Local Strain Approach In Nonproportional Loading

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Synopsis

After briefly reviewing the state of the art of the local strain approach in proportional loading the method is extended to fatigue life predictions of components under nonproportional loading. Special attention is given to approximate notch stress-strain analysis where the solutions proposed are compared with Finite-Element calculations. Fatigue life predictions were carried out for components with locally uniaxial as well as multiaxial stress states.

Introduction

In the last two decades a method which is based on the assessment of the stresses and strains at critical locations — the local strain approach — has become an useful instrument for fatigue life evaluation of structures and components [1,2,3,4].

According to this concept two principal steps have to be carried out:

1. The calculation of the local stress-strain history, and
2. the evaluation of the fatigue damage caused by these stresses and strains.

The initial version of the local strain approach still had some shortcomings which limited the range of general applicability. However, this has been changed by several modifications and extensions being developed over the past years concerning

- nonlinear damage accumulation [5,6],
- cyclic softening and hardening [7],
- residual stresses [8,9],
- welds [8], and
- transferability of smooth specimen data to material elements in components (i.e. size effects, surface roughness etc.) [10,11].

Moreover, fatigue life evaluation have been successfully carried out even for components under proportional multiaxial loading. Fig. 1 shows e.g. fatigue life predictions of the "SAE notched shaft" under constant and variable amplitude loading [12,13].

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This paper outlines how the local strain approach can be applied to structures under nonproportional loading. In the first part, a special case is discussed where an uniaxial stress state prevails at the critical location (notched plate under tensile and torsional loading). The second part deals with the more general case of locally multiaxial stress states illustrated by the example of a notched shaft under out of phase bending and torsional loading.

Although the state of the art like in uniaxial fatigue has not been reached yet some promising results are obtained.

NONPROPORTIONAL LOADING WITH LOCALLY UNIAXIAL STRESSES

Notch stress-strain estimates

The examples given in Fig. 2 represent a special case of nonproportional loading as the critical locations are only uniaxially stressed. For these cases, fatigue life predictions by the local strain approach require only a restructuring of the notch stress-strain analysis. As we still deal with uniaxial stress states at the notch root damage assessment can be done in the usual way.

Recalling the standard algorithms for external uniaxial loading, namely

- Masing and Memory model and
- approximation formulas describing load-notch strain relationships like Neuber's Rule

the following questions arise:

- is there a way to model the cyclic structural behavior like in uniaxial fatigue and
- is it still possible to apply the known approximation formulas?

In order to solve these questions a notched plate was investigated by the Finite Element method employing ADINA with the kinematic hardening module. The material's stress-strain curve was approximated by a bilinear curve ($E_T/E = 0.05$).

Starting with the basic solution, Fig. 3 shows the load – notch strain hysteresis for pure tensile cycling (the amplitude value can be taken from Tab. 1). The theoretical elastic notch stress $\sigma_e = K_t \cdot S$ has been used as load parameter ($K_t = 2.89$). As known from uniaxial fatigue, the cyclic FE-solution can be modeled by just taking the monotonic load – notch strain curve and applying the Masing-model i.e. each load reversal point defines an origin for monotonic loading (now for a doubled stress-strain curve).

Furthermore, Fig. 3 reveals that the notch strain amplitudes are very well approximated by Neuber's Rule[2] [14].

$$E \cdot \sigma \cdot \varepsilon = \sigma_e^2 \tag{1}$$

Turning now to nonproportional loading, Fig. 4 shows load-strain hysteresis for out-of-phase tensile loading ($S_{1,a} = S_{2,a} = \sigma_Y$, $\varphi = 90°$). Using again the theoretical elastic notch stress

$$\sigma_e(t) = 2.89 \cdot S_{2,a} \cdot f(\omega t) - 0.96 \cdot S_{1,a} \cdot f(\omega t \pm \frac{\pi}{2}) \tag{2}$$

[2] Here, Neuber's Rule is given for elastic nominal strains. In case of nonlinear nominal strains a correction factor has to be introduced [15]

as load parameter similar curves as in the uniaxial case are obtained. The approximate solution (FE monotonic plus Masing-Memory) is still based on the monotonic curve for pure tensile loading.

For combined tensile loading the critical locations can only lay on the horizontal or vertical axis. However, for combined tensile and torsional loading the critical location can be at each point of the notch root depending on the load ratio and phase shift. For 90°-out-of-phase tensile and torsional loading ($S_{2,a} = 0.75 \cdot \sigma_Y$, $S_{3,a} = 0.82 \cdot \tau_Y$) the elastic solution leads to a maximum stress amplitude at an angle of 22,5° with

$$\sigma_e(t) = 1.83 \cdot S_{2,a} \cdot \sin \omega t + 1.48 \cdot S_{3,a} \cdot \cos \omega t \tag{3}$$

For all three hysteresis-loops depicted in Fig. 5 the above equation has been used for calculating the load parameter. Although the cyclic FE-solution delivers maximum strain amplitudes at a slightly different location (angle of 11.3°) than the elastic solution the approximate solutions still yield sufficiently accurate notch strain estimates.

From the examples given above and further FE-calculations [16] it can be concluded that nonproportional loading can be handled by known algorithms quite accurately, as long as the critical locations are uniaxially stressed. Of course, this statement has to be limited to the regime of local plasticity. In the range of general yielding losses in accuracy have to be expected.

Application

The general solution scheme for fatigue life predictions of structures with uniaxial stresses at critical locations can be summarized as follows:

0. The structure shall be loaded by $i = 1$ to n load components S_i with given time histories $S_i(t)$.
1. Establish a monotonic load – notch strain (stress) relationship

$$\varepsilon = \varepsilon(\sigma_e) \quad ; \quad \sigma = \sigma(\sigma_e) \tag{4}$$

   e.g. by Neuber's Rule taking the material's stress-strain curve into account.
   Run through the following steps for each point at the notch root:
2. Determine the elastic load – notch stress relationship for each load component S_i[3]

$$K_{t,i}(s) = \frac{\sigma_{e,i}(s)}{S_i} \tag{5}$$

3. Calculate the theoretical elastic notch stress history

$$\sigma_e(s,t) = \sum_{i=1}^{n} K_{t,i}(s) \cdot S_i(t) \tag{6}$$

4. Use Rainflow-Counting of $\sigma_e(s,t)$ to detect closed hysteresis loops and calculate notch stress-strain amplitudes under consideration of Eq. 4 and the Masing-Memory model.
5. Assess and accumulate damage for closed hysteresis loops as known from uniaxial fatigue.

[3] Definition of S_i arbitrary, as only product $K_t \cdot S$ enters Eq. 6

The point with the largest damage sums determines the most critical location and the expected fatigue life.

The above procedure has been applied to study the influence of phase shift and different frequencies on the fatigue life of notched plates under multiaxial loading [16]. Details of the fatigue analysis were:

- plate with circular hole
- material: mild steel St 37, cyclic data from [17]
- mean stress parameter according to Smith/Watson/Topper
- notch stress-strain analysis according to the outlined solution scheme (where Neuber's Rule was employed as Eq. 4)
- linear damage accumulation without consideration of an endurance limit.

Fig. 6 shows lines of constant fatigue life ($1.2 \cdot 10^8$ and $1.0 \cdot 10^5$) for out-of-phase biaxial loading. The allowable nominal stresses have been normalized by the allowable stresses for uniaxial loading. Compared to uniaxial loading, proportional loading leads to a reduction of the strain amplitudes at the critical locations. Therefore, the allowable nominal stresses increase.

For a phase shift of 90° biaxial loading doesn't influence the fatigue life. The most critical phase shift is given by 180° (equivalent to pure shear) leading to a significant reduction of the allowable stress amplitudes.

The influence of a shear component is illustrated in Fig. 7. Here, in-phase loading is most damaging. The lines of constant fatigue life in Fig. 6 and 7 do not show any dependency on the absolute number of cycles (i.e. plasticity doesn't influence these interaction curves).

Biaxial constant amplitude loading with different frequencies leads to locally variable amplitude loading. Thus, rainflow counting and damage accumulation have been used for deriving the fatigue maps given in Fig. 8. Compared to the cases studied previously, different frequencies have a minor effect on the fatigue life.

Examples of service loading are shown in Fig. 9 to 11. A Gaussian load sequence with 10^4 number of cycles was investigated. The fatigue maps for small phase shifts, $\varphi = 0° - 360°$, are similar to the ones for constant amplitude loading (compare Fig. 9 with 6 and Fig. 10 with 7). "Large" phase shifts by multiples of the cycle time — where load reversals occur at the same time — have a less significant influence on the allowable stresses, Fig. 11.

The examples of this section demonstrated the capability of the local strain approach. By this method interaction formulas given by construction codes for multiaxial loading can be examined, and, if necessary improved [16].

NONPROPORTIONAL LOADING WITH LOCALLY MULTIAXIAL STRESS STATES

Basic considerations

Nonproportional loading linked with locally multiaxial stress states leads to a lot more complex situation than the case discussed in the previous section. Employment of the local strain approach requires a multiaxial damage and deformation theory as well as techniques for estimating notch stresses and strains, Fig. 12.

Concerning the damage assessment of arbitrarily multiaxial stress-strain histories critical plane theories (e.g. [18]) or integral approaches (e.g. [19]) have been proposed where defining a damage event for variable amplitude histories is still problematic for the latter one.

In order to describe the multiaxial deformation behaviour an incremental theory of plasticity in general is necessary. As long as stabilized conditions are considered Mroz's model leads to reliable results [20].

At the present state, notch stress-strain histories are most accurately predicted by the Finite Element method using a suitable deformation theory. However, thinking on service loading with more than a million load reversal points, there is a need for easy to handle approximate solutions which do not need so much computational power and experience.

