

The Role of State of Stress for the Determination of  
Life-Time of Turbine-Components

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1. Introduction

Life time prediction of components is usually performed on the basis of stress/strain calculations and material data. In case of components acting at elevated temperatures: yield stress, stress rupture, creep, low cycle fatigue and fracture toughness are the important material properties. In almost all cases those data are gathered under well defined and uniaxial loading conditions whereas the component itself is subjected to rather complex and mainly multiaxial loading conditions (figure 1). Besides the state of stress elastic plastic material behaviour is important especially at stress concentrators such as notches. For practical life time evaluation of rotors the loads are usually determined by two dimensional elastic finite element calculations. Start-up, steady state and shut-down modes of the machine are normally considered. A typical stress distribution in a rotor is shown in figure 2.

Elastic-plastic considerations for notches are performed with the NEUBER-rule or with similar approaches. Creep fatigue interactions are handled with the linear - damage rule whereby lower bound values of the scatterbands of the respective material data are used. Correlation of uniaxial data with the multiaxial loading conditions occurring in the component are done by applying some equivalent stress criteria. It is obvious that for such a procedure only a limited accuracy for the calculated life can be achieved.

One point of uncertainty concerns the application of laboratory data to the multiaxial loading condition in the component. This will be elaborated here more in detail.

## 2. Equivalent Stress Concepts

### 2.1. Creep

In this section we confine ourselves only to the treatment of elastic loading below the yield strength and no occurrence of notch effects. Such problems will be touched upon in the next sections. It is well accepted today that the v.-MISES stress can be satisfactorily used for the description of secondary creep [1]. A few stress rupture results on a typical CrMoV-steel shown in figure 3 indicate that at least for relatively short testing times the v.-MISES criterion can also be used to handle stress rupture.

For long term creep the first principal stress is sometimes treated as the responsible rupture stress. Other concepts with combinations of the v.-MISES, first principal stress and hydrostatic stress also exist [2]. From the technical point of view the pure v.-MISES approach provides an acceptable solution. Uncertainties are however to be covered by taking some safety factor into consideration, especially for notches as being described later.

### 2.2. Low-Cycle-Fatigue (LCF)

In the case of LCF the situation is much more contradictory. Many proposals for equivalent quantities exist as listed e.g. in [3; 4].

From a technical point of view the

- v.-MISES stress criterion
- modified v.-MISES stress criterion
- TRESCA criterion
- First Principal Stress criterion (RANKINE)
- Gamma-plane approach

can be considered to be of relevance.

Experimental evidence exists that the v.-MISES stress can correlate LCF data [4]. For life consideration the v.-MISES stress:

$$\sigma_{v.-MISES} = \frac{\sqrt{2}}{2} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} \quad (1)$$

( $\sigma_1, \sigma_2, \sigma_3$  = principal stresses) has often to be converted into an equivalent strain:

$$\epsilon_{e,q} = \frac{1}{\sqrt{2(1 + \mu)}} \sqrt{[(\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2 + (\epsilon_3 - \epsilon_1)^2]} \quad (2)$$

For ductile materials the TRESKA-criterion (crit. max. shear stress):

$$\tau_{max} = 0.5 (\sigma_1 - \sigma_3) \quad (3)$$

indicates results which are comparable to the v.-MISES concept, but the v.-MISES criterion written as  $\sigma, \epsilon$  or  $\epsilon$  is more appealing from a continuum, calculational and experimental point of view.

However for brittle materials the yield theories are unsatisfactory and attention was turned to other approaches of strength, for instance RANKINE's theory of the principal stress

$$\sigma_1 = \text{constant.} \quad (4)$$

Equation (4) was found to be more suitable for cast-materials. Due to the nature of this criterion it is occasionally applied to blade design. Besides that the first principal stress is the predominant driving force for cracks and has therefore to be employed to fracture mechanical considerations.

Another theory of relevance is the  $\Gamma$ -plane approach introduced by MILLER et.al. [6;7]. Originated from the MOHR's circle of strain, the coordinates of the highest point of the largest MOHR circle represent two strains, which may be derived in terms of principal strains (see figure 4).

Finally it should be stressed, that one major assumption for the validity of the above mentioned approaches is the occurrence of in-phase loading which is fairly well fulfilled for rotors (figure 2).

