The Relationship between the Fatigue Strength Coefficient and the Three Dimensional State of Stress at a Hole

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Abstract

In this paper the main causes of differences between the fatigue strength coefficient K_f and the theoretical stress concentration coefficient K_t are discussed, and the effects of three dimensional state of stress and the stress gradient at a hole on K_f are also investigated using a micromechanics model. A formula to predict K_f is obtained based on the model of fatigue nucleation postulated by Wood. This formula indicates that under high cycle fatigue conditions the stress gradient has almost no effect on K_f, but the three dimensional stress and the smooth specimen size determine the difference between K_f and K_t. From this formula, it can be deduced that K_f will increase as the cycles to failure increase for cyclic hardening materials, on the contrary, K_f will decrease as the cycles to failure increase for cyclic softening materials. This deducation has been checked by experiments on cyclic hardening materials Ly12-cs and Ly12B-az Al alloys. An example of predicting K_f for a plane with a hole is also given in this paper.

Introduction

The mechanism of fatigue nucleation has been investigated widely and the results of investigation indicate that fatigue nucleation is caused by the plastic deformation accumulated with

the increase of fatigue cycles[1,2]. There are many models[1-4] to describe this mechanism and among the models, the extrusion and intrusion model postulated by Wood[3] is popularly accepted now.

There are two sourses of local plasticity. One is the external, for example, the stress concentration caused by notches, etc.. The other is the internal, that is, the scatter of the material structures.

In gerenal, the fatigue strength coefficient K_f is substituted for theoretical stress concentration coefficient K_t in the local strain method of predicting the fatigue life, to take the three dimensional stress and stress gradient at a notch into consideration. K_f can be defined as follows,

$$K_f = \frac{\text{the fatigue strength of smooth specimens at one fatigue life}}{\text{the fatigue strength of notched specimens at the same fatigue life}} \qquad (1)$$

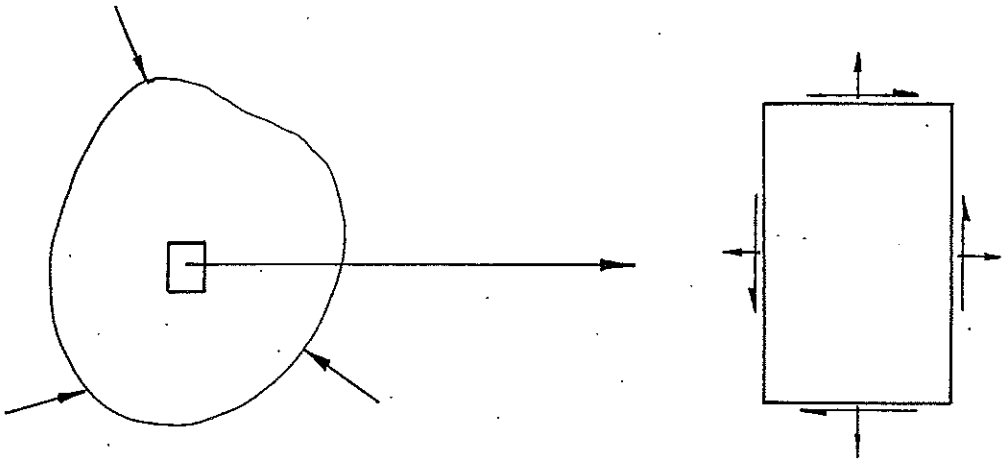
Obviously K_f is a function of fatigue life. There are many formulae to estimate K_f, and among which the Neuber's and Perterson's are well-known and widely employed today[5], but all these estimations didn't consider the effect of the fatigue life on K_f.

In this paper the main causes of differences between K_f and K_t are discussed. According to the above defination, a formula to estimate K_f is presented and from this formula some important results are deduced and checked. An example of predicting K_f is also given to show that the estimation is not complicated and the methodology has a adaptability.

Computational Micromechanics Model

The computational micromechanics model which was given by one of the authors[6] is one of the micromechanics models, in order to provide a new approach to quantitative analysis of the mechanical behaviour of engineering materials in consideration of microstructures. This model deems that materials have a microstructure, all of the field quantities are random variables or statistic functions, and all features which break the homogenity and continuity of the material structure are called "defects" which play an important role in the behaviour of materials.