As the material's deformation behaviour requires an incremental theory it would be desirable to develop an incremental load – notch stress-strain relationship. Differentiating Neuber's rule for local plasticity, Eq. 1

$$\sigma_e d\varepsilon_e = \frac{1}{2}(\sigma \cdot d\varepsilon + \varepsilon \cdot d\sigma) \tag{7}$$

leads to a new interpretation of Neuber's Rule allowing an extension to multiaxial stress states: The theoretical elastic work increment $\sigma_e \cdot d\varepsilon_e$ (with $d\varepsilon_e = d\sigma_e/E$) is equal to the mean of the elastic-plastic work increment $\sigma \cdot d\varepsilon$ and the corresponding complementary work increment $\varepsilon \cdot d\sigma$. Applying Eq. 7 to a multiaxially stressed notch at a traction free surface would lead to

$$\sigma_{z,e} d\varepsilon_{z,e} + \sigma_{\theta,e} \cdot d\varepsilon_{\theta,e} + \tau_{z\theta,e} \cdot d\gamma_{z\theta,e} = \frac{1}{2}(\sigma_z d\varepsilon_z + \sigma_\theta d\varepsilon_\theta + \tau_{z\theta} d\gamma_{z\theta} + \varepsilon_z d\sigma_z + \varepsilon_\theta d\sigma_\theta + \gamma_{z\theta} d\tau_{z\theta}) \tag{8}$$

Alternatively the strain energy density approach [21]

$$\sigma_e \cdot d\varepsilon_e = \sigma \cdot d\varepsilon \tag{9}$$

could also serve as incremental load – notch strain relationship.

Analogous to the approximate solution for proportional loading, the key problem is finding two suitable equations for the notch boundary conditions allowing to solve the set of equations (given by Eq. 8 and the constitutive stress-strain equations). Where in proportional loading the assumptions

$$\frac{\varepsilon_\theta}{\varepsilon_z} = \left(\frac{\varepsilon_\theta}{\varepsilon_z}\right)_e = \text{const.} \tag{10}$$

$$\frac{\gamma_{z\theta}}{\varepsilon_z - \varepsilon_\theta} = \left(\frac{\gamma_{z\theta}}{\varepsilon_z - \varepsilon_\theta}\right)_e = \text{const.} \quad \text{(fixed principal axes)} \tag{11}$$

worked quite successfully [22,23,24], Eq. 11 doesn't hold anymore for nonproportional loading.

Because of the problems finding a suitable substitute for Eq. 11, and the relatively high numerical effort necessary for solving the set of differential equations the following simplified procedure is proposed:

1. Apply the approximation procedure derived for proportional loading [12,13,22,24,23] separately for the acting load components (load separation).
2. Perform a subsequent compatibility iteration to improve the accuracy of the notch stress-strain estimates.
3. Assess damage by the critical plane method.

Notch stress-strain estimates

In order to verify the simplified notch analysis proposed in the previous section a series of Finite Element calculations were carried out [25]. Fig. 13 and 14 gives a survey on the structure and nonproportional loading histories investigated.

The basic idea of the "load separation" method is illustrated in Fig. 15 showing a comparison of the Finite Element results with the approximate solution for 90°-out-of-phase tensile and torsional loading (case H, Fig. 14). The load – notch strain hysteresis loops $\sigma_n - \varepsilon_z$ and $\tau_n - \gamma_{z\theta}$ were calculated separately by the approximate solution developed for proportional loading. A formula proposed by Seeger [26] was applied for correlating nominal stress $\sigma_n(\tau_n)$ with notch equivalent strain ε_q. Computation of the stress and strain components was achieved by Hencky's equations. It has to pointed out that the elastic-plastic interaction between axial force and torque is not been taken into account by load separation method.

Despite this simplification, the approximate solution delivers an acceptable estimation of the amplitudes of notch stress and strain. Obviously the phase shifting effect reducing the correlation between the components in the amplitudes corresponds to some degree with the load separation.

Loading with different frequencies of axial and torsional load (case N in Fig. 14, $f_N/f_T = 2.0$) the load separation procedure yields less successful results, Fig. 16 and 17. The changing interaction between the components leads to complex notch stress-strain paths and an increase of the strain amplitudes in comparison with frequent out-of-phase loading (same frequencies). Especially, the shear strain amplitudes are strongly underestimated with differences up to 30%.

The experience with the load separation method can be summarized as follows:

- For the first time, the method enables application of the local strain approach to nonproportional loading and includes the limiting case of elastic behaviour.
- The method is easy to handle as it employs the techniques known from proportional loading, and it doesn't require an incremental solution.
- Despite the rude simplification — neglecting plastic interaction of the external loads — notch strain estimates are sufficiently accurate for out-of-phase loading (same frequencies).
- For loading histories where the maxima of the load components are reached at the same point of time (one special case is proportional loading), and a large amount of plasticity is induced load separation can lead to an underestimation of notch strains up to 30 %.

To improve the accuracy a subsequent compatibility procedure was developed. Because of the neglected interaction load separation leads to too high stresses and too small strains in comparison with the relevant curve, Fig. 18. Hence, the load separation solution may be interpreted as the solution of a stiffer material.

A load-neutral compensation of the incompatibilities of notch stress and strain states might be achieved by the incremental relations of the known notch stress-strain functions, e.g. the constant strain energy density formulation $\sigma_i^A \cdot \Delta\varepsilon_i^A = \sigma_i^B \cdot \Delta\varepsilon_i^B$, here – different from Eq. 9 – applied for each component separately, Fig. 18

$$\sigma_i^{LSC} \cdot \Delta\varepsilon_i^{LSC} = \sigma_i^{LSC} \cdot \alpha\Delta\varepsilon_i^{LS} = \sigma_i^{LS} \cdot \Delta\varepsilon_i^{LS} \tag{12}$$

The indices LSC and LS refer to the load separation solution with and without the compatibility procedure.

To ensure compatible stress-strain states the multiaxial deformation law of plasticity for kinematic hardening has to be satisfied

$$\left\{\Delta\sigma^{LSC}\right\} = \mathbf{E}^* \left\{\Delta\varepsilon^{LSC}\right\} \tag{13}$$

($\mathbf{E}^* \equiv$ elastic-plastic stiffness matrix of material). In the formulation of Mroz [20] individual material behaviour can be taken into account.

Fig. 19 to 21 shows that the compatibility procedure effects a considerable improvement of the approximate solution particularly in the case of different frequencies. The resulting stress-strain paths now reproduce relatively good the FE hysteresis loops, the amplitudes as well as the complex general course.

Application and Discussion

At present state following procedure could be advised (Fig. 22):

1. Notch stress-strain analysis by load separation i.e.
   for each load component calculate local stresses/strains by approximation procedure for proportional multiaxial loading using Masing Memory model, known local notch strain approximation formulas and Hencky's equations.
2. In case improvement of accuracy is required (e.g. large amount of plasticity is to be expected) do a subsequent compatibility iteration.
3. Apply critical plane approach for damage assessment (see [18])
   i.e. define a damage parameter e.g. Kandil/Brown/Miller strain including a mean stress term, calculate strain and stress according to the chosen parameter for all possible plains using the calculated notch stresses and strains (either from step 1 or step 2) and perform a damage analysis for each plane like in uniaxial fatigue.

The outlined procedure had been employed for fatigue life prediction of the SAE shaft under out-of-phase bending and torsion. For this type of loading strain estimations by load separation are sufficiently accurate. The compatibility iteration leads only to minor changes of the strain amplitudes. For the basic load – notch (equivalent) strain the formula proposed by Seeger is used.

As damage parameter serves the parameter of Kandil/Brown/Miller. The mean stress term can be ignored as fully reversal loading is considered.

The comparison of predicted with actual life in Fig. 22 shows that the procedure presented gives reasonable results quite comparable to the usual life estimation of uniaxially stressed notched structures. The differences remain within a factor of two.

Taking into account that a technical crack initiation (e.g. 0.5 mm) is hard to detect for the notched shaft [27] the first results may not be overestimated. More experimental verification of the method is necessary including constant amplitude as well as variable amplitude loading. Concerning variable amplitude loading suitable accumulation theories are most important.

Last but not least, it has to be pointed out that methods for loading history analysis are still missing for nonproportional loading i.e. methods for reduction of field measurements, reconstruction of reduced histories and extrapolating of short term measurements

Time has to be taken into account as additional parameter in order to describe interaction of the load components.

Conclusions

1. The theoretical elastic notch stress is the key loading parameter for nonproportionally loaded structures with locally uniaxial stress states. Plotting this parameter versus notch strain similar hysteresis loops like the ones of uniaxial fatigue are obtained. Therefore, all the known techniques of the local strain approach can be applied without modifications to these cases.
2. Locally multiaxial stress states under nonproportional loading require a restructuring of the local strain approach. First estimates of notch stresses and strains can be achieved by separating the external load components. Applying a subsequent compatibility procedure allows an improvement in accuracy.