### 3. Life Time Evaluation (The v.-MISES Approach)

In this section we briefly introduce the method of life-time evaluation currently employed. The start-up and shut-down modes are responsible for the exhaustion of fatigue-life while the steady-state load corresponds with the creep deformation.

Fatigue at notches is most interesting and will therefore be considered here in detail. The main problem is to convert elastic stresses into a situation where cyclic plasticity occurs and where cyclic strain ranges have to be determined.

For the evaluation of plastic strain the NEUBER-rule or similar approaches are used. The cyclic relaxation of the mean stress as well as the occurrence of residual stresses have to be taken into consideration. All different methods assume the validity of the MASING-rule as cyclic material behaviour. This means that the stabilized hysteresis loop can be derived from the cyclic stress/strain curve by scaling the latter with a factor of two. To account for mean stress a procedure as proposed in [5] can be used (figure 5a.). Since experimental evidence exists that the hysteresis loop turns out to be symmetric with respect to the zero mean stress after a few cycles we prefer a construction of the stresses and strains as shown in figure 5b.

This construction contains the conversion of the calculated equivalent elastic stress range into the equivalent plastic strain range (figure 5b). Assuming proportional loading the elastic cyclic stress range is calculated on a cycle basis (figure 2). The range  $\Delta\sigma_i^{(e)}$  of each stress component is given by the respective extreme values:

$$\Delta\sigma_i^{(e)} = \sigma_{i,t_1}(\max) - \sigma_{i,t_2}(\min) \quad (5)$$

index  $i = 1, 2, 3$  (principal stresses)

$$\Delta\sigma_{e,1..n} = \frac{1}{\sqrt{2}} \sqrt{[(\Delta\sigma_1^{e1} - \Delta\sigma_2^{e1})^2 + (\Delta\sigma_2^{e1} - \Delta\sigma_3^{e1})^2 + (\Delta\sigma_3^{e1} - \Delta\sigma_1^{e1})^2]} \quad (6)$$

This  $\sigma_{e,1}$  is used for the determination of the local stress/strain ranges  $\Delta\sigma$  and  $\Delta\epsilon$  with the NEUBER-rule.

Other methods dealing with the construction of the local strain ranges are available in literature e.g. [8;9]. They

often however need actual strains to be calculated which is not possible with a pure elastic calculation method. As a lack of constitutive equations for the cyclic deformation behaviour of the materials, elastic plastic calculations are currently not expected to provide a real advantage.

Our experience has shown a good correlation between the start-up and shut-down stresses, with the effect, that the shut-down mode not always has to be determined. In such cases it is only necessary to multiply the start-up stresses with a factor  $> 1$  in order to calculate the respective elastic stress range.

It is important that the steady state stresses at the notch roots are adjusted according to the cyclic stresses. Since the whole procedure contains many uncertainties an extra safety factor has to be build in.

Until now we were mainly concerned with pure creep and LCF effects. Life time evaluation however means combination of both of them. Although several sophisticated models for creep fatigue exist for technical applications the linear damage rule is mainly used. The reason for that is that the transfer of those models to actual components behaviour is even more complex and uncertain than a simple linear damage rule:

$$\sum_i \frac{N_i}{N_f^{(i)}} + \sum_k \frac{t_k}{t_r^{(k)}} = D \quad (7)$$

- $N_i$  : actual number of cycles under cyclic load type  $i$
- $N_f^{(i)}$  : Number of cycles to crack initiation under cyclic load type  $i$
- $t_k$  : actual creep time under  $k^{\text{th}}$  stress level
- $t_r^{(k)}$  : stress rupture life under  $k^{\text{th}}$  stress level
- $D$  : Number indicating failure under creep and fatigue

The pure creep part is given by the effective steady state operating hours and the fatigue part is described by the start-up and shut-down-modes as already mentioned. Since the start-up and shut-down cycles might take a relatively long time compared to typical fatigue test frequencies, a creep part can be added to the transient part. We shall not discuss this matter furthermore in this paper.

