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The Influence of Multiaxial Stress on Fatigue Failure

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Abstract

A multiaxial fatigue criterion is investigated based on the strain energy density damage law. The key element of this theory is that the damage caused as a result of cyclic loading is a function of the mechanical input into the material. The proposed criterion is hydrostatic pressure sensitive , and is consistent with the concept of crack initiation and subsequent propagation. The proposed method applies to materials which obey and do not obey the idealized Masing type description. A desirable feature of this criterion is its unifying nature for both short and long cyclic lives.

The prediction of the proposed criterion are compared with the experimental results of biaxial fatigue tests for A-516 Gr. 70 low alloy steel and 1018 steel and are shown to be in good agreement.

1. Introduction

This investigation was undertaken to assist in the development of failure prediction techniques for components subjected to general states of intense cyclic strain. Strains of this nature occur in a number of elements in nuclear, aircraft, petrochemical, automobile industries. Many components have design details that involve severe multiaxial stress concentrations. At these highly stressed locations the stress may be well above the fatigue limit. Conditions of this type are extremaly common and

are often the cause of premature failure in pieces of equipment intended for long life applications.
The fatigue life of a component can be devided into a number of distinct phases were different parameter control the process. In the absence of internal defects, fatigue cracks are initiated within the surface material where biaxial stress conditions prevail.In many cases the applied cyclic loading is also multiaxial.

Numerous proposals have been made to correlate multiaxial fatigue test data [1-5]. The multiplicity of the proposed criteria is an indication of the complexity of the problem and lack of an agreement upon unified approach. The challange is therefore to define a damage parameter which can be used to describe fatigue phenomena. Most of the proposed criteria are either stress or strain based and are often for the biaxial conditions. In these approaches the interrelation between the cyclic stress and strain, and fatigue damage process is usually overlooked. Since the fatigue damage is generally caused by the cyclic plastic strain, the plastic strain energy density plays an important role in the damage process [6-9]. In the present study a special form of cyclic strain energy density equal to the sum of plastic strain energy density and tensile elastic strain energy density is used as a damage parameter [10-13].This approach has a number of desirable features such as being consistent with the the crack initiation and subsequent propagation it unifyes both the low- and high-cycle fatigue regimes and can be adopted to nonproportional cyclic loading. The immediate aim of this investigation was to produce a method of estimating the fatigue life under multiaxial strain conditions for both Masing and non-Masing material behaviour.

2. Multiaxial fatigue criterion.

Fatigue failure under multiaxial stress conditions ia a complex subject and numerous criteria ranging from the purely empirical to the theoretical have been proposed.

During cyclic loading , energy is dissipated because of plastic deformations . A part of this energy is converted into heat , and the other part is rendered irrecoverable at every cycle due to the plastic strain energy absorption . The plastic strain energy per cycle , ΔW^p, has a valuable feature that does not vary appreciably with cycles in case of strain-controlled tests [6,7,13]. However , when the strain range , $\Delta\varepsilon$, decreases , $\Delta\varepsilon^p$---- 0 and the corresponding plastic strain energy density, ΔW^p --- 0 . In this case , the macroscopic (bulk) response of the material is quasielastic , altough at the microscopic (grain) level plastic deformation may occur . Therefore , it is assumed that damage due to the cyclic multiaxial loading can be modeled as a function of the absorbed plastic strain energy per cycle and that part of the elastic energy which faciliates the crack growth [10-13] .Thus , both tensile elastic and plastic parts of the strain energy per cycle , have to be determined , i.e.

$$\Delta W^t = \Delta W^p + \Delta W^{e+} . \tag{1}$$

To calculate the input strain energy density we will proceed with calculating the elastic and plastic strain energy densities separately.This is made possible by the virtue of the separation of the total strain increment into the elastic and plastic parts. For an isotropic elastic material, the stress-strain relationship is given by:

$$\varepsilon_{ij} = \frac{1 + \nu}{E} \sigma_{ij} - \frac{\nu}{E} \sigma_{kk}\delta_{ij} , \tag{2}$$

where ν is Poisson's ratio; E Young's modulus; δ_{ij} is the Kronecker delta, equal to unity when i=j and zero otherwise, and a repeated index implies summation over its range, in this case i,j=1,2,3.