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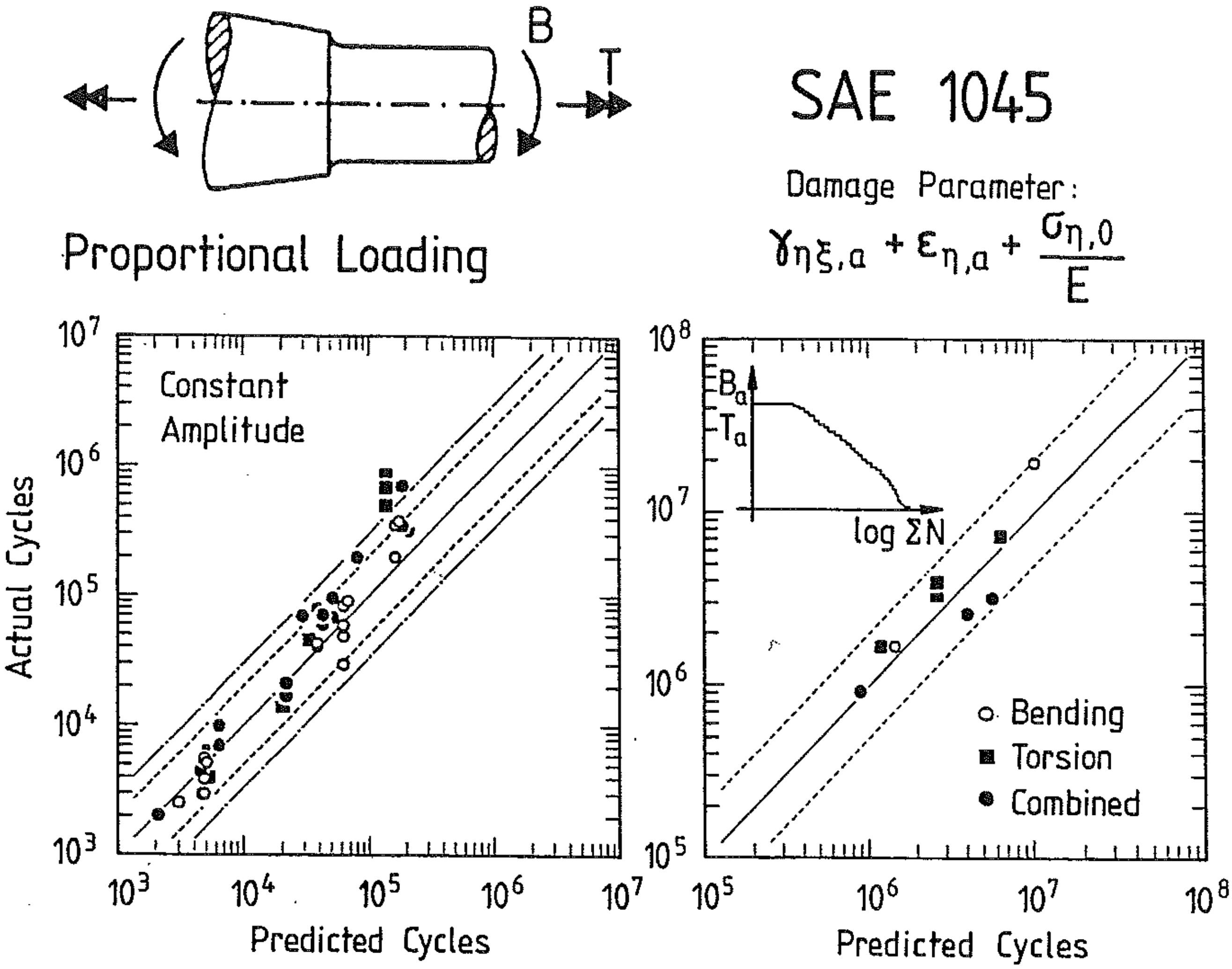


Fig. 1 Fatigue life estimation of the "SAE notched shaft" under constant and variable amplitude loading

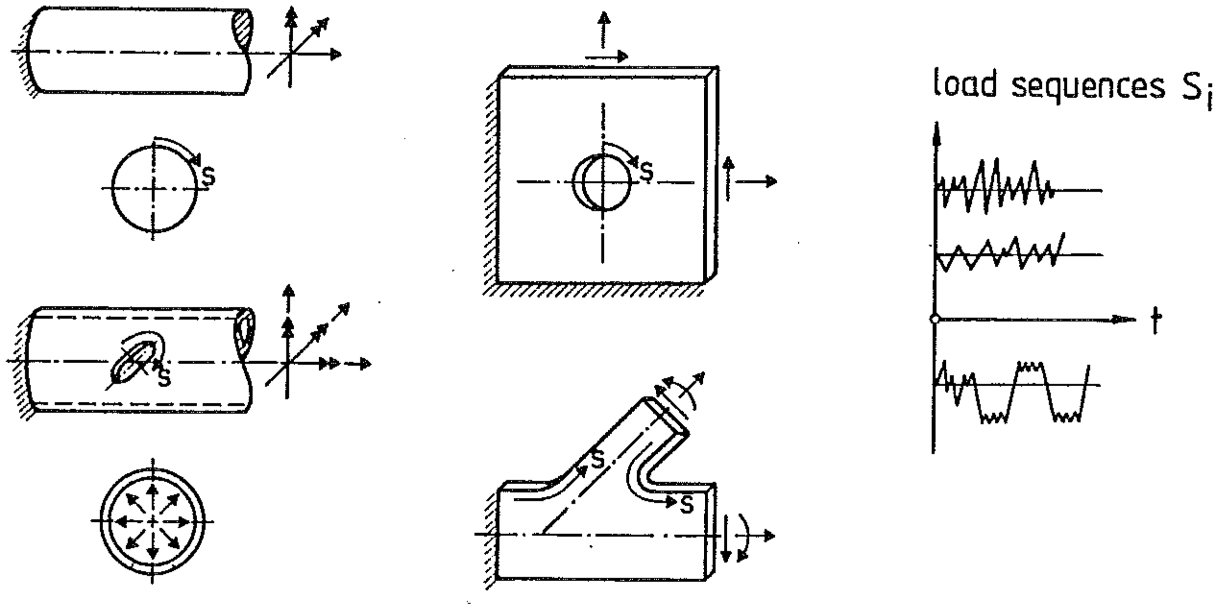
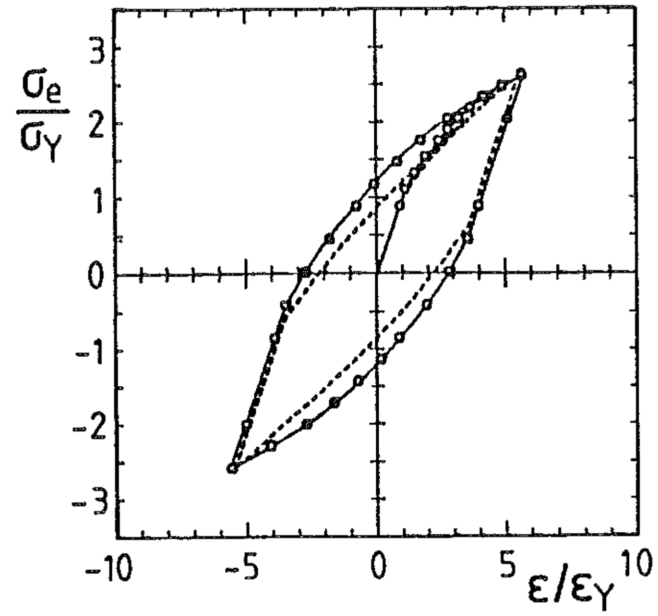


Fig. 2 Examples of structures with uniaxially stressed notch elements



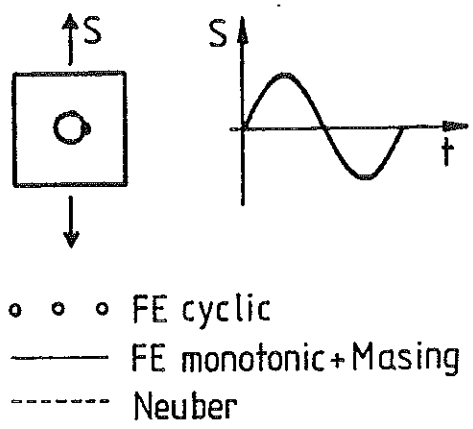
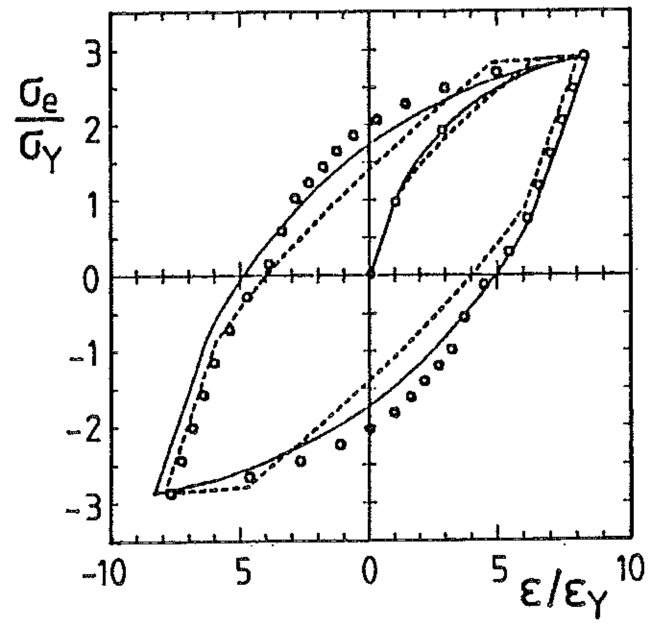


Fig. 3 Load – notch strain hysteresis – single load



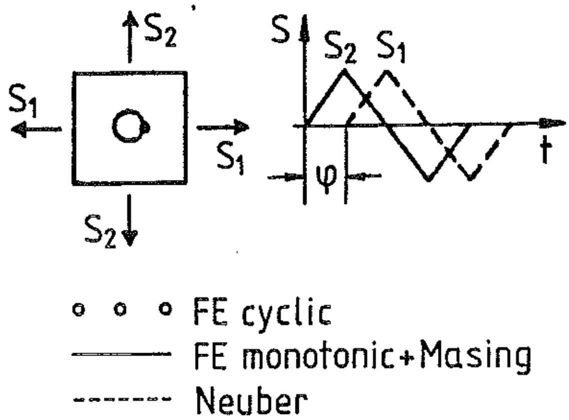
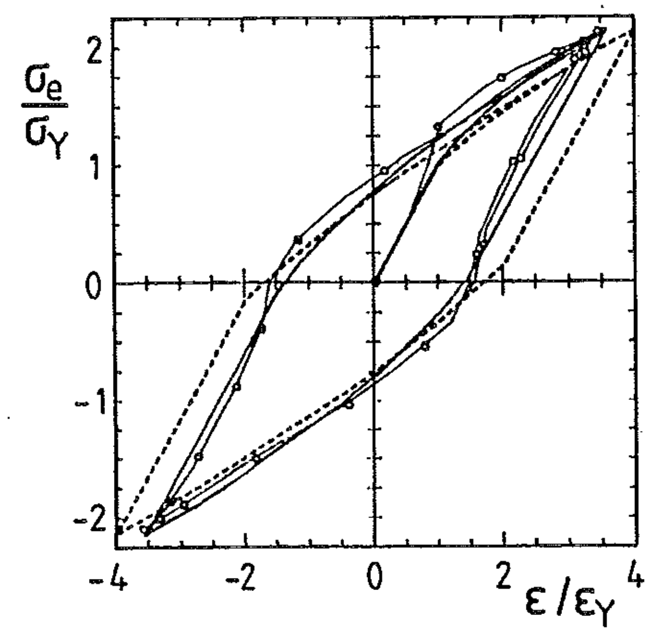