One advantage of the above outlined v.-MISES approach is that the same equivalent stress is used for calculating the fatigue part as well as the creep part. Uncertainties still remain as for instance the concept itself or the treatment

of the creep influence when cyclic stress relaxation occurs. Another point of further investigation is the damage term  $D$ . Quite good correlations were obtained under uniaxial laboratory conditions for CrMoV rotor steels with  $D = 0.75$  [10]. Unfortunately there are currently no data available to proof this for multiaxial loading conditions.

#### 4. Other Concepts

Until now we confined our considerations mainly to the v.-MISES criterion. A similar procedure could be performed using e.g. the octahedral shear strains for fatigue and the v.-MISES stress for the creep part. Basically one would expect rather similar results but the combination of these two methods does not lead to a single equivalent stress for a consistent treatment of creep and fatigue.

The  $\Gamma$ -plane approach is a relatively new concept which rather starts from a fatigue damage point of view than from a yielding criterion. Figure 4 shows the  $\Gamma$ -plane with lines of constant life for different equivalent stresses and strains. It can be seen that the deviation of the various approaches varies within the scatterband of the uniaxial data (with the exception of RANKINE's principal stress criterion). Applying the  $\Gamma$ -plane approach into practice needs similar ways of converting the calculated elastic stresses into equivalent strains and combining creep and fatigue for the v.-MISES concept. In addition to that there is a lack of data to validate the isolife-lines in the  $\Gamma$ -plane. Combining all these facts it turns out that a multiaxial method such as the  $\Gamma$ -plane approach is currently not satisfying technical life time calculations of gas turbine components such as a rotor. It shall however not be excluded that such approaches do have potential for future application.

#### 5. Conclusions

A technical method for the calculation of life-times of turbine rotors is presented. It shows that the v.-MISES stress provides a good basis for both: creep and fatigue loading. Elastic calculations have to be adjusted to handle plasticity which might occur in a notch-root. A NEUBER-type then leads to a satisfactory result. Creep fatigue interactions are treated with the linear damage rule. More sophisticated methods e.g. the  $\Gamma$ -plane approach, describing multiaxial cyclic loading, are in principle attractive, there is however still work necessary in order to apply this method to practical

problems.

6. Literature

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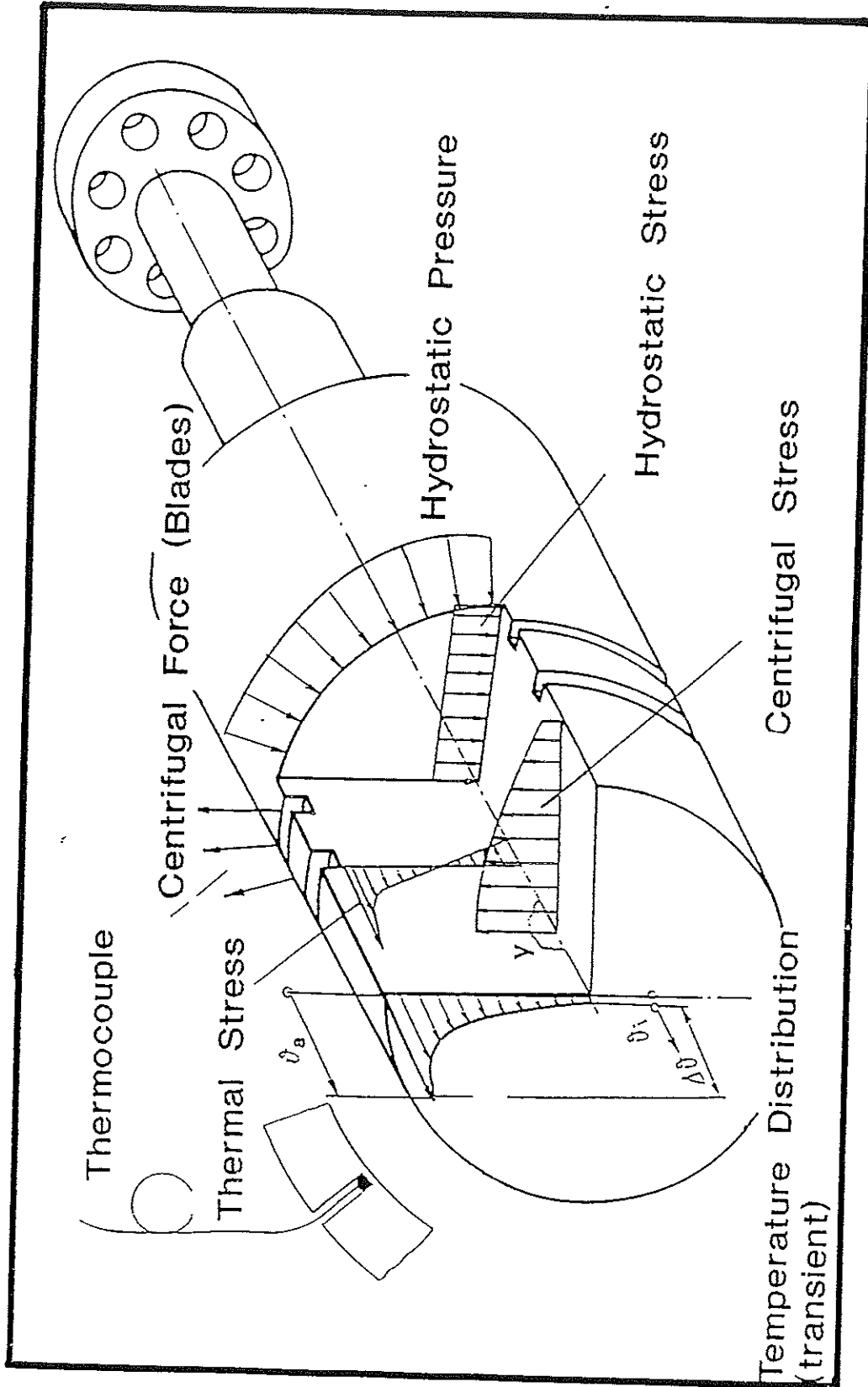