Based on the above basic hypotheses, reference /6/ defines and discusses the computational micromechanics model, reviews and studies the fundamental theories adapted to this model, presents the transition function method to obtain the relations between macroscopic and microscopic quantities, and gives the gerenal steps of solving problems. Here only briefly reviews the steps to analyze a mechanical behaviour of metal: (1)the preparation of the microstructure datum. If these datum are not available, some experiments on the microscopic properties of the material should be done. These datum usually include the compositions of the material, the percentage and the distribution of each composition, the main defects of the material, etc., (2) according to the problem to be solved, an interested local domain which is rather smaller than the specimen but much larger than microscopic scale is separated from the specimen in order to reduce the numbers of elements in numerical analysis. The loads acted on the local domain can be easily computated using macromechanics (Fig.1), (3)then based on the experimental datum, the local domain is separated using Monte Carlo Method, (4)the microscopic parameters, such as voids, dislocations, etc., are introduced into the numerical or theoretical analysis. What microscopic parameters are introduced depends on the micromechanism of the mechanical behaviour concerned, which usually have been investigated, (5)summerize the theoretical and numerical results.



(a) specimen (b) local domain

Fig.1 The local domain

Estimation of K_f

1. Crack nucleation model

Under high cycle fatigue, the fatigue crack nucleation can be described by the extrusion and intrusion model postulated by Wood, and the dislocation dipole model has successfully explained the ratcheting phenomena of fatigue[7], Fig.2.