Fig. 4 Load – notch strain hysteresis – out-of-phase loading $S_1 - S_2$



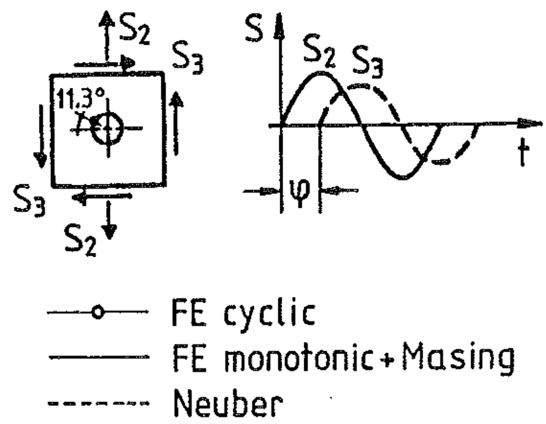


Fig. 5 Load – notch strain hysteresis – out-of-phase loading $S_2 - S_3$

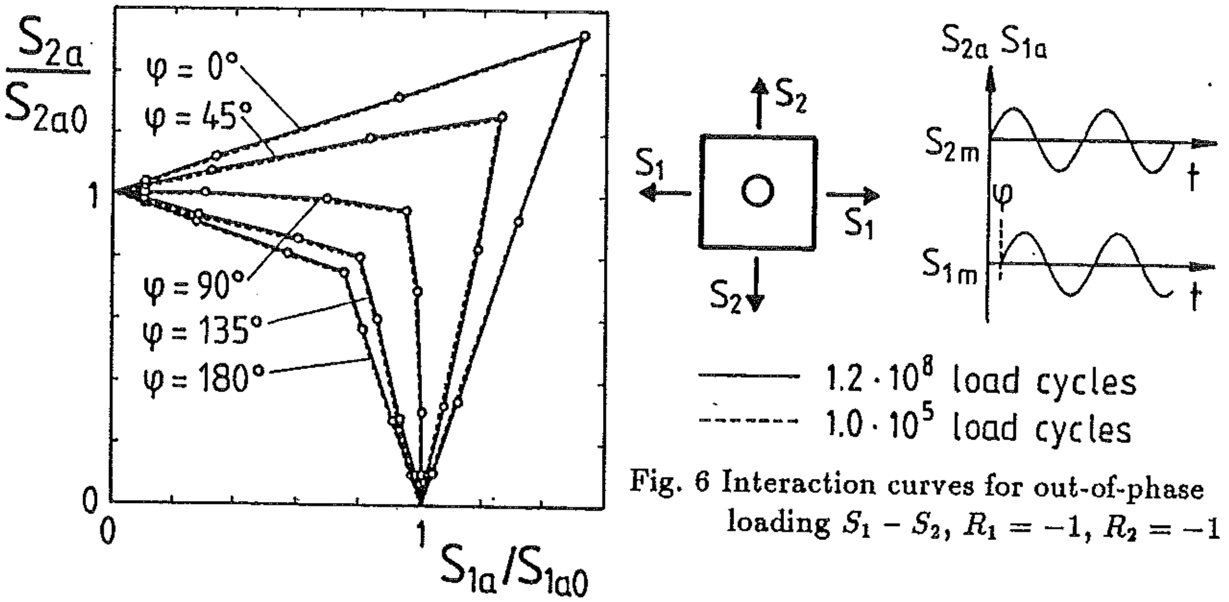


Fig. 6 Interaction curves for out-of-phase loading $S_1 - S_2$, $R_1 = -1$, $R_2 = -1$

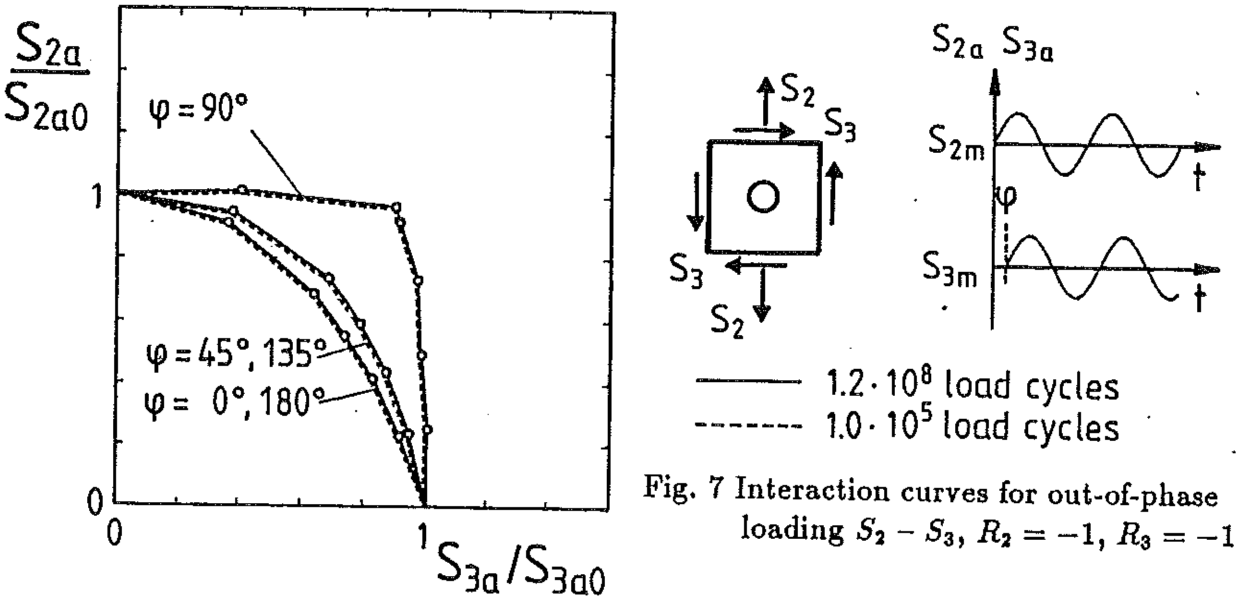


Fig. 7 Interaction curves for out-of-phase loading $S_2 - S_3$, $R_2 = -1$, $R_3 = -1$

