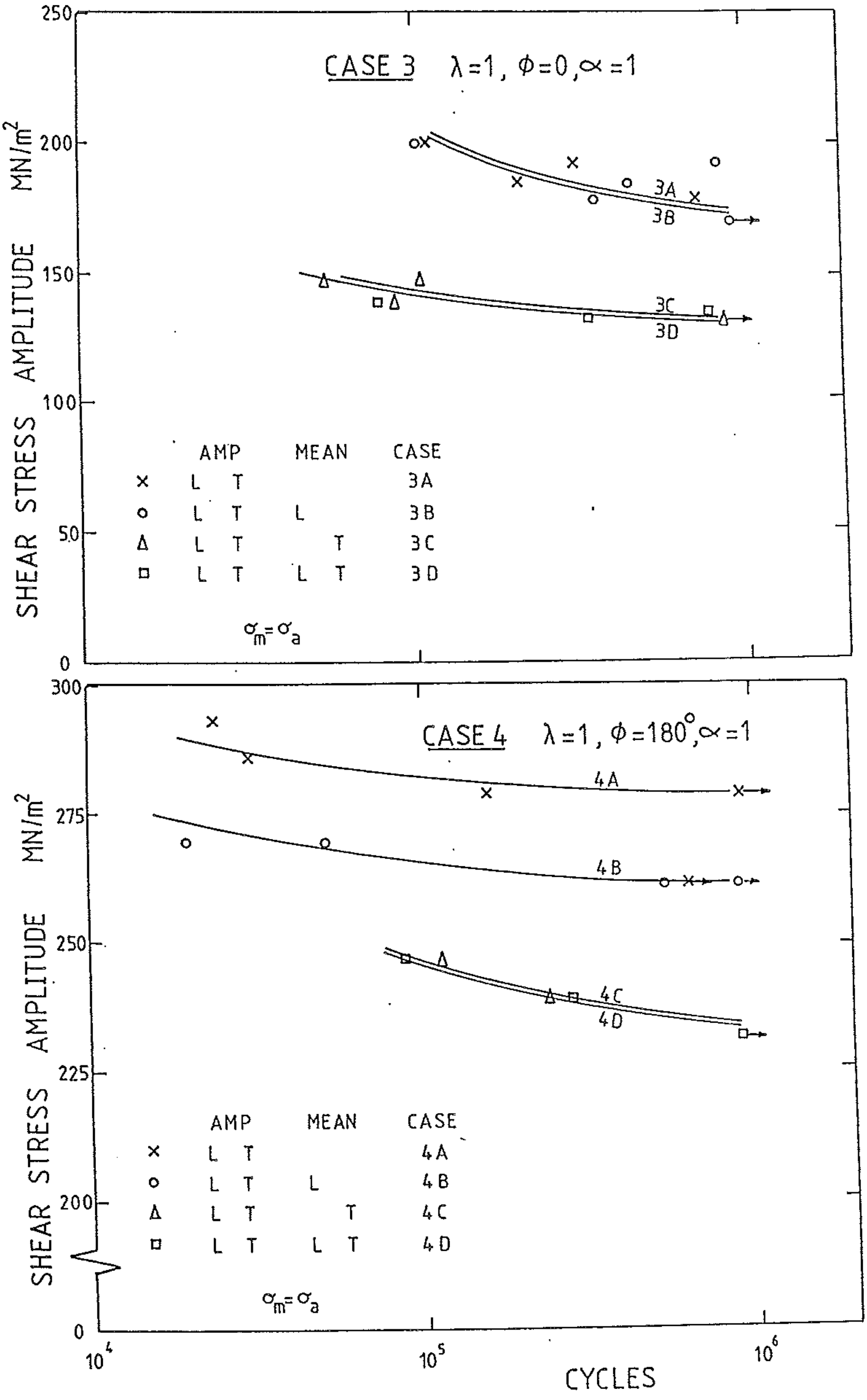
TABLE 1C. Test Conditions and Stresses on Critical Shear Stress Planes for Cases 7 and 8.