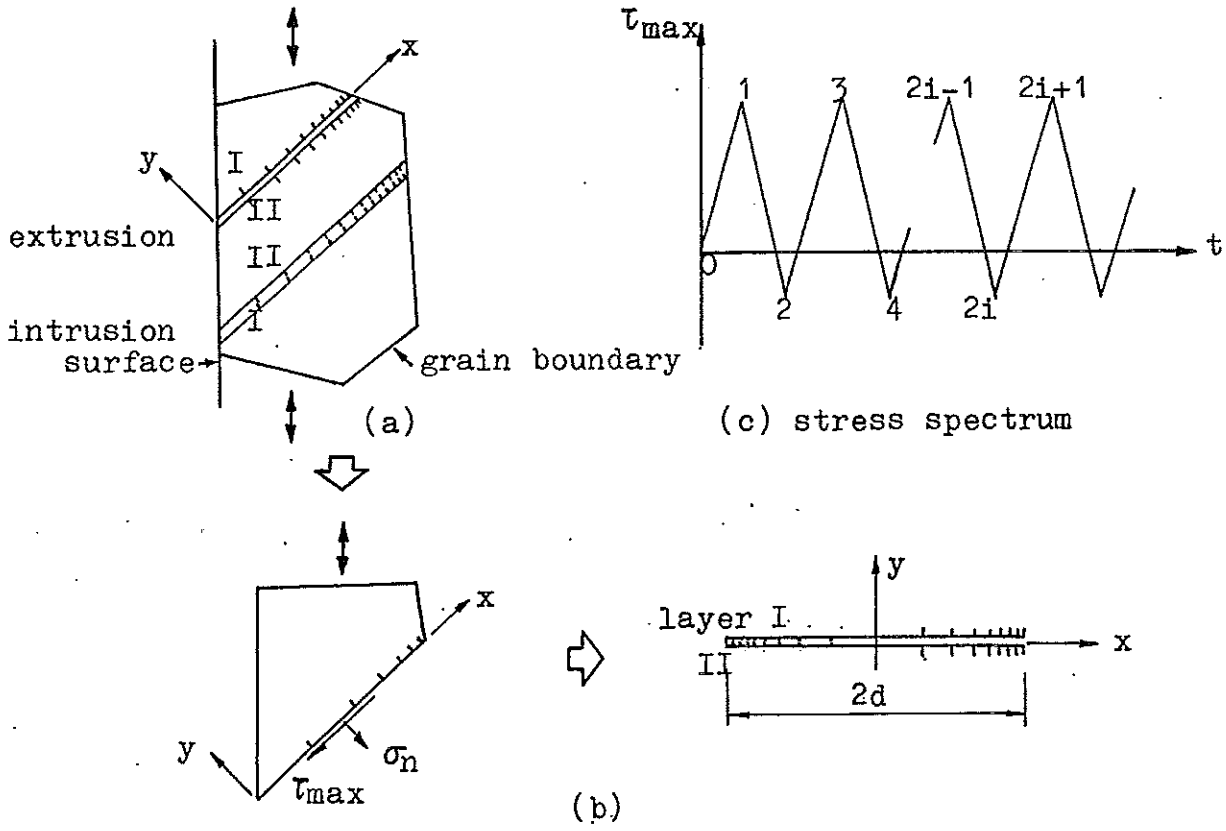


Fig.2 The model of fatigue crack nucleation

A smooth specimen is under a cyclic uniaxial loading. During the first loading, a slip takes place along a line(layer I) inclined by 45° against the axis of the loading. Dislocations are piled up on layer I. For the unloading, a reversal slip takes place on a line(layer II) located close to the layer I. Dislocations with the opposite sign are piled up on the layer II, and in each of the following loadings, more positive dislocations are piled up on layer I, and in each of the following unloadings, more negative dislocations are piled up on layer II, and both the dislocations are piled up against a grain boundary. Since the plas-

tic deformation is irreversible, the dislocation density on layer I is not changed during the unloading and that on layer II is not influenced by the loading. The dislocation density D(x) can be solved using the method given by Leibried[8] for the model as shown in Fig.2(b). The self elastic strain energy of the accumulated dislocations U can also be obtained,

$$D(x) = \pm \frac{(\Delta\tau-2k)}{\pi A} \frac{x}{(d^2-x^2)^{1/2}} i \tag{2}$$

$$U = \frac{(\Delta\tau-2k)^2}{8A} d^2 i \tag{3}$$

where positive sign in equation (2) is taken for layer I, and the negative sign for layer II, k is the frication stress, d is the grain size, $A=G/2\pi(1-\gamma)$ for edge dislocations, $A=G/2\pi$ for screw dislocations, G is the shear modulus and γ is Poisson's ratio. The dislocations monotonically increase as the cycles increase. Physically, it is impossible to have an infinite number of dislocations in the material. So a criterion for the fatigue crack initiation could be a critical value of U,

$$U = W_c d \tag{4}$$

where W_c is the specific fracture energy for a unit area and is a material constant. From equations (3) and (4), it can be obtained

$$8AW_c = (\Delta\tau-2k)^2 nd \tag{5}$$

where n is the cycles to nucleate.

2. Crack initiation at a notch root

Based on the computational micromechanics model as stated above, it is not difficult to find the most dangerous element at a notch root. Usually this element is in three dimensional state of stress as shown in fig.3(a). From this the maximum shear stress and the normal stress can be obtained easily,

$$\begin{aligned} \sigma_n &= \sigma_n(x) \\ \tau &= \tau(x) \end{aligned} \tag{6}$$

$\sigma_n(x)$ and $\tau(x)$ vary with the geometry, external load, microstruc-

ture of the material, etc.. For a small x, equation (6) can be approximated as

$$\begin{aligned} \sigma_n &= \sigma_o - C_\sigma x \\ \tau &= \tau_o - C_\tau x \end{aligned} \qquad (7)$$

where C_σ and C_τ are the gradients of normal stress and shear stress at the root of a notch respectively, σ_o and τ_o are the normal stress and shear stress at the edge of the notch.

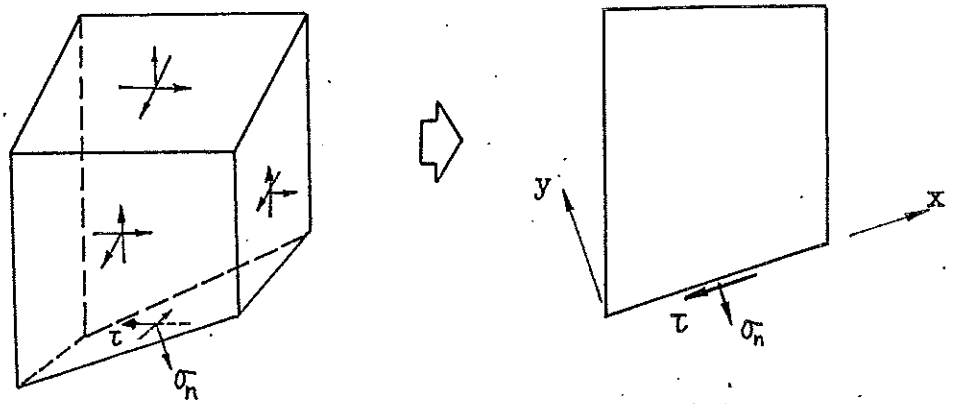


Fig.3 Three dimensional stress state of an element at the notch root

The dislocation density $D_n(x)$ and the self elastic strain energy U_n can be obtained as the above section,