Therefore the cyclic elastic strain energy density for the positive stress parts of the cycle , can be calculated from

$$\Delta W^{e+} = \frac{1}{2E}\left[(I_1^{max})^2 - 2(1+\nu)\, I_2^{max} \right] \tag{3}$$

where $I_1^{max} = \sigma_1^{max}H(\sigma_1^{max}) + \sigma_2^{max}H(\sigma_2^{max}) + \sigma_3^{max}H(\sigma_3^{max})$,

$$I_2^{max} = \sigma_1^{max}\sigma_2^{max}H(\sigma_1^{max})H(\sigma_2^{max}) + \sigma_2^{max}\sigma_3^{max}H(\sigma_2^{max})H(\sigma_3^{max}) + \sigma_1^{max}\sigma_3^{max}H(\sigma_1^{max})H(\sigma_3^{max})$$

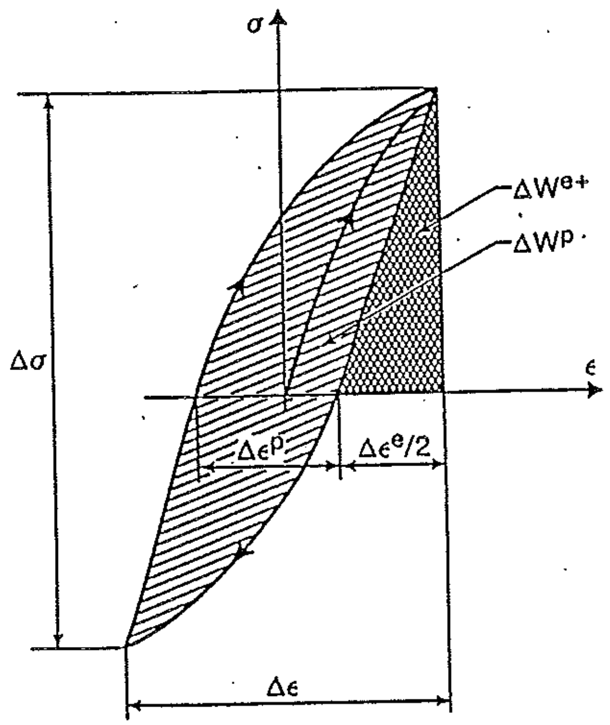

Fig. 1 A typical hysteresis loop and the tensile elastic and plastic strain energies.