Case	Case in Ref (10)	Critical Shear Plane	Crack Type
1	1	12 = 31	Transverse A/B, circular into surface
2	7	23	Longitudinal B, into surface
3	5	23	Longitudinal B, into surface
4	9	12	Longitudinal A, along surface
5	12	12	Longitudinal A, along surface
6	13	12	Longitudinal A, along surface
7	14	12	Longitudinal A, along surface
8	15	12	Longitudinal A, along surface

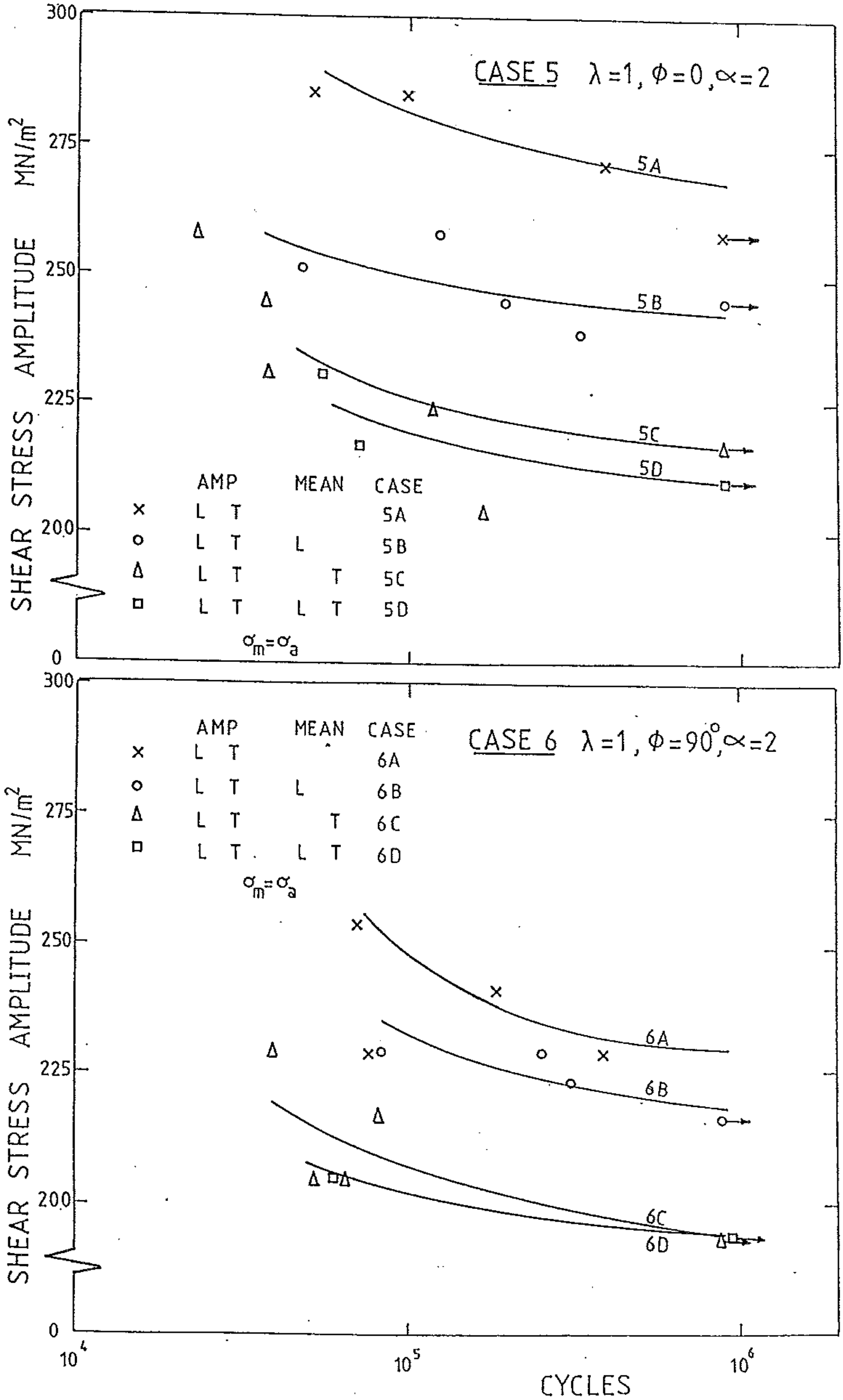
TABLE 2. Critical Shear Planes and Crack Types