$$D_n(x) = \pm \frac{(\Delta\tau_o - C_\tau x - 2k)}{\pi A} \cdot \frac{xi}{(d^2-x^2)^{1/2}} \pm \frac{C_\tau}{2\pi A} \cdot \frac{di}{(d^2-x^2)^{1/2}} \qquad (8)$$

$$U_n = \frac{i}{8A}(\Delta\tau_o - 2k)^2 d^2 - \frac{i}{4\pi A}(\Delta\tau_o - 2k) C_\tau d^3 + \frac{i}{64A} C_\tau^2 d^4 \qquad (9)$$

When $U_n = W_c d$, the crack initiated, and then equation (9) can be written as

$$8AW_c = n(\Delta\tau_o - 2k)^2 d - \frac{2n}{\pi}(\Delta\tau_o - 2k) C_\tau d^2 + \frac{n}{8} C_\tau^2 d^3 \qquad (10)$$

To compare eq. (5) with eq. (10), there is influence of two behind terms in equation (10) on the crack initiation when the stress gradient is considered.

3. Estimation of K_f

From equation (5), the relationship between the load $\Delta\tau$ and the fatigue life n of the crack initiation can be written as

$$\Delta\tau = 2k + \sqrt{\frac{8AW_c}{nd}} \tag{11}$$

When the material microstructure enters in consideration, the external gross stress $\Delta\tau_g$ is different from the internal stress $\Delta\tau$ acted on the most serious element. Suppose

$$\Delta\tau = K_\sigma . \Delta\tau_g \tag{12}$$

where K_σ is called as microstress concentration coefficient of a smooth specimen, and K_σ is equal to 1 for homogeneous materials. When equation (12) is used, (11) is written as

$$K_\sigma . \Delta\tau_g = 2k + \sqrt{\frac{8AW_c}{nd}} \tag{13}$$

Similarly, for a notched specimen, we can solve eq.(10)

$$\Delta\tau_o = 2k + \frac{C_\tau d}{\pi} + \sqrt{\frac{8AW_c}{nd}} \tag{14}$$

and let K_σ^n be the microstress concentration coefficient of a notched specimen, then

$$\Delta\tau_o = K_\sigma^n . \Delta\tau_g^n \tag{15}$$

Obviously, K_σ^n is equal to K_t under conditions of linear elastic stress and homogeneous materials. When eq.(15) is used, eq.(14) is written as

$$K_\sigma^n . \Delta\tau_g^n = 2k + \frac{C_\tau d}{\pi} + \sqrt{\frac{8AW_c}{nd}} \tag{16}$$

According to the definition of K_f eq.(1), the following formula can be obtained from equations (13) and (16),

$$K_f = \Delta\tau_g / \Delta\tau_g^n$$

$$= \frac{K_\sigma^n}{K_\sigma}\left(1 - \frac{C_\tau d}{2\pi k + C_\tau d + \pi\sqrt{\frac{8AW_c}{nd}}}\right) \tag{17}$$

The formula (17) gives an estimation of the fatigue stress coefficient K_f. From this formula it can be seen that K_f is related

to fatigue life n, the size of specimens(K_σ^n,K_σ), the geometry of the notch(K_σ^n,C_t), the material(d,W_c,A,etc.), and so on. From this formula, the following two results can be deduced.

(a) In gerenal, $C_t d \ll 2\pi k$, so the above equation (17) can be written as follows

$$K_f = K_\sigma^n \,/\, K_\sigma \tag{18}$$

that is to say, under high cycle fatigue conditions, stress gradient has almost no effect on K_f, but the three dimensional state of stress at a notch and the smooth specimen size determine the difference between K_f and K_t.

(b) In high cycle fatigue, the root of the notch is in elasticity during cyclic loading, so K_σ^n is a constant and then K_f is expected to be a constant. But if the root of a notch is under elasto-plasticity, K_σ^n will increase as the cycles to failure increase for cyclic hardening materials and K_σ^n will decrease as the cycles to failure increase for cyclic softening materials. So it can be deduced that K_f will increase as the cycles to failure increase for cyclic hardening materials, and on the contrary, K_f will decrease as the cycles to failure increase for cyclic softening materials. Fig.4 presents the experimental results of K_f for hardening material Ly12-cs and Ly12B-cz to check this deducation.