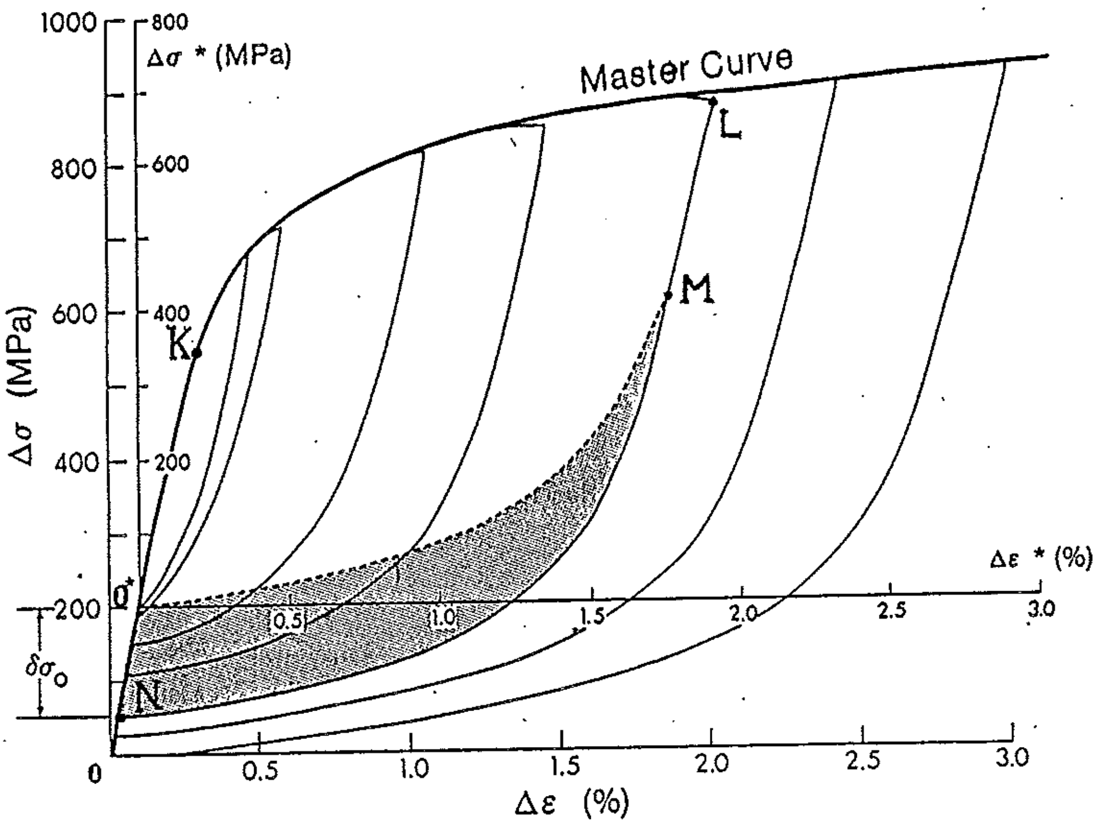

Fig. 2 Definition of master curve.

are the first and second invariants of the stress tensor , H is the Heaviside function.

The cyclic plastic strain energy density , ΔW^p , can be calculated from

$$\Delta W^p = \int_{cycle} \Delta\sigma_{ij} \, d(\Delta\varepsilon_{ij}{}^p) \tag{4}$$

where the integration is carried over the closed cyclic loop . The plastic strain components for a proportional or nearly proportional loading are given by [14] :

$$\Delta\varepsilon_{ij}{}^p = 3(2K^*)^{-1/n^*}(\Delta\sigma^*)^{(1-n^*)/n^*} \Delta S_{ij}{}^* \tag{5}$$

where $\Delta\sigma^* = (3/2 \, \Delta S_{ij}{}^* \, \Delta S_{ij}{}^*)^{1/2}$
and K^* and n^* are material parameters , in general functions of multiaxial stress state. According to the J_2 theory the equivalent plastic strain can be defined as follows

$$\Delta\varepsilon^p = (2/3 \varepsilon^p_{ij} \varepsilon^p_{ij})^{1/2} . \tag{6}$$

Combining equation (5) and (6) we obtain the effective skelton cyclic stress-strain curve [15-17] for multiaxial proportional loading

$$\Delta\varepsilon^p = 2(\Delta\sigma^*/2K^*)^{1/n^*} . \tag{7}$$

The hysteresis loop O^*KLMO^* corresponding to the Masing behaviour is obtained by translating the lower branch LMN by a quantity equal to the increase in the proportional stress range , i.e. $\delta\sigma_o$. The plastic strain energy of the Masing loop O^*KLMO^* would be

$$\Delta W^p_m = \frac{1 - n^*}{1 + n^*} \Delta\sigma^* \Delta\varepsilon^p = \frac{1 - n^*}{1 + n^*} (\Delta\sigma - \delta\sigma_o) \Delta\varepsilon^p . \tag{8}$$

The cross - hatched area NO^*M in Fig. 2 , representing the increase of the plastic strain energy per cycle due to non-Masing behaviour , can be evaluated in the following manner. For the coordinate system with origin at L , the plastic strain energy density for the loading from point L to O^* is given by

$$\Delta W_1^P = \int \sigma' d\varepsilon^P . \quad (9)$$

For loading LMN we have

$$\Delta W_2^P = \int \sigma' d\varepsilon^P + \delta\sigma_o \Delta\varepsilon^P . \quad (10)$$