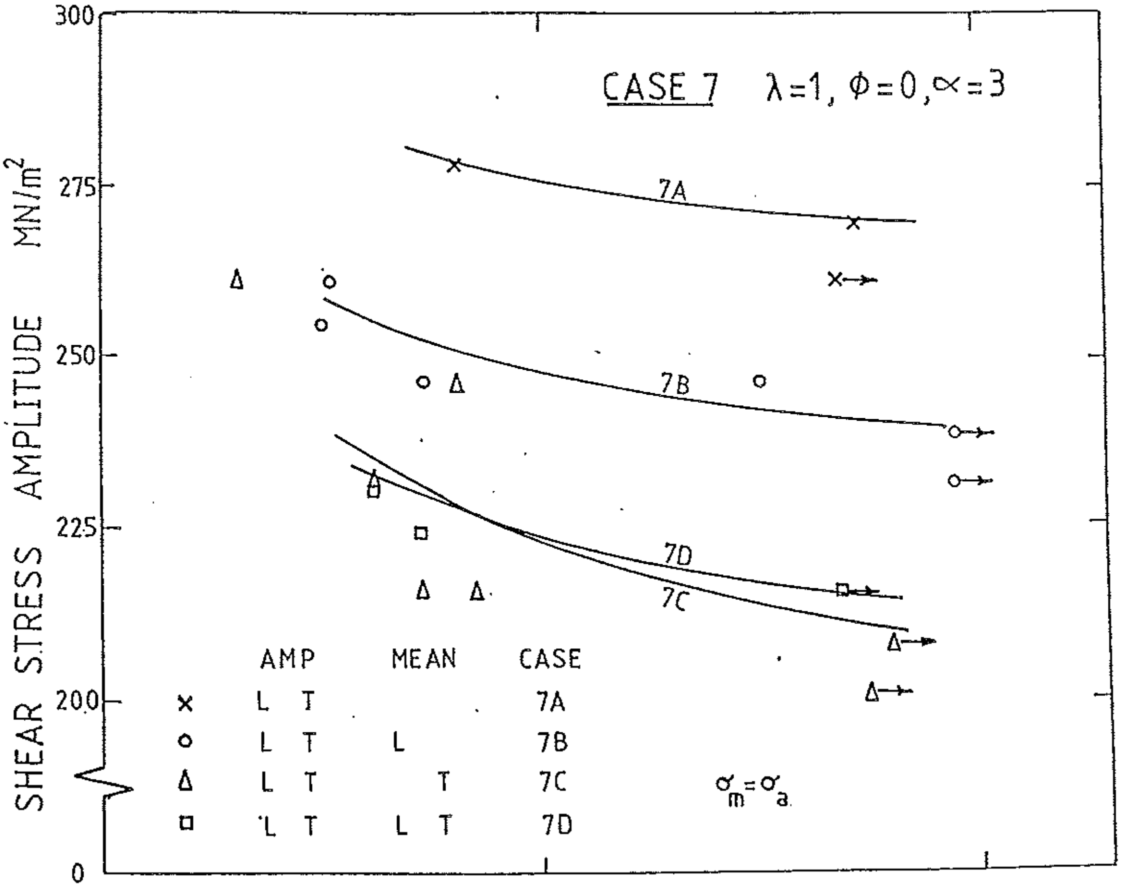
Figs. 1 and 2. Test Results. Cases 1 and 2.