4. Estimation of K_f distribution

From the hypotheses of the computational micromechanics model stated in the above paragraph, it is known that K_σ and K_σ^n are random variables, so K_f is a random function.

Let the probabilistic density function of K_σ be f(x), the probabilistic density function of K_σ^n be $f_n(y)$, then the probabilisitic density function of K_f is

$$f_k(z) = \frac{d}{dz} \iint_D f(x) f_n(y)\, dx\, dy \tag{19}$$

where D is integral domain enclosed by $\frac{x}{y} \leqslant z$ and intervals of the random variable x value and the random variable y value.

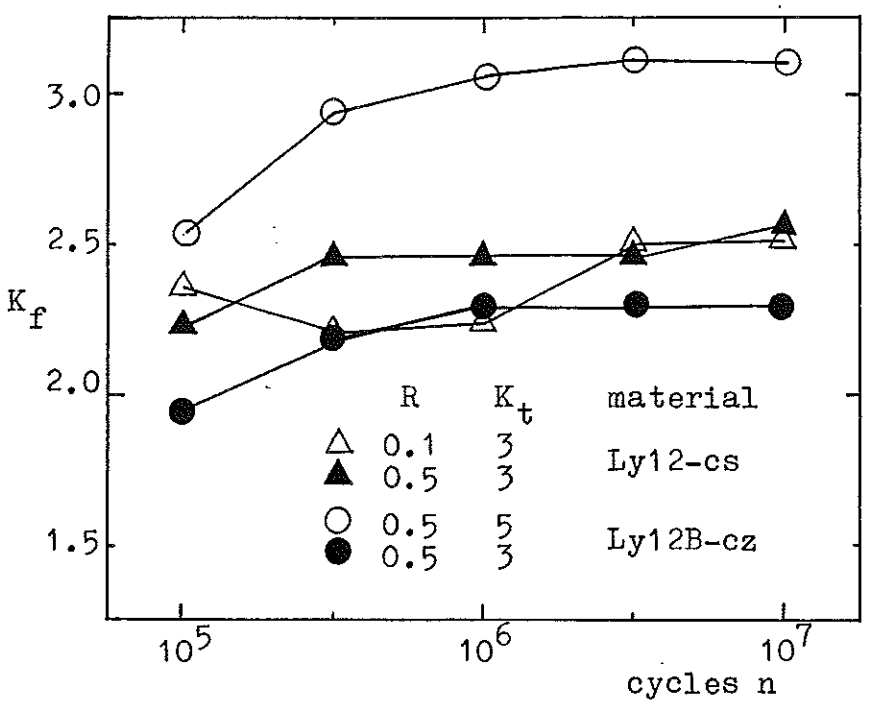


Fig.4 The experimental results of K_f of Ly12-cs and Ly12B-cz Al alloys

A Numerical Example

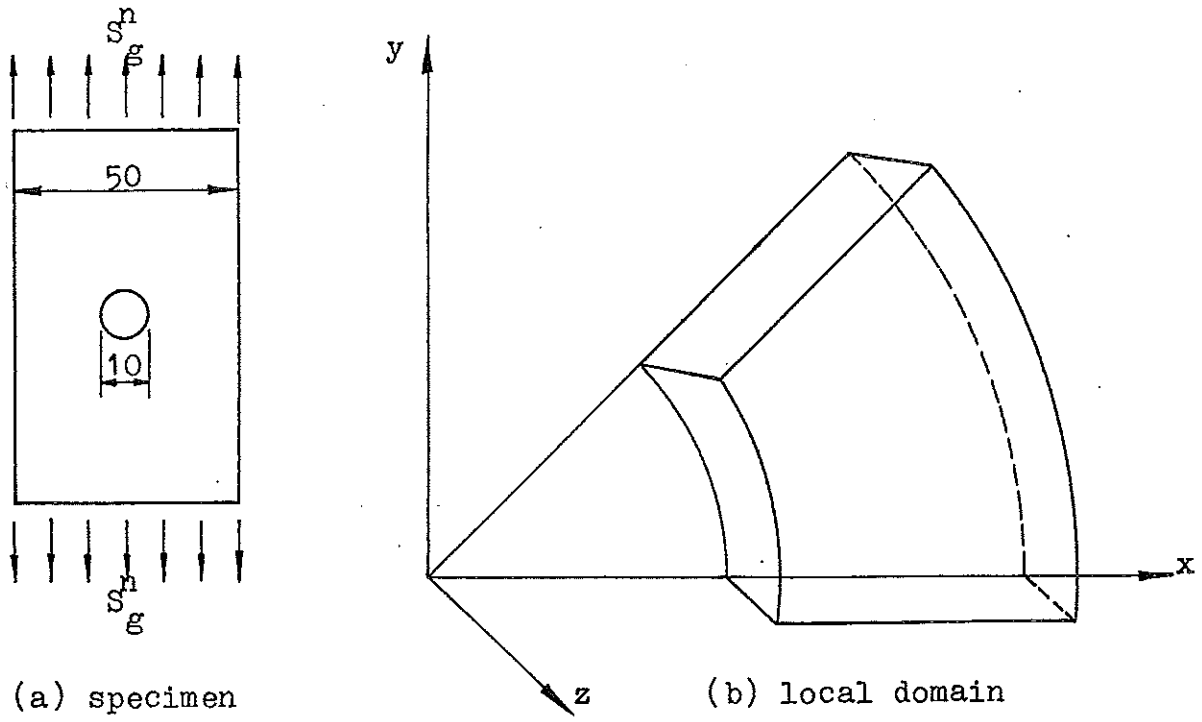
As a numerical example, the K_f of a plane with a central hole (Fig.5, K_t≐3.25) is estimated. Here only gives the estimated results.