Therefore the area of NO^*MN is equal to

$$(NO^*MN)_{area} = \Delta W_2^P - \Delta W_1^P = \delta\sigma_o . \quad (11)$$

Then the plastic strain energy density of the hysteresis loop NO^*KLMN is given by

$$\Delta W^P = \frac{2(1-n^*)(2K^*)^{-1/n^*}}{(1+n^*)} (\Delta\sigma^*)^{(1+n^*)/n^*} + 2(2K^*)^{-1/n^*} \delta\sigma_o (\Delta\sigma^*)^{1/n^*} \quad (12)$$

where $\delta\sigma_o$ is the increase in the elastic part due to multiaxial loading history , a measure of the multiaxial cyclic hardening (or softening) and is given by

$$\delta\sigma_o = \Delta\sigma - \Delta\sigma^* = \Delta\sigma - 2K^* (\Delta\varepsilon^P/2)^{n^*} \quad (13)$$

where $\Delta\sigma = \left[3/2 \, (S_{ij}^{(2)} - S_{ij}^{(1)})(S_{ij}^{(2)} - S_{ij}^{(1)}) \right]^{1/2}$ is the effective range stress range , and subscripts (2) and (1) denote the maximum and minimum value of the deviatoric stress components.

The increase in the proportional range $\delta\sigma_o$ may also be estimated from the effective stress on the cyclic curve ($\Delta\sigma/2$ for each controlled strain) and the minimum proportional stress σ_o , i.e.

$$\delta\sigma_o = \Delta\sigma/2 - \sigma_o . \quad (14)$$

However, relation (13) is more accurate.
It is to be noted that for ideal Masing behaviour, the skeleton curve and cyclic curve will coincide, i.e. $n^{*}=n'$ and $\delta\sigma_{o}=0$, and equation (12) reduces to

$$\Delta W^{p} = \frac{1 - n'}{1 + n'} \Delta\sigma\Delta\varepsilon^{p} \quad (15)$$

Thus the total damaging cyclic strain energy density, ΔW^{t}, is obtained by combining Eq. (3) and (12), i.e.

$$\Delta W^{t} = \Delta W^{e+} + \Delta W^{p} = \frac{1}{2E}\left[(I_{1}^{max})^{2} - 2(1+\nu)\, I_{2}^{max} \right] +$$

$$\frac{2(1-n^{*})(2K^{*})^{-1/n^{*}}}{(1+n^{*})} (\Delta\sigma^{*})^{(1+n^{*})/n^{*}} + 2(2K^{*})^{-1/n^{*}} \delta\sigma_{o} (\Delta\sigma^{*})^{1/n^{*}} \quad (16)$$

The failure criterion [10] in the case of multiaxial fatigue can be expressed as

$$\Delta W^{t} = \varkappa(\rho) N_{f}^{\alpha} + C(\rho) \quad (17)$$

where $\varkappa(\rho)$ and $C(\rho)$ are the functions of ρ, defined as triaxiality constraint factor

$$\rho = (1 + \nu^{*}) \frac{\varepsilon_{1}}{\gamma_{max}} \quad (18)$$

where $\varepsilon_{1} = \max\,[\varepsilon_{a}$ or $\varepsilon_{t}]$;

$\gamma_{max} = \max\,[(\varepsilon_{a}-\varepsilon_{r})$ or $(\varepsilon_{t}-\varepsilon_{r})]$.

In equation (18) ε_{a} is axial strain, ε_{t} is tangential strain and ε_{r} is radial strain respectively. The effective Poisson's ratio, ν^{*} is calculated from

$$\nu^{*} = \frac{\nu_{p}(1-\nu_{e})(\varepsilon_{a}+\varepsilon_{t}) + (\nu_{e}-\nu_{p})(\varepsilon_{a}^{e}+\varepsilon_{t}^{e})}{(1-\nu_{e})(\varepsilon_{a}+\varepsilon_{t}) + (\nu_{e}-\nu_{p})(\varepsilon_{a}^{e}+\varepsilon_{t}^{e})} .$$