Fig. 1: Typical Loading Conditions of a Gas-Turbine Rotor



Fig. 2: Typical Loading of a GT-Rotor

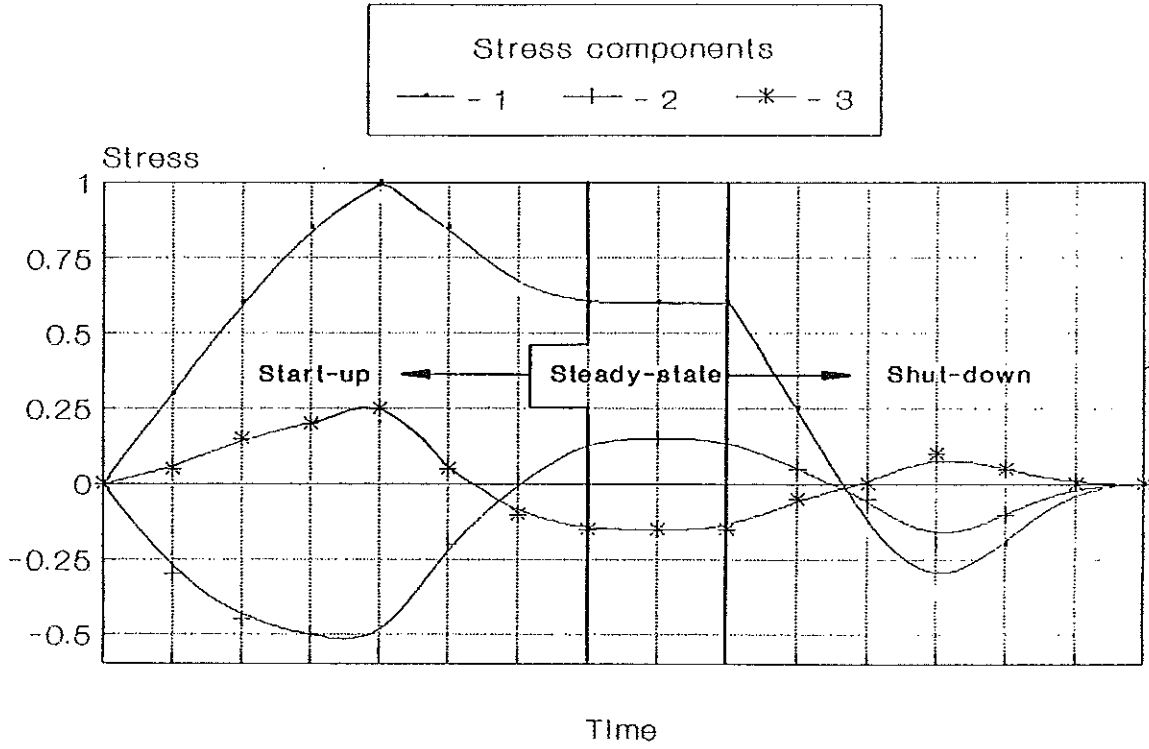