First the microstress distribution in the smooth specimen is obtained througth the numerical analysis according to the computational micromechanics model. Table I presents the 45 maximum microstress values for 45 random distribution of the microstructure of the carbon steel (σ_b≐65kg/mm^2) under the gross stress S_g=10kg/mm^2. For the datum of table I, τ_{max} can be approximated to the shift Log-Normal distribution,

$$f(\tau_{max})=\begin{cases}\dfrac{1}{\sqrt{2\pi}\,\sigma_\tau(\tau_{max}-\tau_o)}\exp\left(-\dfrac{(\mathrm{Ln}(\tau_{max}-\tau_o)-\tau_m)^2}{2\,\sigma_\tau^2}\right) & \tau_{max}\geqslant\tau_o\\ 0 & \tau_{max}<\tau_o\end{cases} \quad (20)$$

where τ_o=0.68250, τ_m=1.66128, σ^2=0.012001.

Similary, the microstress distribution can also be computated



(a) specimen (b) local domain

Fig.5 A specimen with a central hole

for the local domain as shown in Fig.5 under the gross stress $S_g^n=10kg/mm^2$. The numerical results are listed in table II. τ_{max} can be approximated to

$$f_n(\tau_{max}) = \begin{cases} \frac{1}{b-a} & b \leq \tau_{max} \leq a \\ 0 & \text{others} \end{cases} \qquad (21)$$

where a=9.58209, b=22.98031.

Then using eq.(19), we can obtain the density function of K_f,

$$f_k(z)=\begin{cases} \int_{a/z}^{b/z} \frac{t}{b-a}\, h(t)\, dt & 0 < z \leq a/\tau_o \\ \int_{\tau_o}^{b/z} \frac{t}{b-a}\, h(t)\, dt & a/\tau_o < z \leq b/\tau_o \\ 0 & \text{others} \end{cases} \qquad (22)$$

$$\text{where } h(t)=\frac{1}{\sqrt{2\pi}\sigma_t(t-\tau_o)} \exp\left(-\frac{(Ln(t-\tau_o)-\tau_m)^2}{2\sigma_t^2}\right)$$

The curve of $f_k(z)$ is presented in Fig.6. For eq.(22) it is calculated that the mean value of K_f is 2.748, and this value is fairly coincided with the prediction of Perterson's, $K_f \doteq 2.7$.