As a first approximation, the functions $\varkappa(\rho)$ and $C(\rho)$ can be expressed in a linear form

$$\varkappa(\rho) = a\rho + b \qquad (19)$$
$$C(\rho) = e\rho + f$$

where a,b,e,f are material constants. Therefore, the explicit form of the failure criterion is obtained by combining Eqs. (17) and (19), i.e.

$$\Delta W^{t} = (a\rho + b)\ N_f^{\alpha} + (e\rho + f). \qquad (20)$$

It is noted that the multiaxial fatigue criterion (20) can be expressed as,

$$F(I_1^{max}, J_2) = G(N_f, \rho) \qquad (21)$$

The proposed criterion is hydrostatic pressure sensitive (I_1^{max}) and has an invariant property, i.e. it is frame indifferent criterion.

3. Comparison with the experimental results

To examine the applicability of the investigated criterion and determine the material constants sufficiently detailed results of multiaxial fatigue tests with good consistency are required. In the published literature we can find some biaxial fatigue test data of varying types. For example, results of fatigue tests for tension-torsion cyclic loading are given in [18-19] and those for tension-pressure cycling in [20-21]. However, the data presented in most of these papers for our purpose are incomplete, and could not be directly used for a comparative study. To be able to make a comparison we would require data on uniaxial cyclic stress-strain curves, history of stress and strain during cycle, and number of cycles to failure. In other words we need data minimum for two strain (or stress) ratio in this one can be uniaxial case. In the present paper we used experimental data obtained for A-516 Gr.70 low alloy steel [13,17,22,23] and