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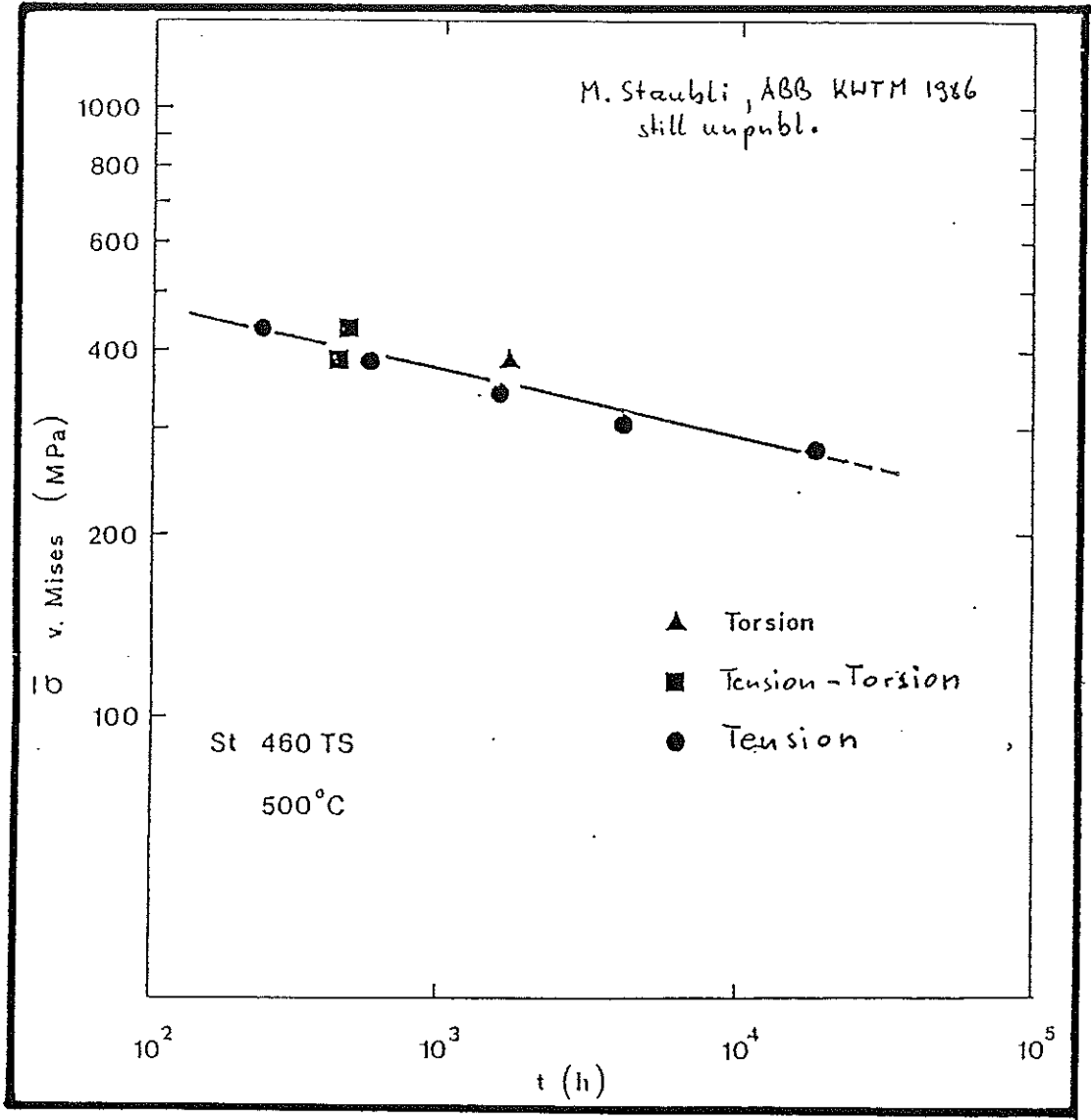


Figure 3: Correlation of multiaxial stress rupture data with the v. Mises stress.