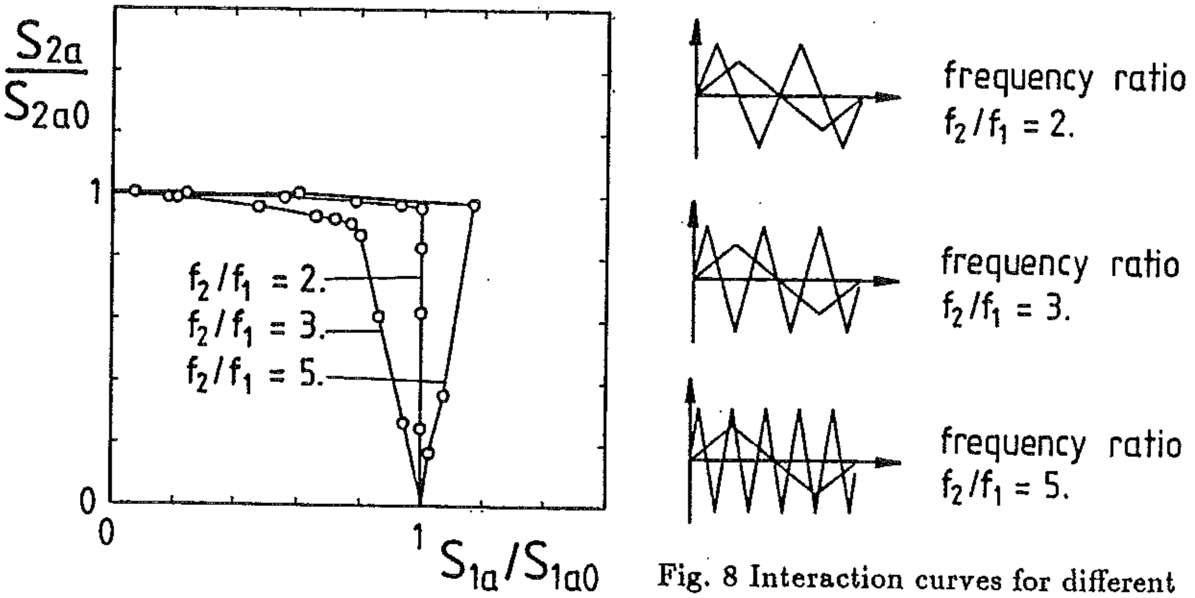


Fig. 8 Interaction curves for different frequencies – $R_1 = -1$, $R_2 = -1$

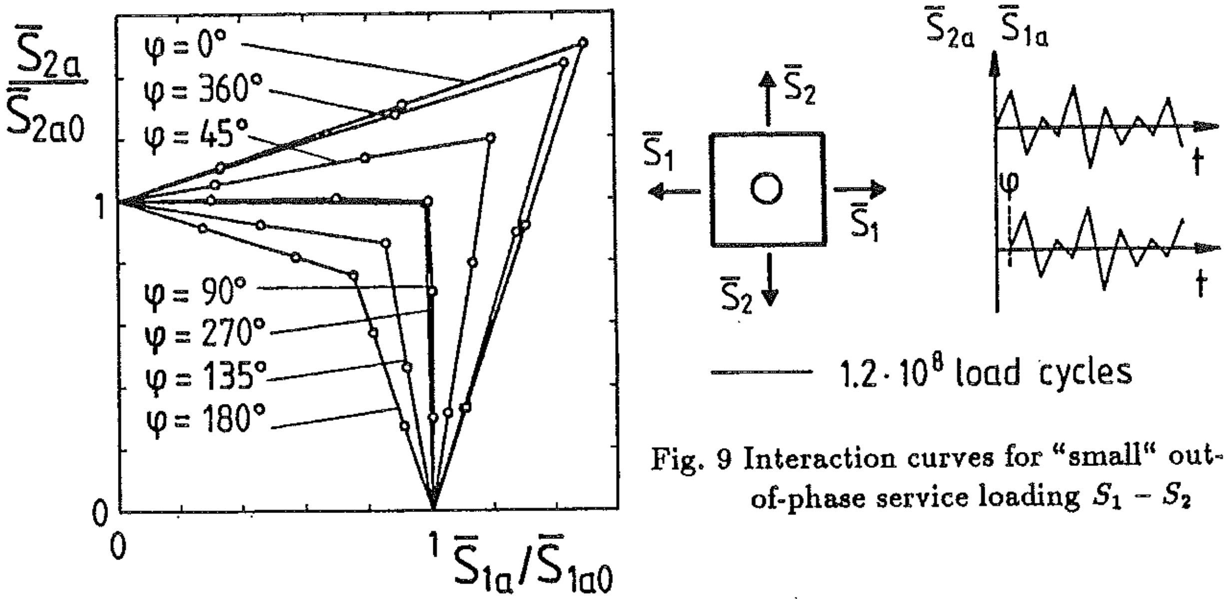


Fig. 9 Interaction curves for "small" out-of-phase service loading $S_1 - S_2$

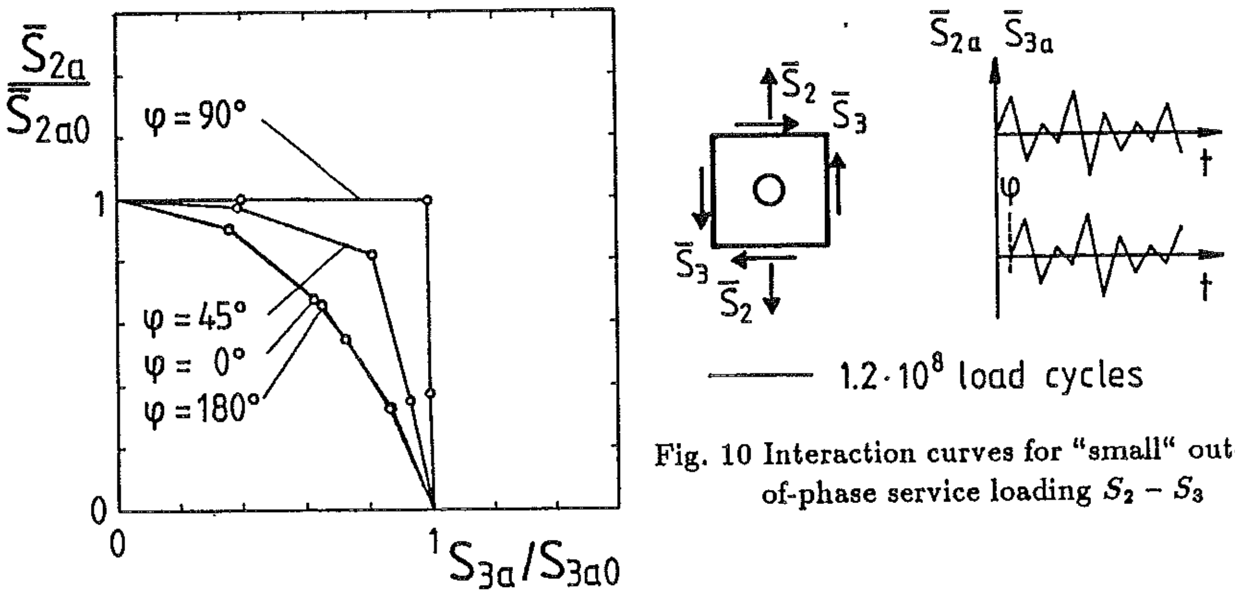


Fig. 10 Interaction curves for "small" out-of-phase service loading $S_2 - S_3$

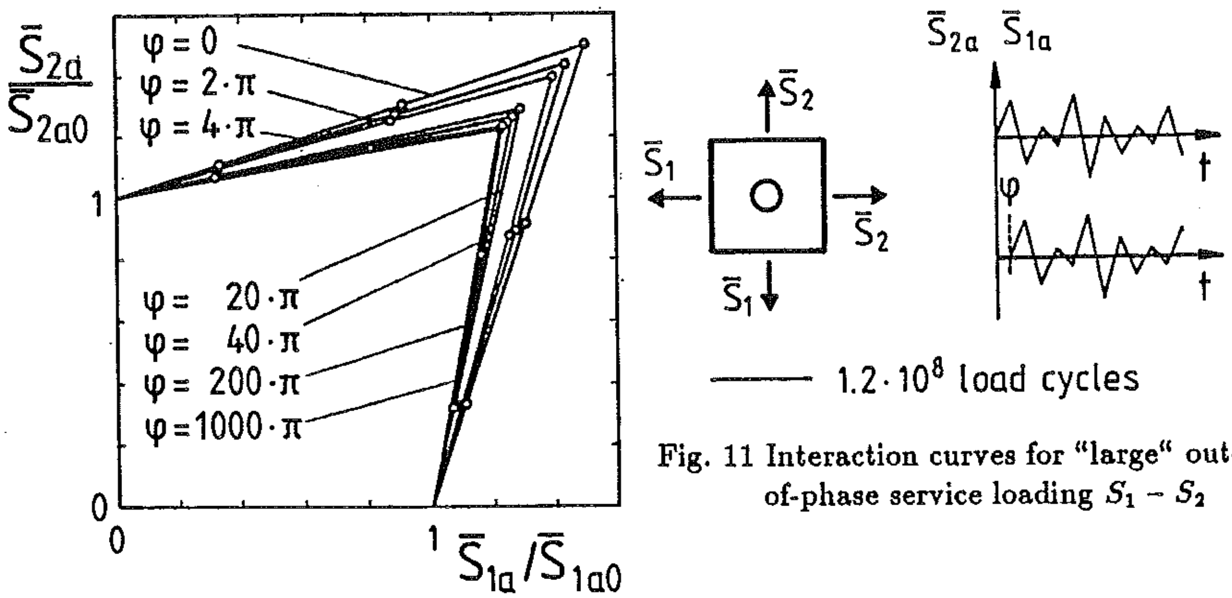


Fig. 11 Interaction curves for "large" out-of-phase service loading $S_1 - S_2$