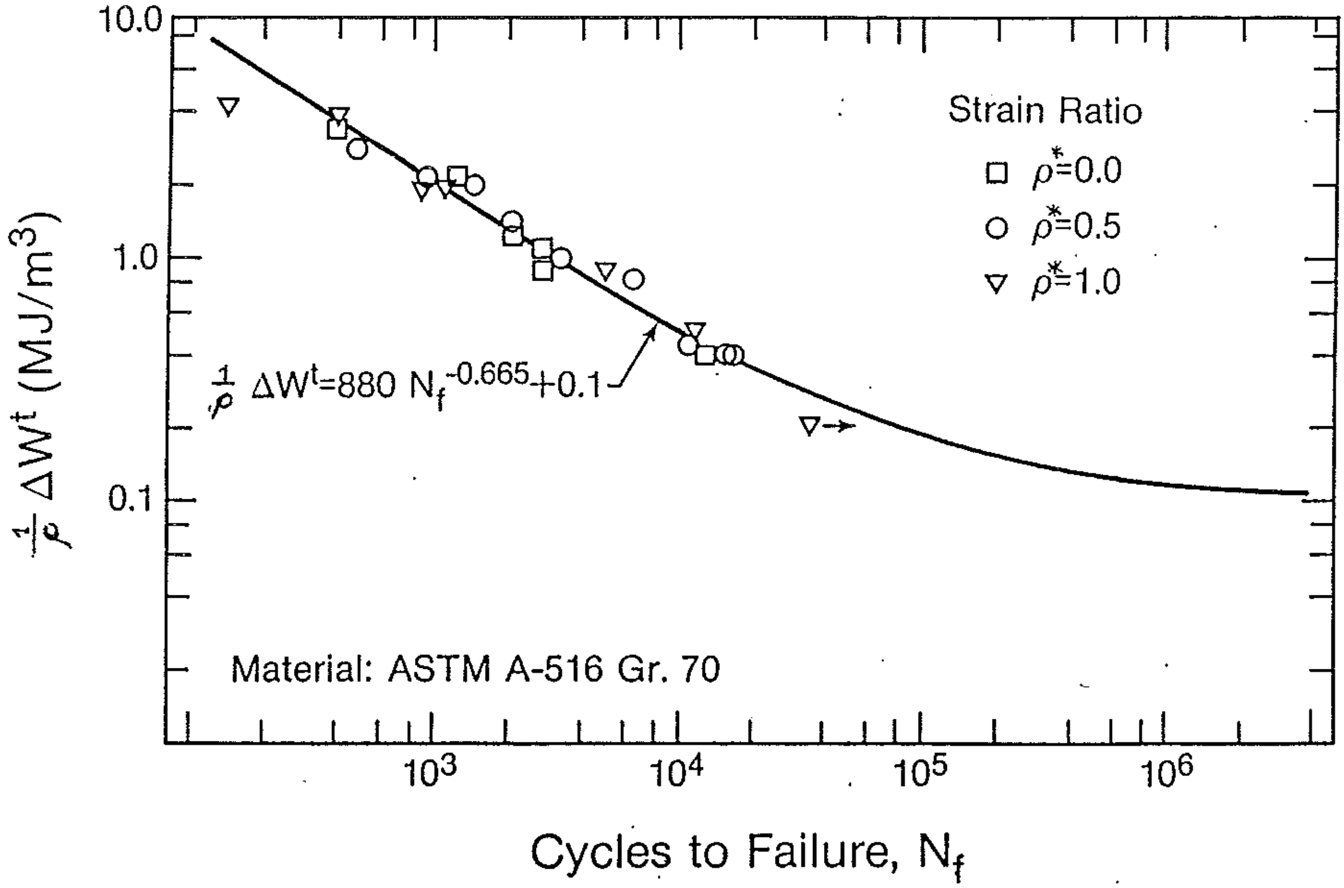

Fig. 3. Total strain energy density, $\Delta W^t/\rho$, versus number of cycles to failure, N_f, for A-516 Gr.70 low alloy steel.

normalized 1018 mild steel. The mechanical and cyclic properties of the examined materials are presented in Table 1 and Table 2, respectively. In both casesfully reversed strain-controlled tests at different strain ratios ($\rho^*=\varepsilon_t\backslash\varepsilon_a$) have been carried out on thin-walled tubular specimens using an electro-hydraulic servo-controlled testing system. Specimens were cyclically loaded in axial direction while pressures were applied to the inside and outside alternatively during each half cycle.. The tests were conducted in oil and the definition of failure was based on the detection of small crack,normally indicated by a distorsion in the hysteresis loop. The predictions of the presented criterion are compared with the experimental data for different strain ratio in Fig.3 and Fig.4. It worth to mention that for examined materials only one set of experimental data, e.g. uniaxial case (ρ=1) is sufficient to specify $\varkappa(\rho)$ and $C(\rho)$. For both steels we obtained that b=f=0 and a= $\varkappa_{uniaxial}$ and e= $C_{uniaxial}$. The value of $C_{uniaxial}$ is that part of total strain energy density which

TABLE 1 Mechanical and Cyclic Properties of A-516 Gr.70 Low Alloy Carbon Steel.

Property	E(MPa)	$\sigma_{y(0.2\%)}$(MPa)	$\sigma'_{y(0.2\%)}$(MPa)	σ_{lim}(MPa)
Value	204000	190	325	205
Property	K'(MPa)	K^*(MPa)	n'	n^*
Value	1067	630	0.193	0.144
Property	$\varkappa_u$(MJ/m^3)	C_u(MJ/m^3)	a(MJ/m^3)	e(MJ/m^3)
Value	880	0.1	880	0.1

TABLE 2 Mechanical and Cyclic Properties of Normalized 1018 Mild Steel.

Property	E(MPa)	$\sigma_{y(0.2\%)}$(MPa)	$\sigma'_{y(0.2\%)}$(MPa)	σ_{lim}(MPa)
Value	207000	200	210	160
Property	K'(MPa)	K^*(MPa)	n'	n^*
Value	948	602	0.2315	0.1765
Property	$\varkappa_u$(MJ/m^3)	C_u(MJ/m^3)	a(MJ/m^3)	e(MJ/m^3)
Value	232	0.06	232	0.06