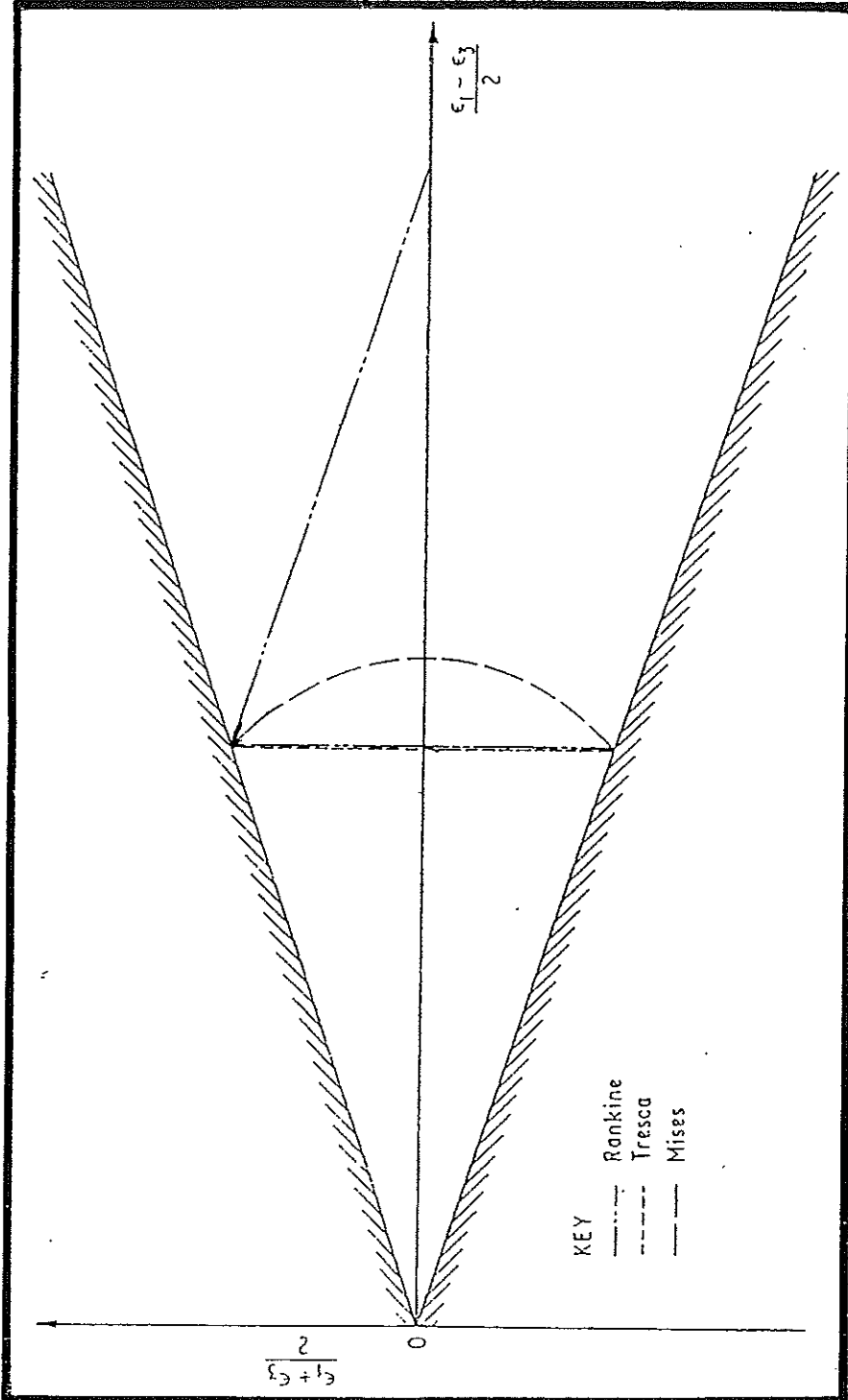


Fig.4 Gamma-plane with classical failure criteria [6]