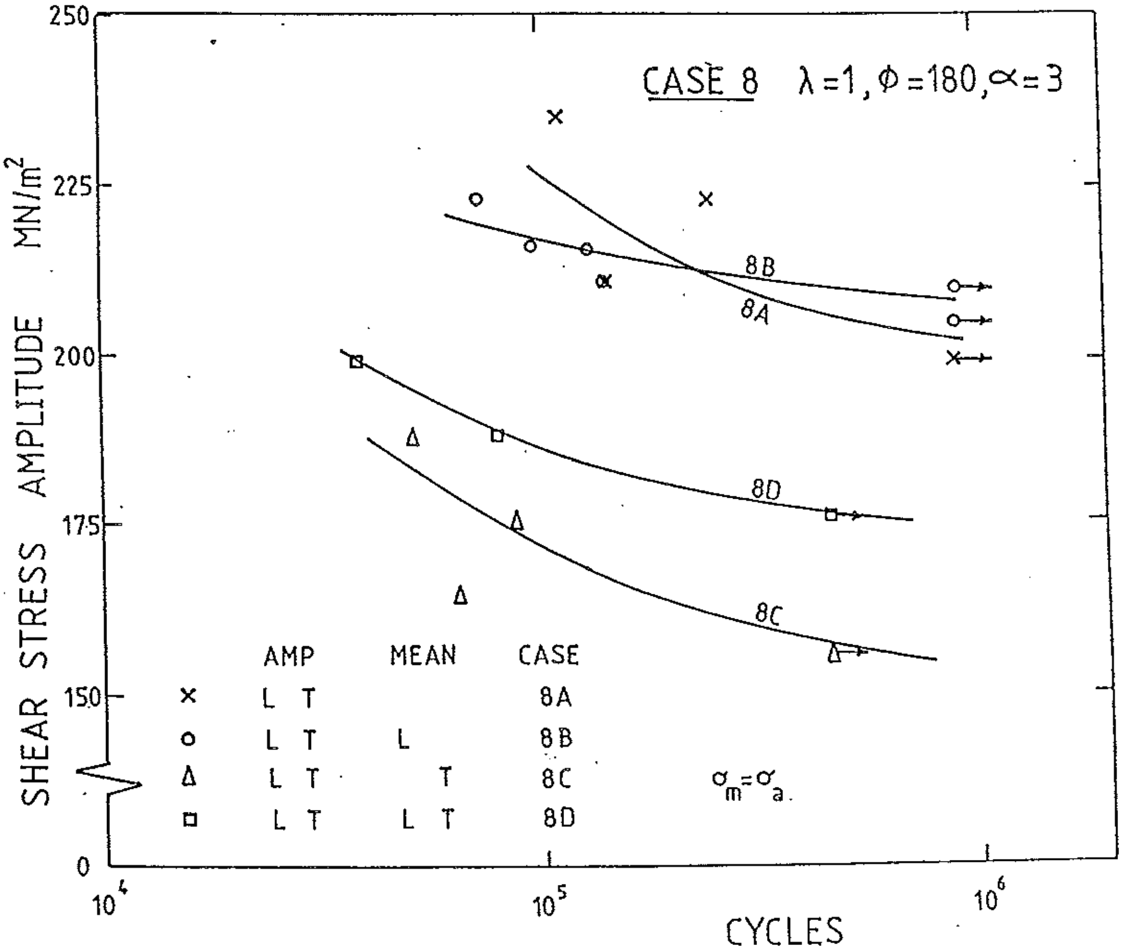


Figs. 3 and 4. Test Results. Cases 3 and 4.



Figs. 5 and 6. Test Results, Cases 5 and 6.





Figs. 7 and 8. Test Results. Cases 7 and 8.