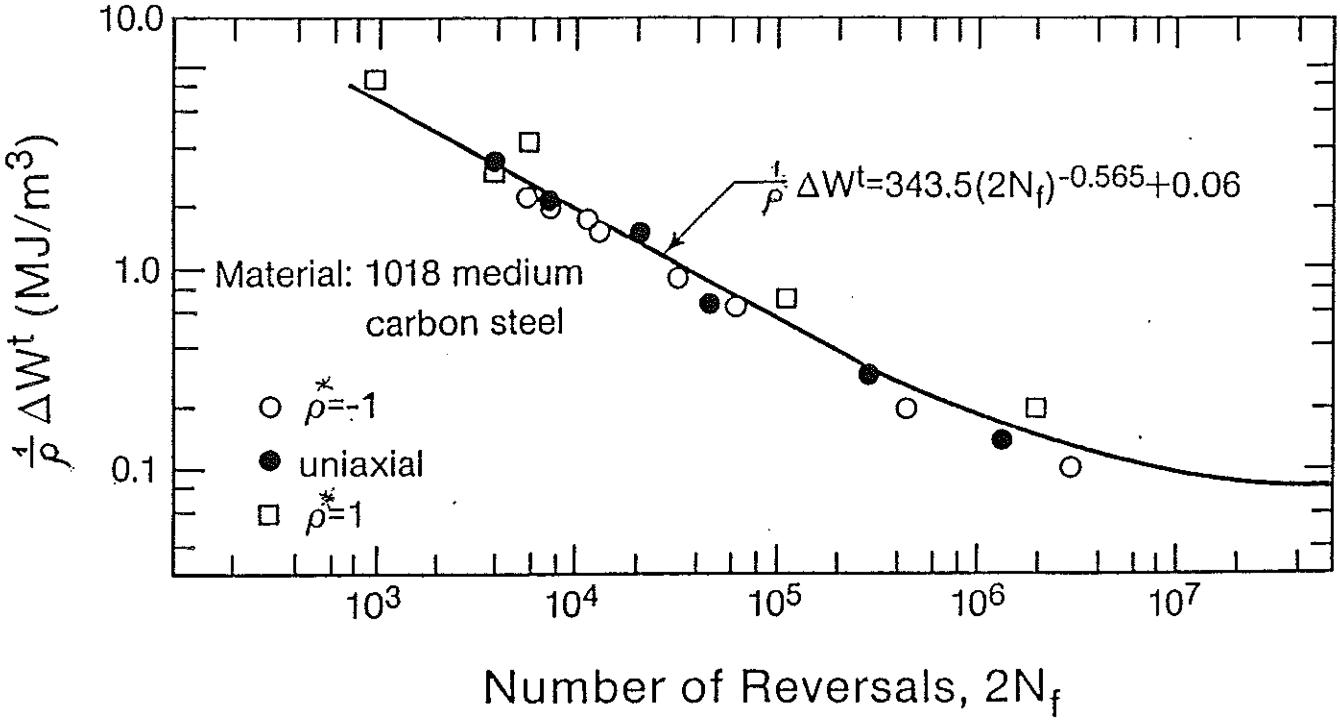

Fig. 4 Total strain energy density, $\Delta W^t/\rho$, versus number of reversals to failure, $2N_f$,for normalized 1018 mild steel.

does not cause the damage and can be calculated as

$$C_{uniaxial}=(1/2E)(\sigma_{lim})^2 . \qquad (22)$$

However, in general we would require two sets of data. One can observes that the correlation between criterion and experimental data for examined materials is fairly good.

4. Conclusions

An energy based criterion for the multiaxial fatigue failure has been investigated for proportional or nearly proportional cyclic loading. A form of the cyclic strain energy density equal to the sum of plastic and tensile elastic strain energy densities is used as a damage parameter for multiaxial fatigue failure. The total strain energy density approach has a physical interpretation , and is consistent with the concept of crack initiation and subsequent propagation.

The proposed criterion is hydrostatic pressure sensitive, applies to materials which do not obey the idealized Masing type description and can describe both long and short fatigue lives. The predictions of the investigated method are compared with the experimental results for ASTM A-516 Gr. 70 carbon low-alloy steel and normalized 1018 mild steel. The agreement for examined materials is found to be fairly good.

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