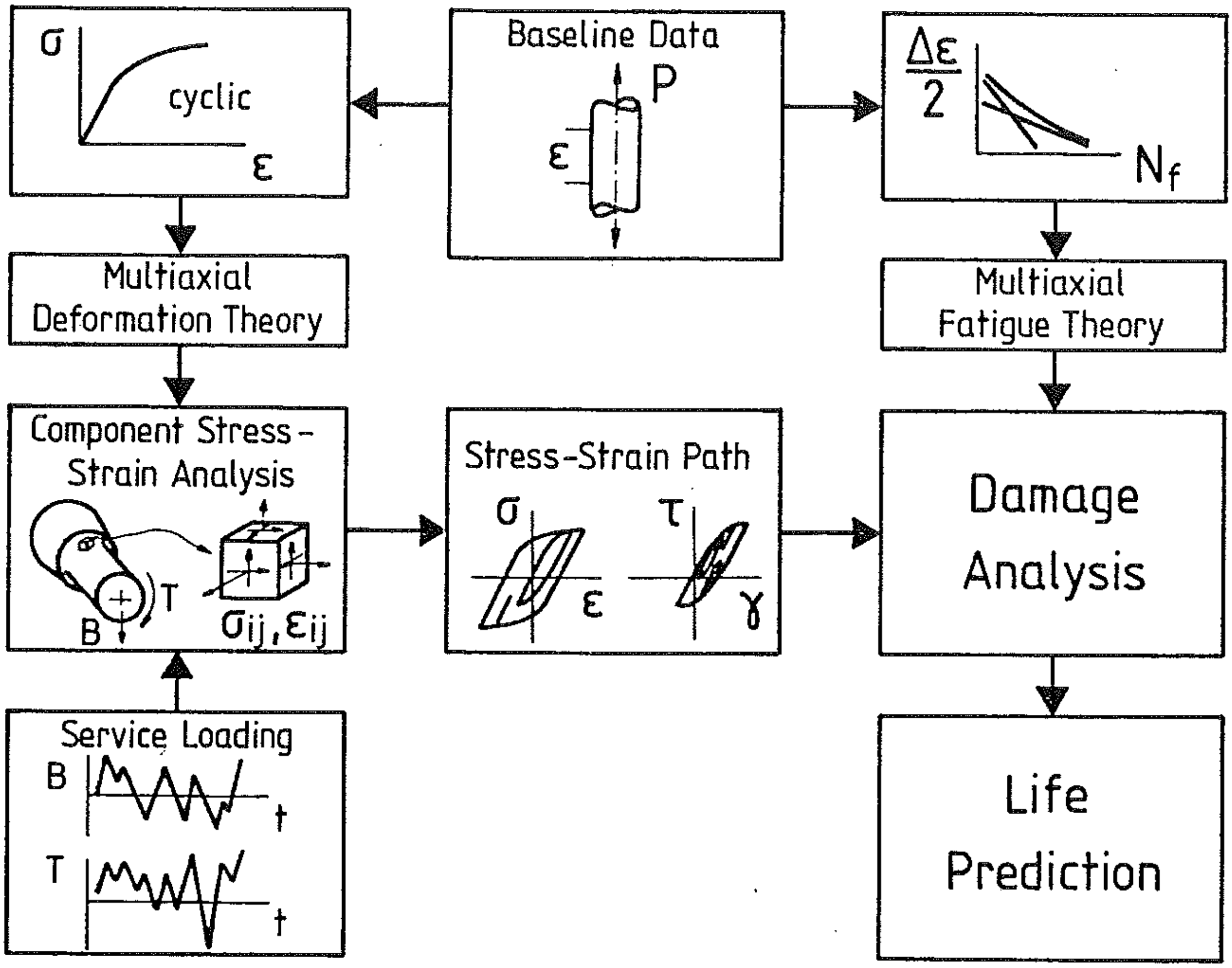


Fig. 12 Local strain approach in multiaxial fatigue

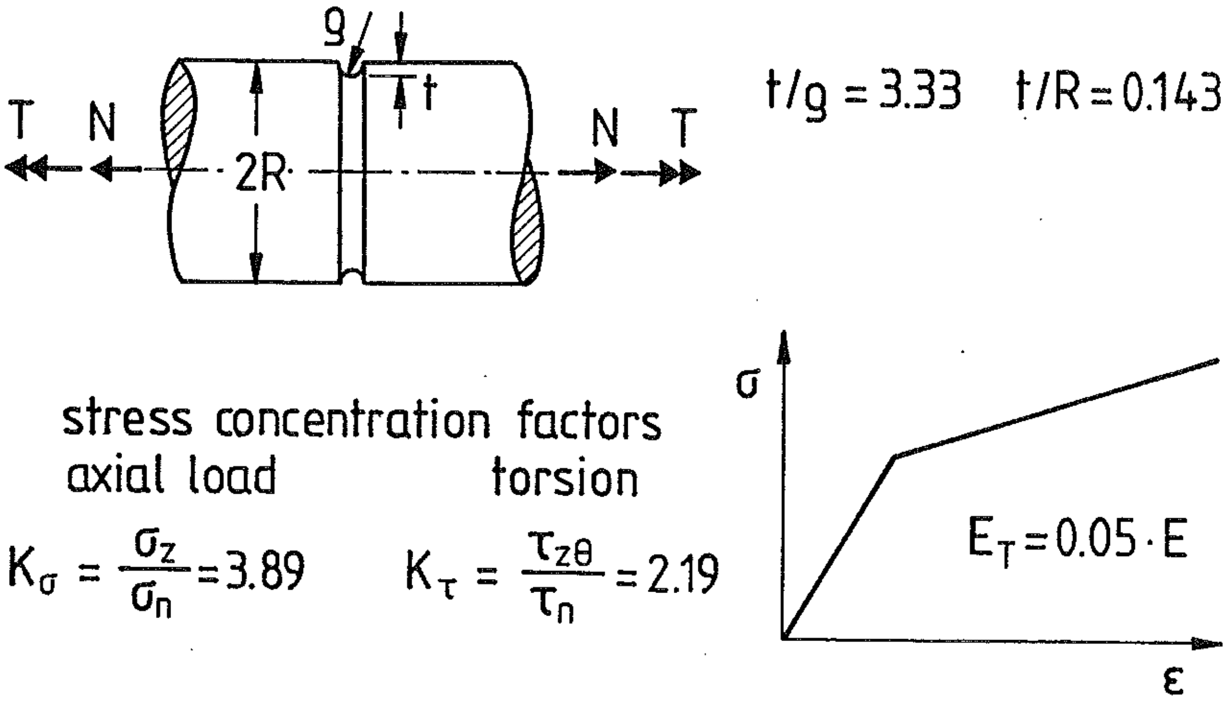


Fig. 13 Structure and σ-ϵ-curve of FE-computations

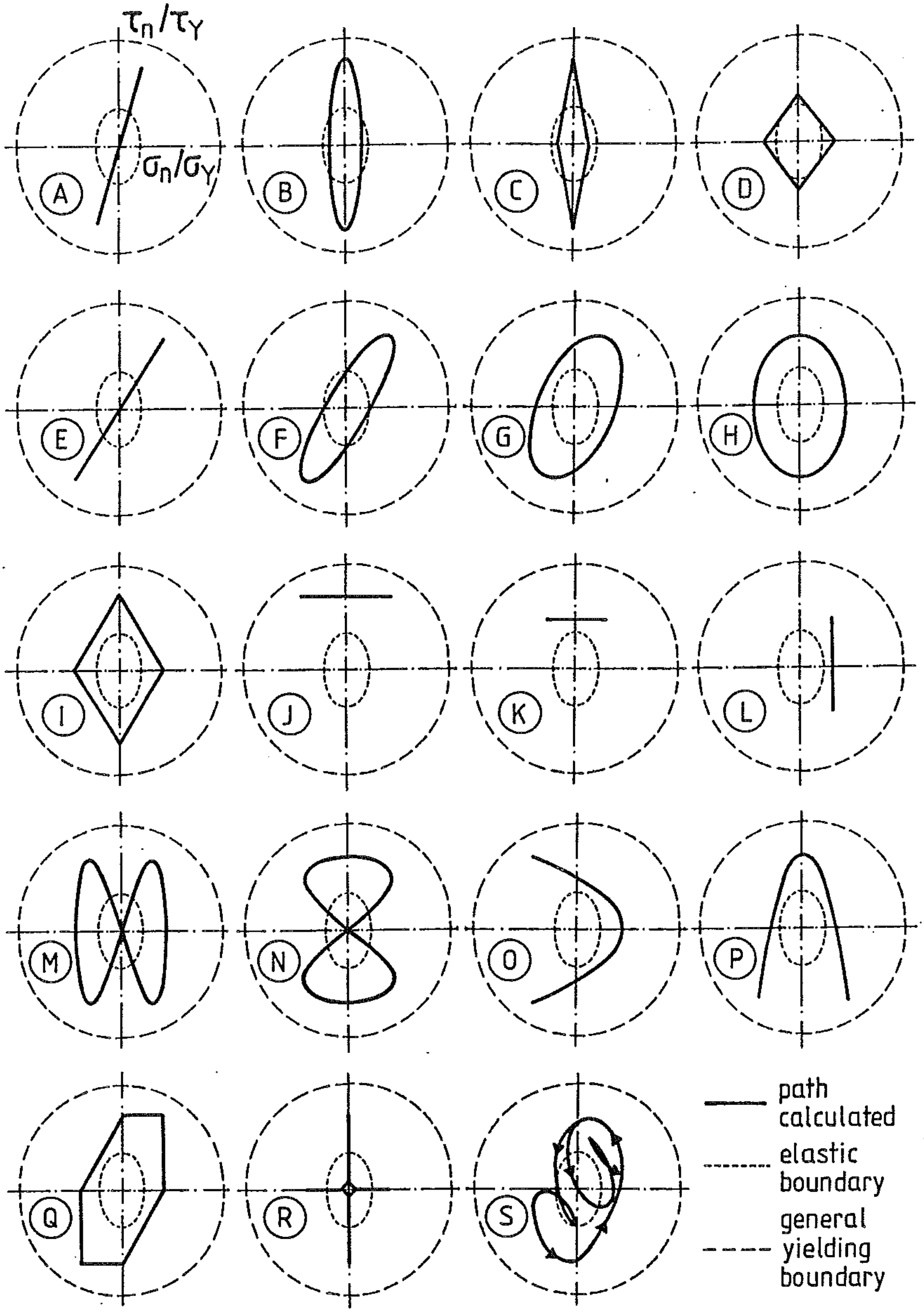


Fig. 14 Survey of the loading histories investigated

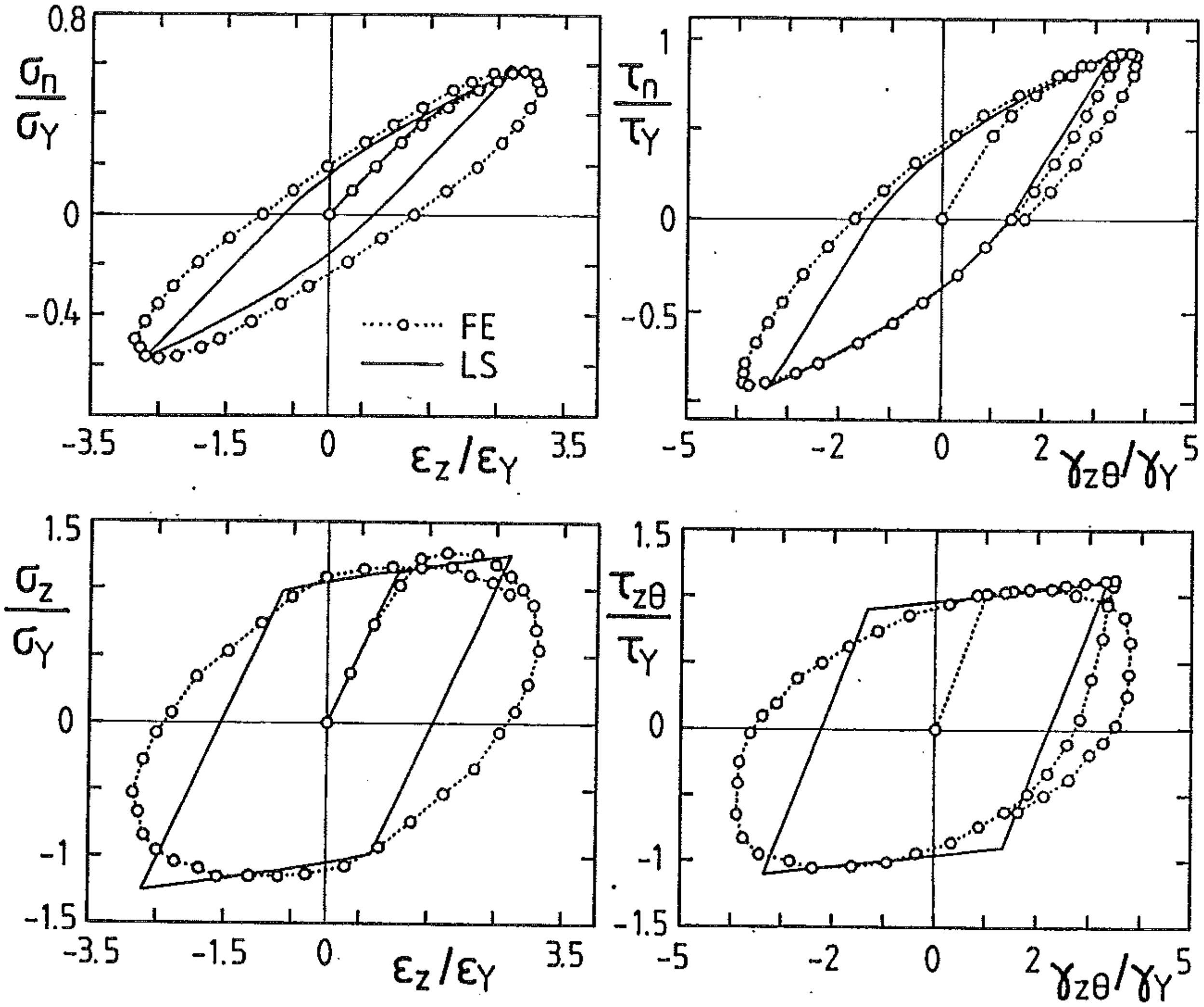


Fig. 15 Axial and torsional load – strain and stress – strain hysteresis loops for 90° out-of-phase loading – comparison of FEM and load separation method (LS)