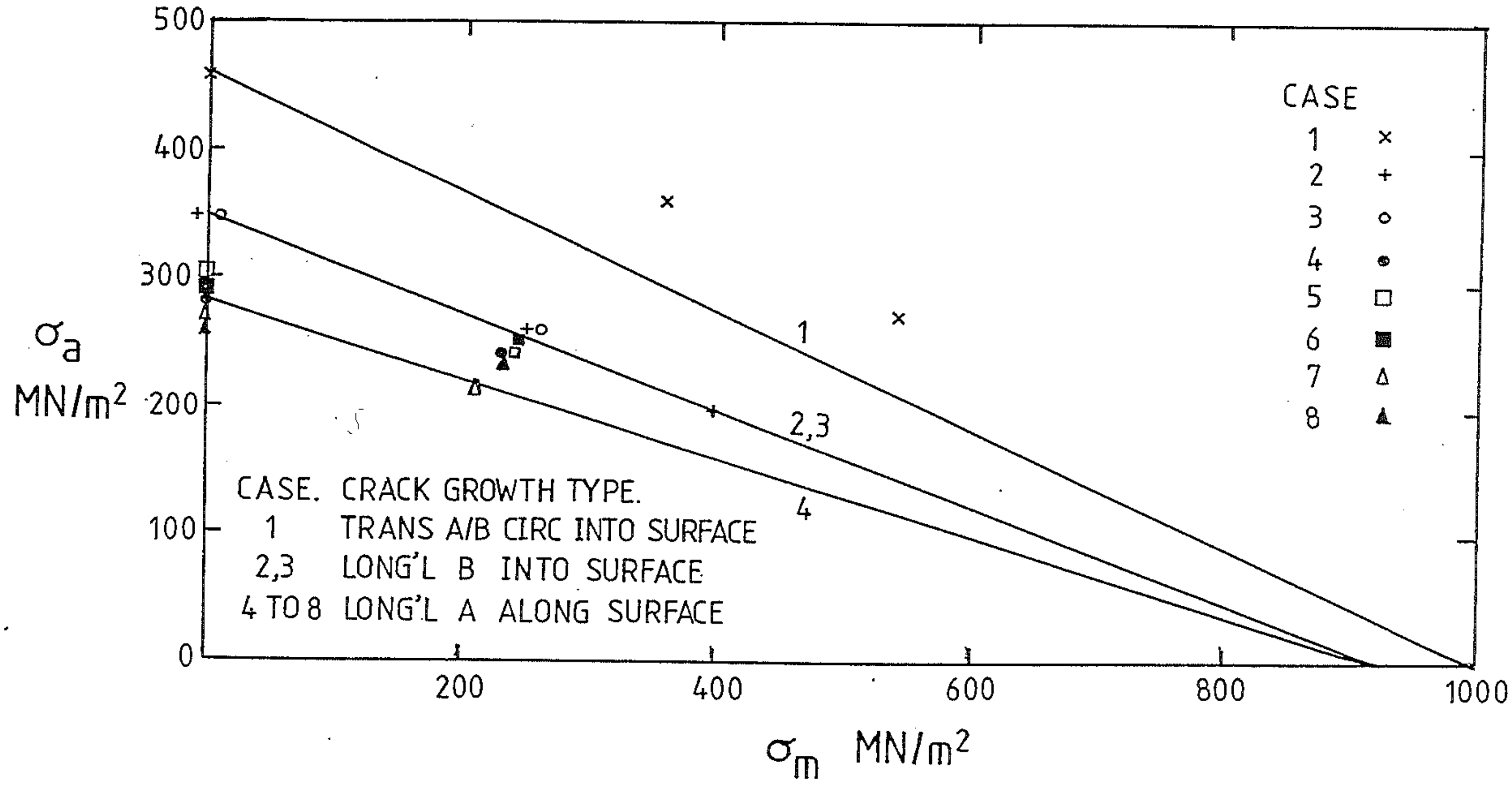


Fig. 9. The effect of mean stress - modified Goodman Line.

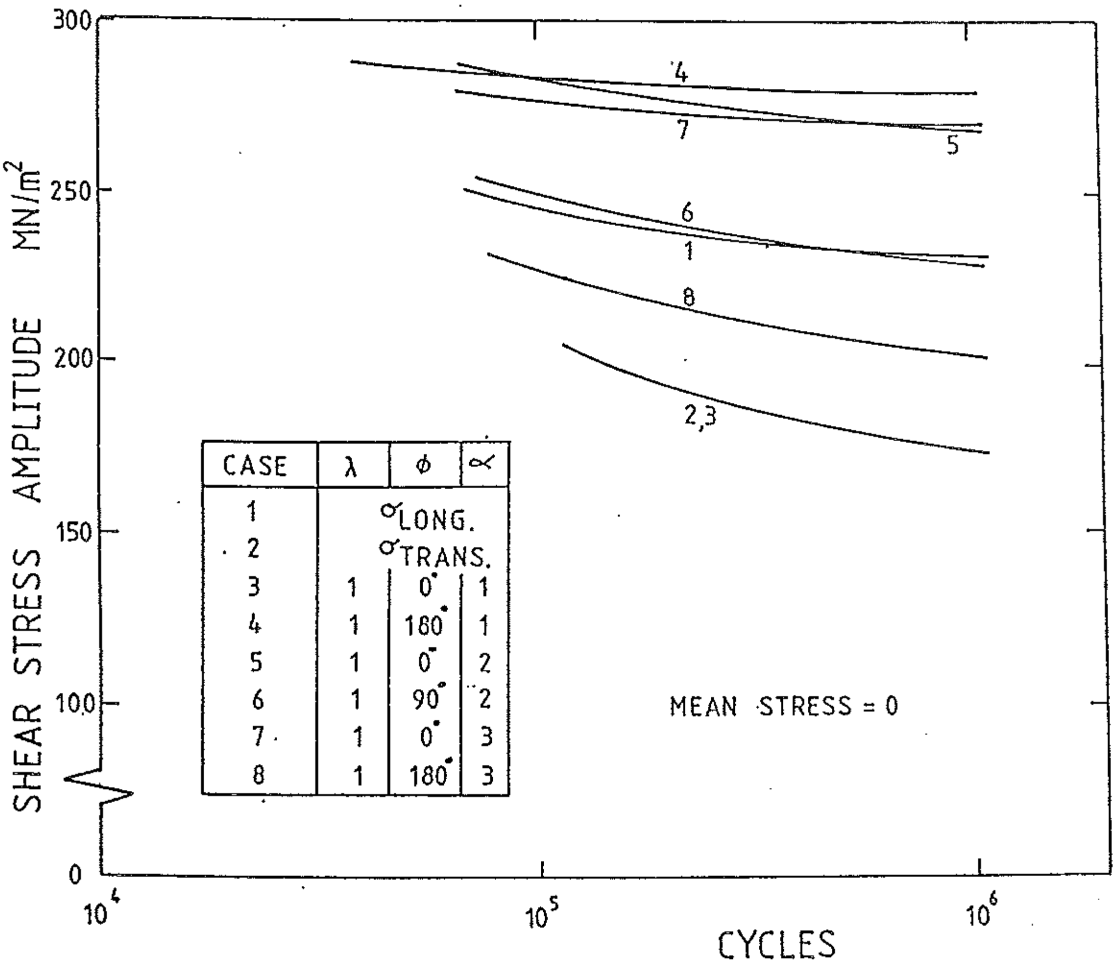


Fig. 10. Test Results. Cases 1A to 8A.