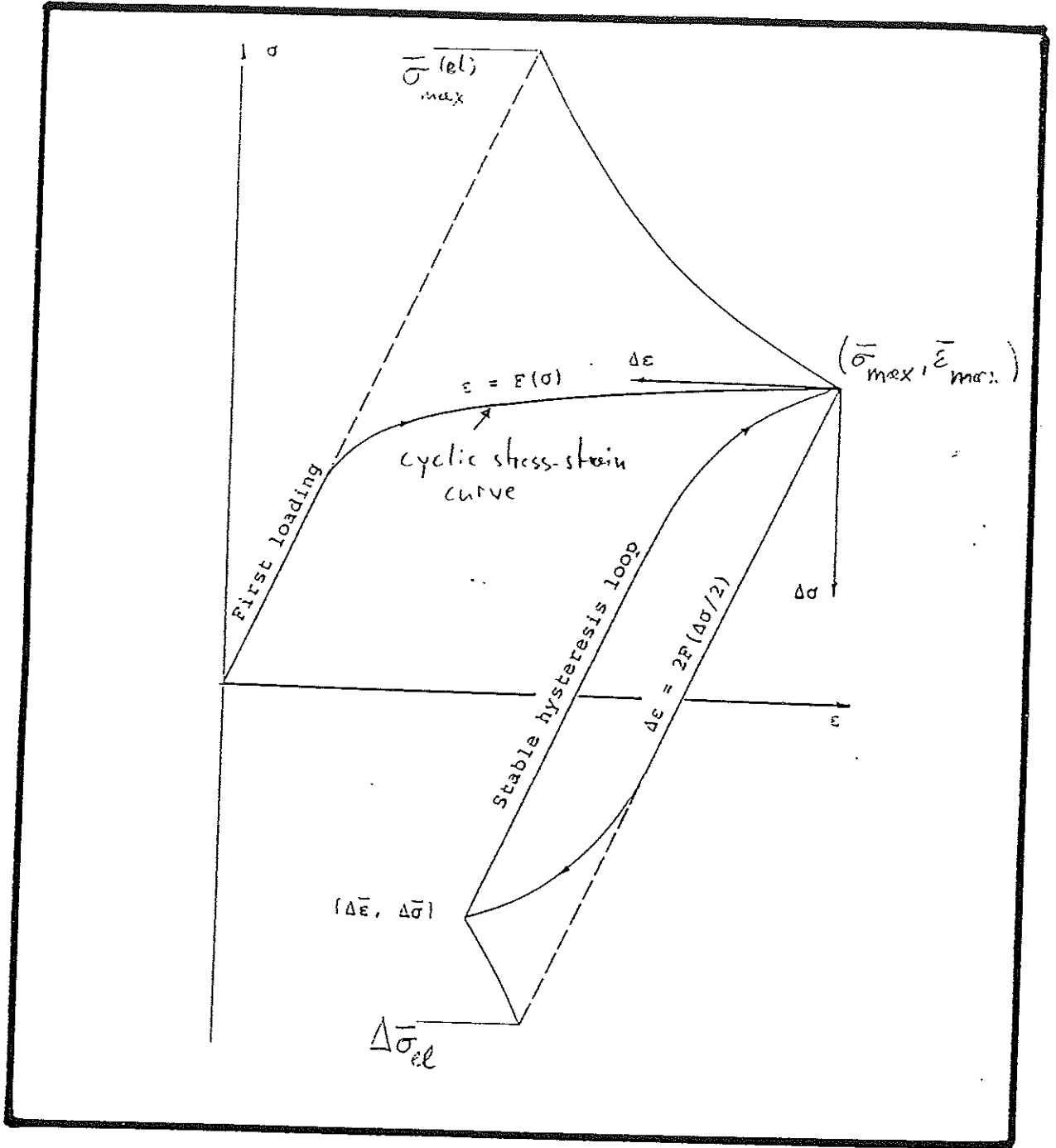


Figure 5a: Determination of the equivalent strain range with the Neuber-rule taking mean stress effects into consideration. The hysteresis loop is obtained by doubling the cyclic stress-strain curve.