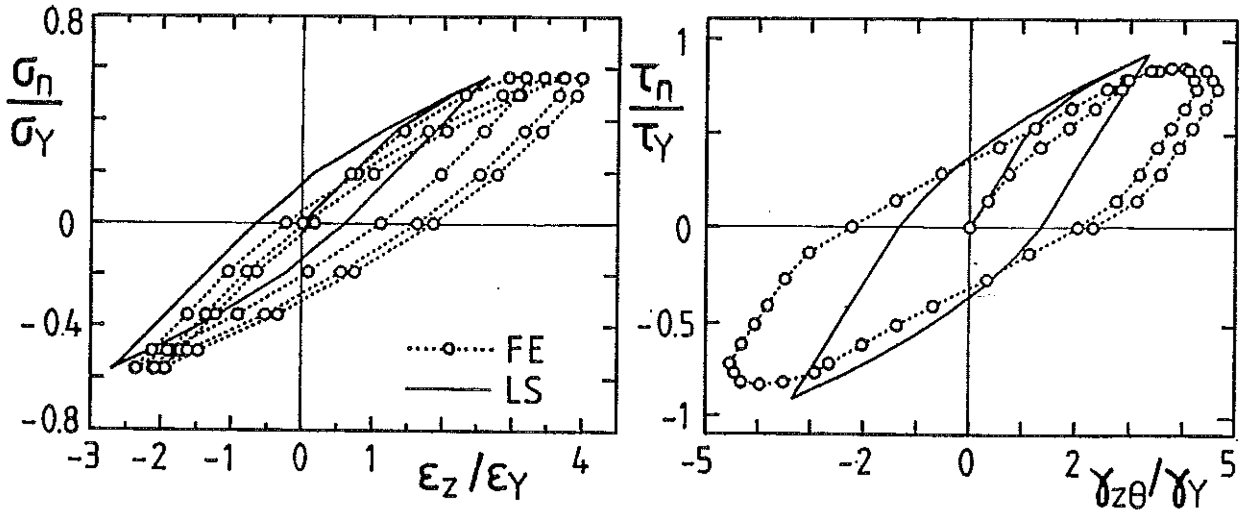


Fig. 16 Axial and torsional load – strain hysteresis loops for different load frequencies – comparison of FEM and load separation method (LS)

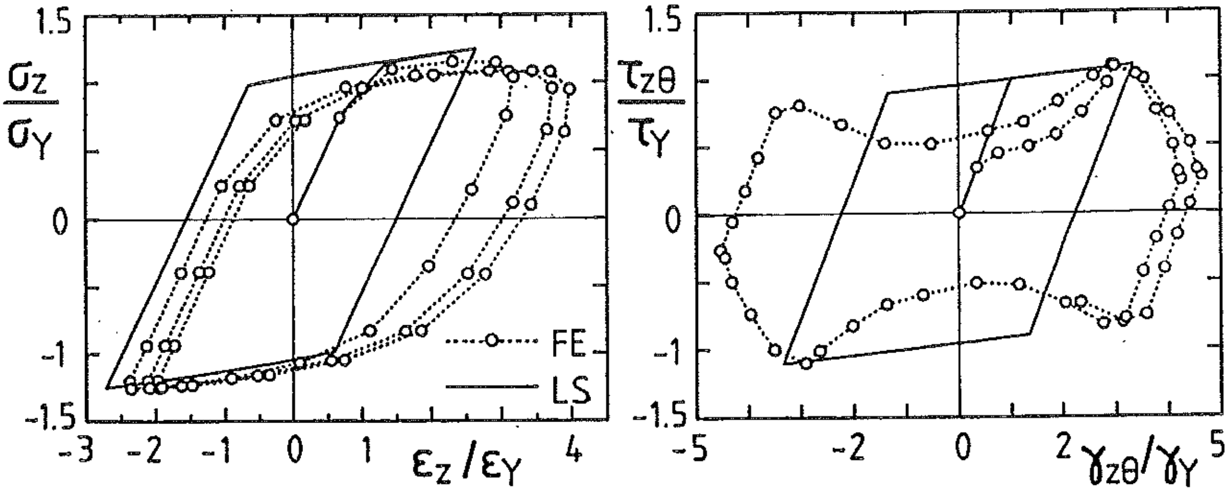


Fig. 17 Axial and torsional stress – strain hysteresis loops for different load frequencies – comparison of FEM and load separation method (LS)

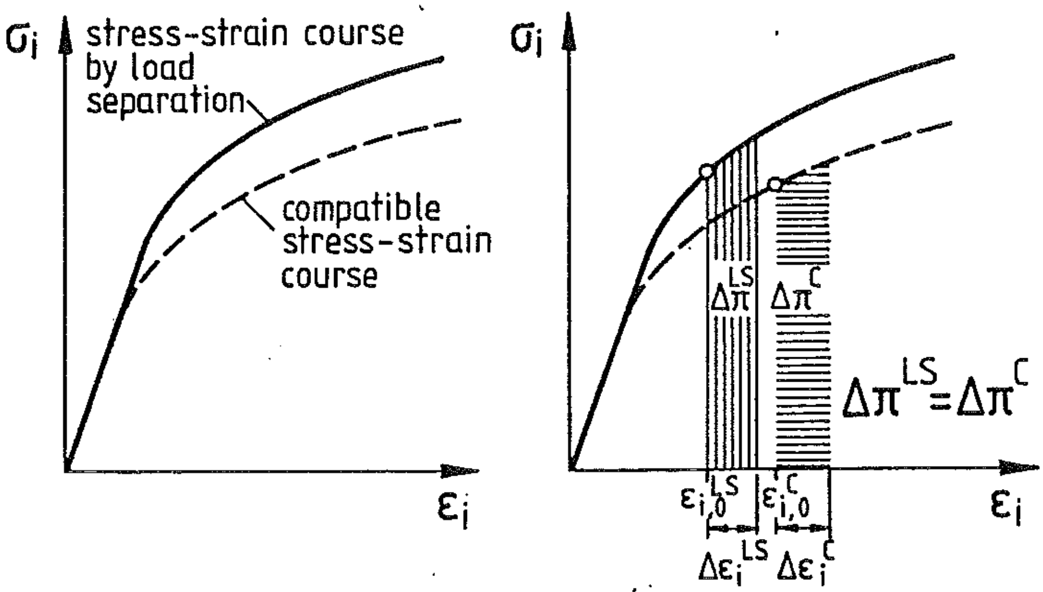


Fig. 18 Load separation solution without and with compatibility procedure

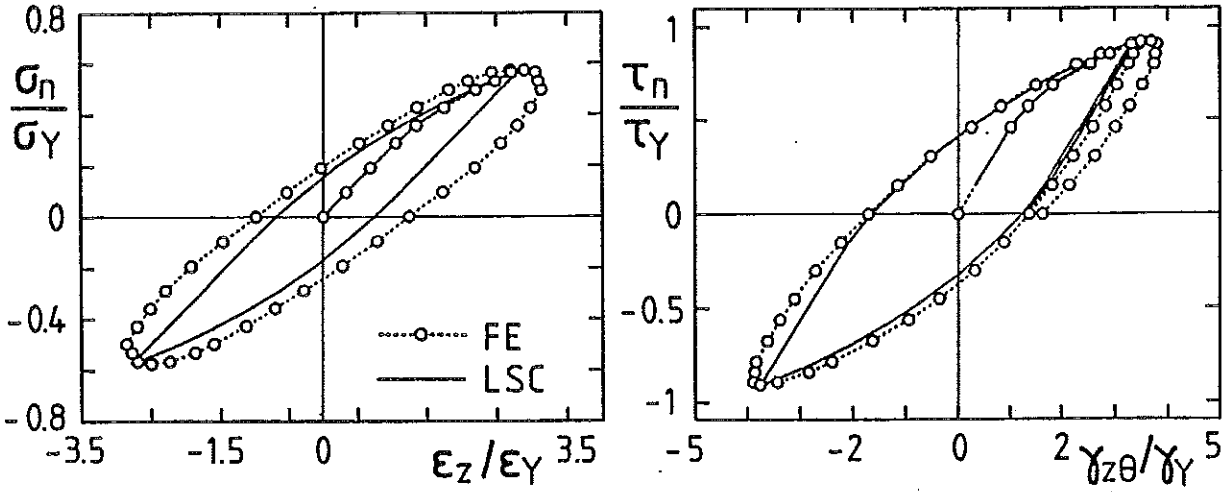


Fig. 19 Axial and torsional load – strain hysteresis loops for 90° out-of-phase loading – comparison of FEM and load separation with compatibility procedure (LSC)

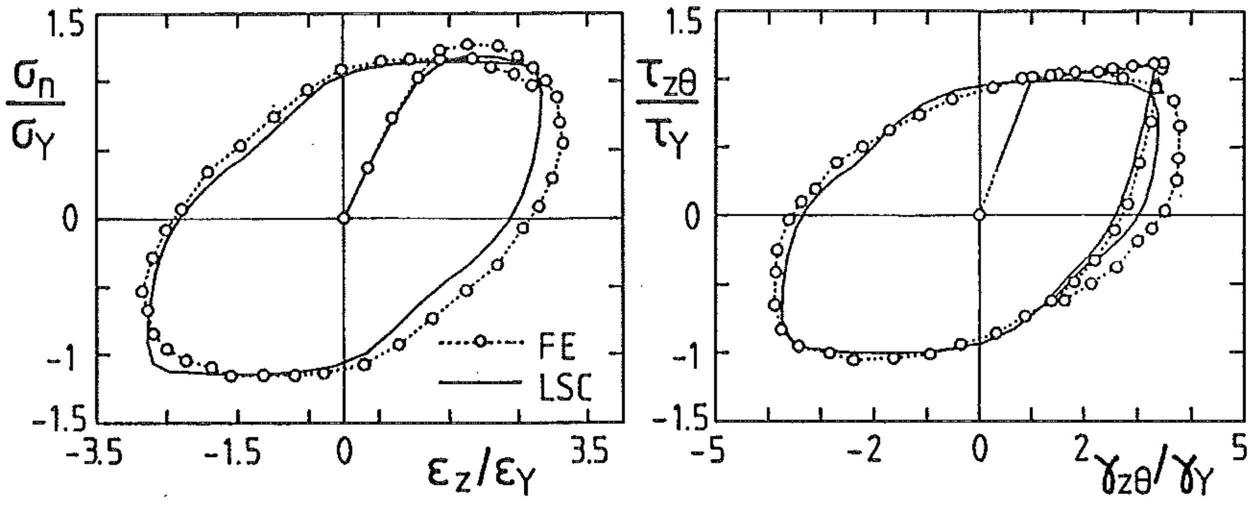


Fig. 20 Axial and torsional stress – strain hysteresis loops for 90° out-of-phase loading comparison of FEM load separation method with compatibility procedure (LSC)