Table I Datum of the microstress in a smooth specimen

No.	σ_3	σ_2	σ_1	τ_{max}
1	-1.61336	0.59456	8.73987	5.17662
2	-1.60788	0.48199	8.76099	5.18444
3	-1.47955	0.66840	9.00151	5.24053
4	-2.00066	0.32291	8.61931	5.30999
5	-0.48355	1.15925	10.28063	5.38209
6	0.52533	1.59127	11.29239	5.38353
7	-1.23093	0.33595	9.60970	5.42031
8	-2.09433	0.62894	8.89480	5.49459
9	0.34311	1.45777	11.39429	5.52559
10	-2.05669	0.31895	9.01135	5.53402
11	-2.42485	0.03707	8.64542	5.53513
12	-2.94036	0.15362	8.23606	5.58821
13	0.41707	1.82492	11.62684	5.60488
14	-2.51336	-0.06750	8.75228	5.63282
15	-0.98730	0.56040	10.31837	5.65283
16	0.39010	1.52939	11.83952	5.72471
17	-0.69126	1.19181	10.80080	5.74603
18	-0.89540	0.80998	10.67526	5.78533
19	-0.88049	1.11418	10.70233	5.79141
20	-4.30346	-0.78936	7.28866	5.79606
21	-0.77206	0.98654	10.84720	5.80963
22	-0.92089	0.81311	10.73482	5.82785
23	-2.90005	0.00996	8.87684	5.88845
24	-0.87172	0.87893	10.91073	5.89123
25	-2.50218	0.16887	9.32148	5.91183
26	-1.97325	0.60859	9.86623	5.91974
27	-0.74223	0.73164	11.13479	5.93851
28	-2.88111	-0.21423	9.05587	5.96849
29	-3.15351	-0.09561	8.78432	5.96891
30	-0.78423	0.83183	11.16008	5.97215
31	-2.88836	-0.08844	9.13781	6.01309
32	-0.98597	1.49865	11.14172	6.06384
33	-1.34998	1.09336	10.77777	6.06388
34	-0.60209	0.89191	11.67587	6.13898
35	-2.22719	0.66866	10.39232	6.30976
36	0.49114	1.20563	13.28293	6.39589
37	0.65367	1.19012	14.60220	6.47427
38	0.20490	1.16288	13.21196	6.50353
39	0.25872	1.27493	13.83243	6.78685
40	-2.98073	0.46153	10.61173	6.79623
41	-3.26709	0.26928	10.40424	6.83567
42	-2.68410	0.49842	11.26611	6.97510
43	0.34042	0.48477	14.52277	7.09117
44	0.75461	1.04370	15.57885	7.41212
45	0.86779	1.01178	16.16002	7.64611

Table II Datum of microstress at the root of the hole

9.17506	10.45957	10.60805	12.15397	12.42493	12.72018
12.82807	12.84184	13.47050	13.49480	14.67484	14.73700
15.24181	15.63136	16.22176	16.69189	16.92330	17.46455
17.61833	18.66148	19.42490	19.70090	19.82032	20.26209
21.02320	21.55563	21.89124	21.92070	22.52271	

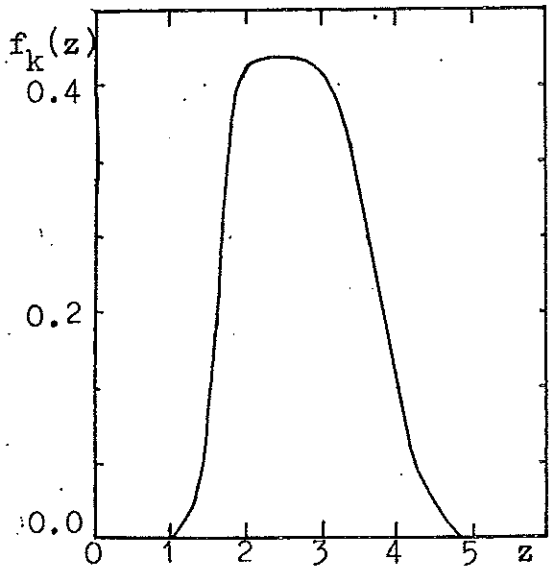


Fig.6 The probabilistic density of K_f

Conclusion

The computational micromechanics model is briefly reviewed. The main causes of the difference between K_f and K_t are discussed. A formula to estimate K_f, especially under high cycle fatigue, is given, and then the distribution of K_f is roughly investigated which is rarely studied until now. The following conclusions are derived from these results.

1. Under high cycle fatigue conditions, the stress gradient at a notch has almost no effect on K_f, but the three dimensional stress at the hole and the smooth specimen size determine the difference between K_f and K_t.
2. K_f is a statistic function which is mainly related to the material microstructure and the geometry of the notch.
3. K_f increases as the cycles to failure increase for the cyclic hardening materials, and K_f decreases as the cycles to failure increase for the cyclic softening materials.

Ackowledgement

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