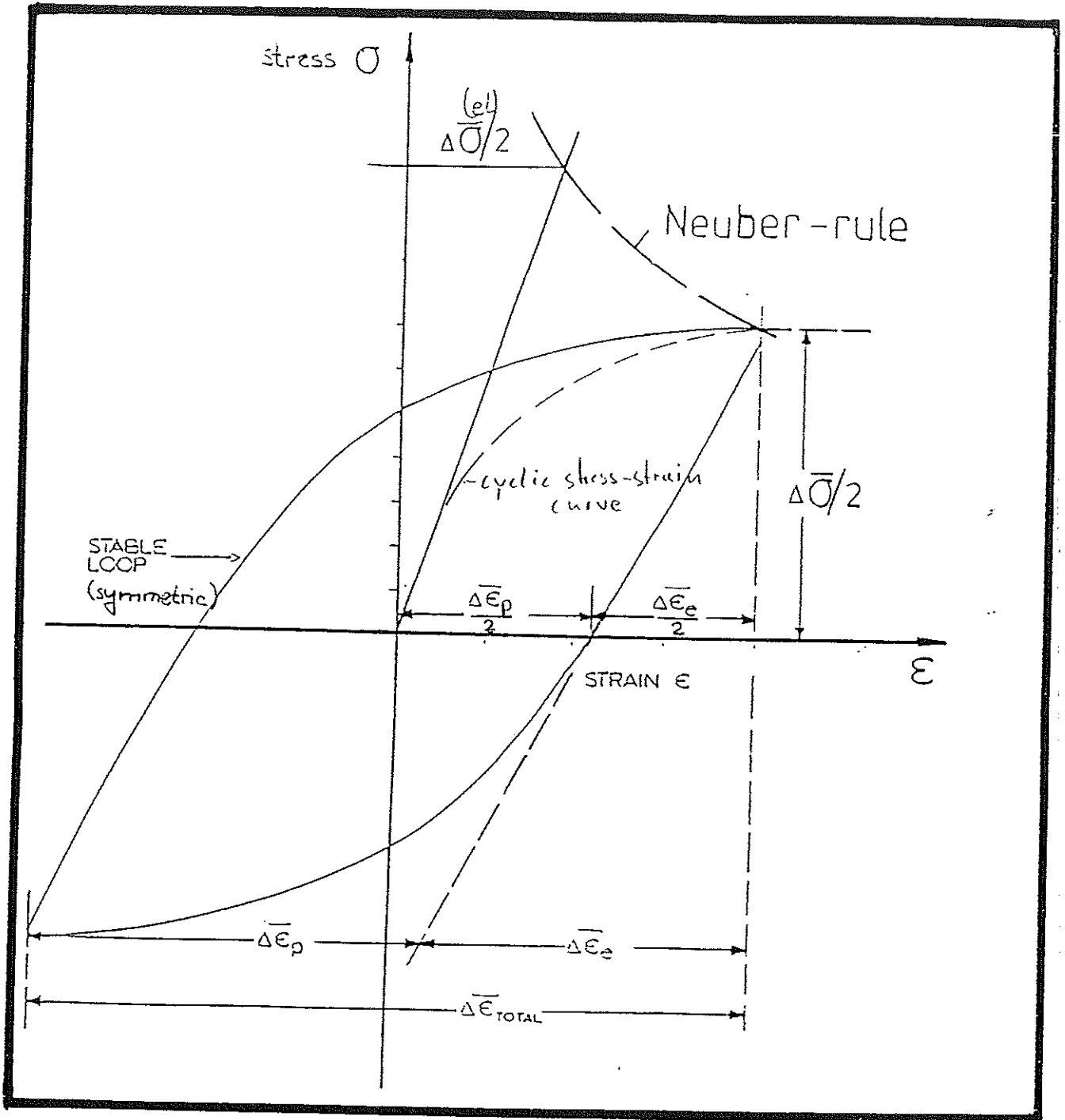


Figure 5b: Determination of equivalent strain ranges with the Neuber-rule assuming a symmetric hysteresis loop being present after a few hundred cycles. The hysteresis loop is obtained by doubling the cyclic stress-strain curve. (after [5]).