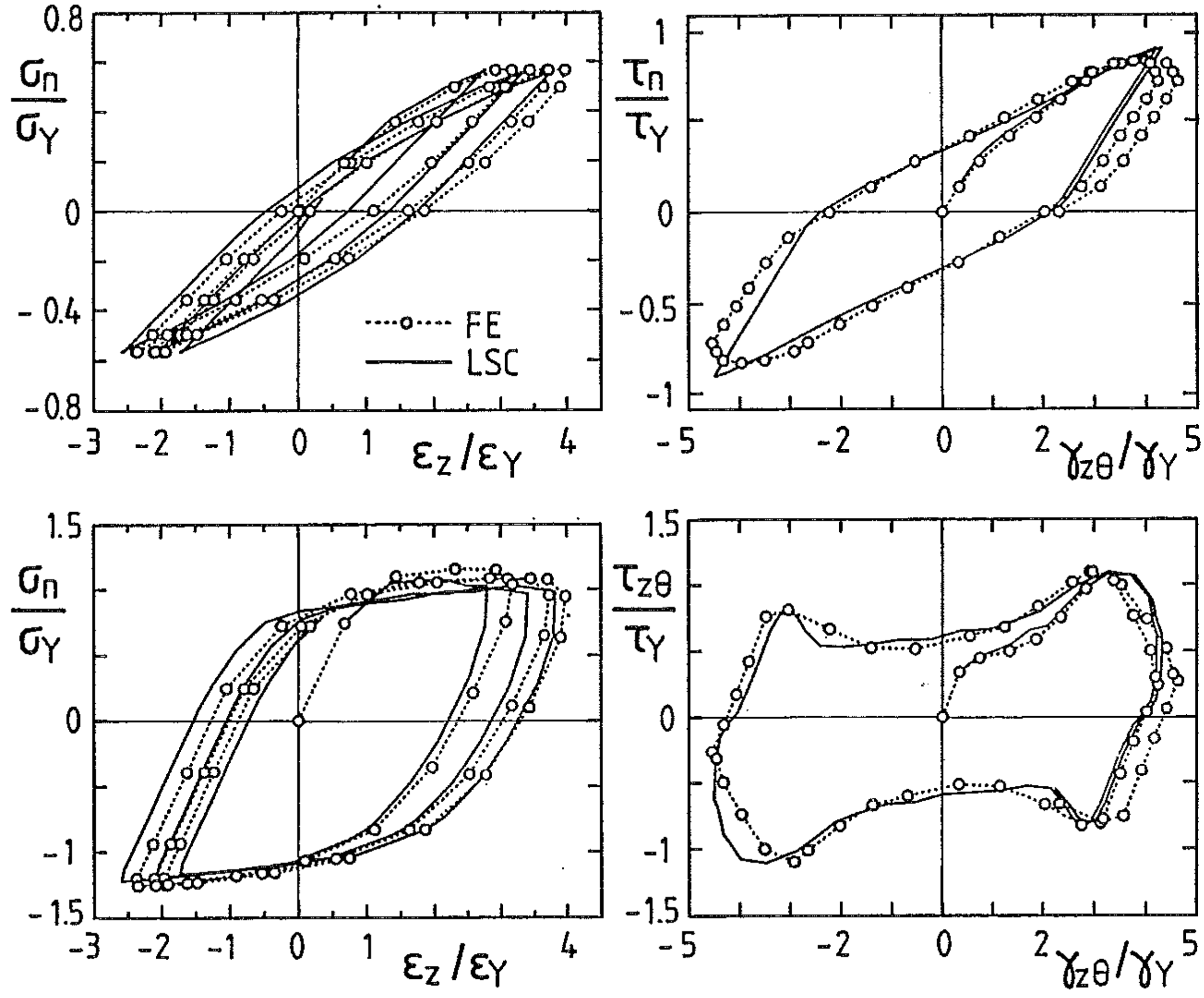


Fig. 21 Axial and torsional stress – strain hysteresis loops for different load frequencies – comparison of FEM and load separation with compatibility procedure (LSC)

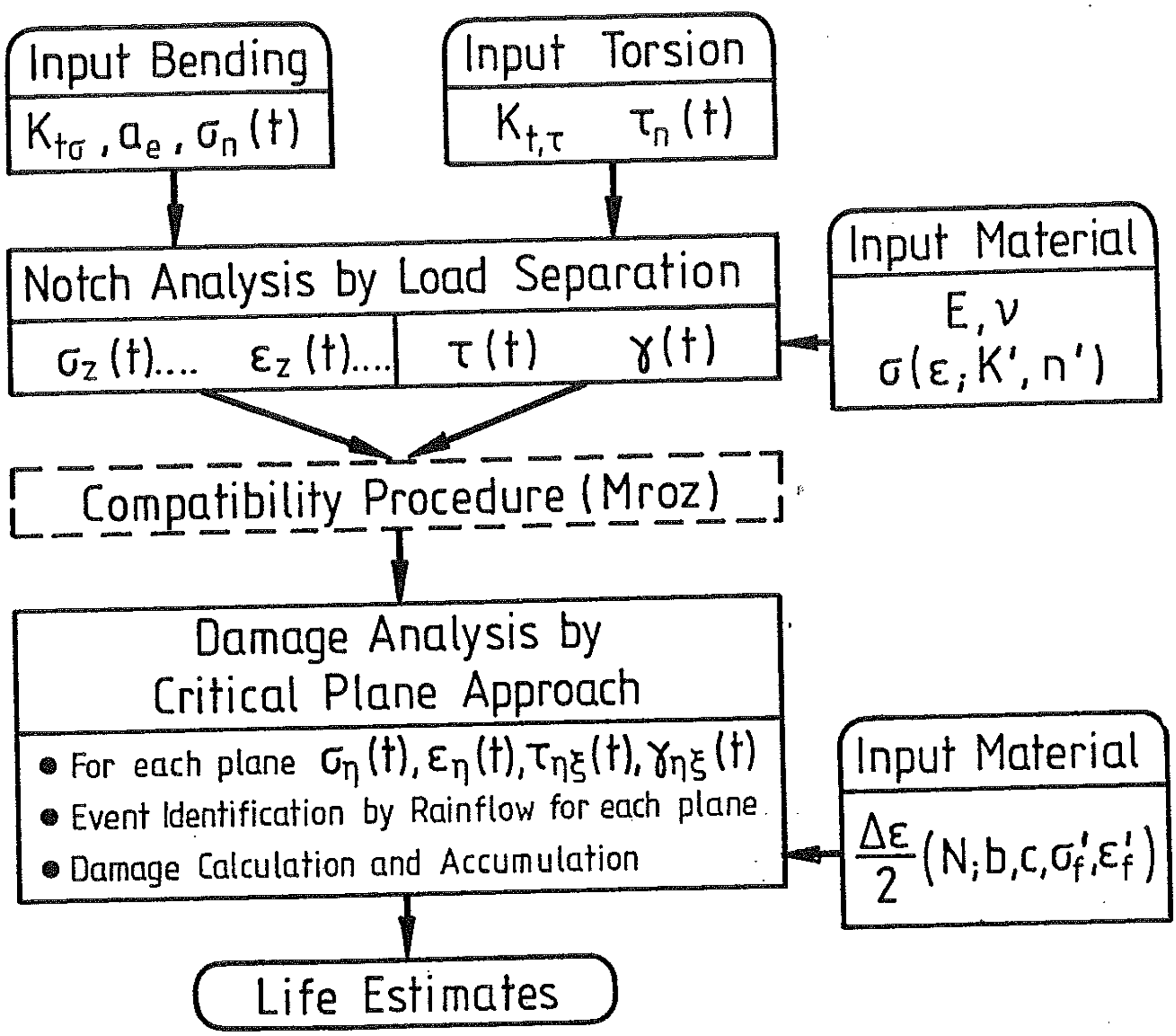
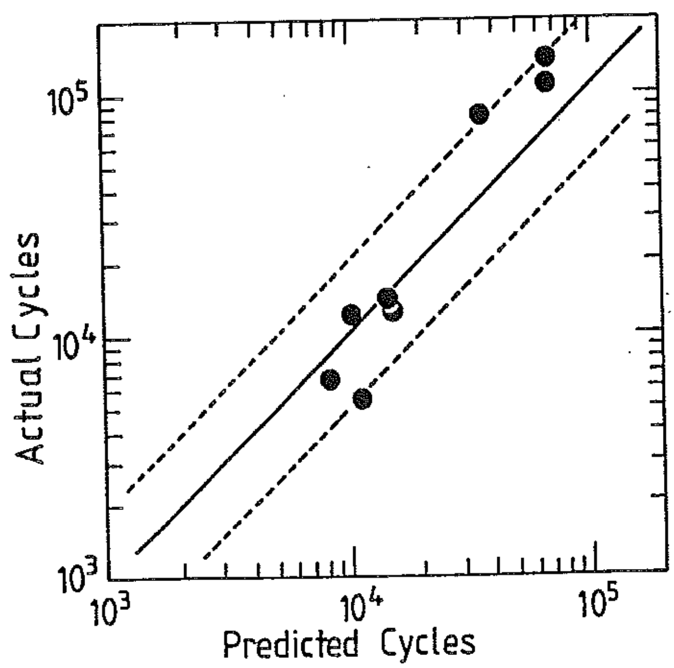


Fig. 22 Fatigue life estimation scheme proposed



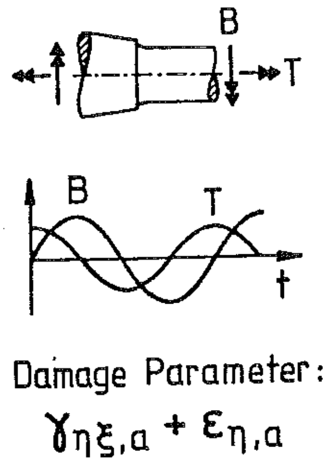


Fig. 23 Fatigue life estimation of the "SAE notched shaft" under constant amplitude 90° out-of-phase loading