Numerical Evaluation of Multiaxial Static Fatigue in Ceramics

S. Lauf, K. Turan

Third International Conference on Biaxial/Multiaxial Fatigue
April 3-6, 1989, Stuttgart, FRG

The theoretical/numerical treatment of multiaxial static fatigue strength uses finite element analysis of stress and deformation fields. The results are used for life time prediction by means of statistics within an integrated software system for design and analysis. The approach for life time evaluation considers mechanical and thermal loading.

1. Introduction

The use of structural ceramics has increased in the recent years. Excellent material properties like high temperature strength, good corrosion resistance and thermal shock behaviour makes them a candidate for high temperature applications. But there are some characteristics, which must be taken under consideration when designing with structural ceramics. The fundamental characteristics of ceramic materials are that they show no plastic deformation before failure and they have little toughness. Under static tension stress conditions in air ceramic materials show delayed fracture, i.e. a stady decrease of strength with time. This behaviour is called "static fatigue" and determines the strength from room temperature up to temperatures above 1 000°C until distinct creep occurs, which is yet neglected. Furthermore the strength of ceramic materials shows a great scatter which reflects on the time dependent strength.

Therefore when designing with ceramics an important factor is the computation of the reliability or probability of survival of a structural component under certain loading conditions. The computation of stresses using finite element analysis is the basis for this.

2. Statistical strength description

In a controlled strength test with a batch of nominally identical test pieces of a brittle material, it is found that the strength values lie on some distribution curve about an average value, Fig.1. This value is usually the one quoted as strength.

The narrower the distribution, the more reproducible the strength of the material, and the more reliable the material will be under load. If the distribution is wide, particulary if there is a long tail down to low strength, the material has highly unreliable strength and should be used with great caution.

Many factors affect the distribution width, including the way the material is made, the surface finish produced, and the frequency of occurence of large microstructural defects, the last, of course, having a profound effect on material reliability.

It has been found that the distribution function which describes the test results best is that due to Weibull.

The basic Weibull equation describing the probability of failure P_f, as a function of fracture stress. σ_f, is given by the equation.

$$P_f(\sigma) = 1 - \exp\left\{ - \frac{V}{V_0} \left(\frac{\sigma - \sigma_u}{\sigma_0}\right)^m \right\} \qquad (1)$$

σ_u characterizes the lower level of stress at which the probability of failure falls to zero. σ_o is a scaling factor and m is kown as the Weibull modulus, describing the width of the distribution. The higher m is the smaller the scatter of the distribution. The ratio v/v_o describes the influence of volume under uniform stress. The larger the volume under stress, the greater is the chance of there being a large flaw in it. Usually the lower stress parameter σ_u is set zero and for a batch of identical test samples the volume factor is set equal one. Setting σ_u equal zero follows for practical reasons.

An equivalent statistical description uses the survival probability.

$$P_s = 1 - P_f \tag{2}$$

3. Fatigue in ceramic materials

Brittle materials, particularly glasses and polycrystalline ceramics containing an intercrystalline glassy bonding phase, weaken with time under load. Therefore life time under constant stress is limited.

This behaviour is called "static fatigue" as opposed to metals where fatigue is a cyclic effect.

The physical explanation is that cracks or flaws grow slowly with time under subcritical stress until the crack reaches a critical length.

Slow crack growth occurs as a result of a stress-aided thermally-activated event and by chemical attack from the environment within the crack.

The rate of crack propagation, v, with a stress intensity factor K_I at the crack tip can be represented by following equation:

$$v = v_o \exp\left(-\frac{E_o - BK_I}{kT}\right) \tag{3a}$$

with E_o as an effective activation energy, the Boltzmann factor k, while v_o and B denote crack extension parameters.

For materials which show significant slow crack growth the relationship may not be as simple as presented.

Fig. 2 shows schematically the type of relationship between v and K_I in a logarithmic plot.

In the general case there are a number of distinct regions as shown in the figure.

- Region I is governed by slow crack growth and is highly dependent on environment.
- In region II crack growth is limited by diffusion of the environment.
- In region III crack growth is independent of environment.

In region III intensity factor has nearly the magnitude of fracture toughness K_{IC}, the crack accelerates very fast and instantaneous failure occurs.

The slow crack growth limit or static fatigue limit, K_o, occurs at such low velocities in ceramic materials, smaller than 10^{-10} m/sec, that its existence has not been generally proven.

Real ceramics often show only region I behaviour over the measurable range of velocity. The velocity is highly dependent on the measuintensity factor K_I. Therefore the exponential function is approximated by one term of the exponential series and yields that the crack extension, v, is given by

$$v = AK_I^n \qquad (3b)$$

with the crack extension parameters A and n.

Most data are plotted as log v versus log K giving a slope n. The crack extension parameter n indicates how liable the material is to fatigue. The lower the value of n the more susceptible the material is to slow crack growth.

Therefore it is desirable to have a high n value for high performance structural ceramics. For fatigue calculations it is usual to make the following assumptions:

- Subcritical crack extension is given by the power law of equation (3).

- No or negligible creep reactions occur and therefore linear elastic fracture mechanics is valid to calculate the stress distribution at material defects.

- For basic calculations a simple shaped crack with the length 2a and sharp crack tips is considered, Fig. 3. Plane stress conditions are assumed. This means the applied stress o is perpendicular to the crack surface. The stress intensity factor K_I is described by /2/

$$K_I = \sigma \sqrt{a}\, Y \tag{4}$$

is the applied stress, a half the crack length and Y a geometrical factor.

Failure occurs when the critical condition is reached.

$$K_I = K_{IC} \tag{5}$$

This is either if the crack length reaches a critical value a_c or by performing an inert strength test, where no crack extension occurs.

$$K_{IC} = \sigma \sqrt{a_c}\, Y$$
$$K_{IC} = \sigma_{IC} \sqrt{a}\, Y \tag{6}$$

- For all conceivable applied stresses the same subcritical crack extension mechanism is valid.

From these assumptions it follows that the time of failure t_f is given by the upper limit of the time integral over the applied stress.

$$B\sigma_{IC}^{n-2} = \int_0^{t_f} \sigma^n(t)dt \qquad (7)$$

This integral must be equal to a constant which contains sub-critical and critical parameters of the considered material

$$B = 2/(n-2)AY^2K_{IC}^{n-2} \qquad (8)$$

The strength S after time t and uniform uniaxial tension is given by /3,4/

$$S(t) = \left(\sigma_{IC}^{n-2} - \frac{1}{B}\int_0^t \sigma^n(t')dt'\right)^{1/(n-2)} \qquad (9)$$

4. The statistical fatigue failure approach

The failure probability of a body subjected to a non-uniform uniaxial stress can be expressed as /5/

$$P_f = 1 - \exp\left\{-\{\Gamma(1+\frac{1}{m})\}^m \bar{\sigma}_v^{-m} v^{-1} \int_V \{\sigma/H(\sigma)\}^m dV\right\} \qquad (10)$$

with the unit tensile strength $\bar{\sigma}_v$, the unit volume v and $H(\sigma) = 1$ for positive (tensile) values of σ and $-\alpha$ for negative (compressive) values of σ. characterizes the modulus of the ratio of the mean failure stress of unit volume in uniaxial compression, to that for uniaxial tension.

In general, the stress state at a point in a real component is not uniaxial; it is characterized by three principal stresses. When extending Equation (10) to cover the body subjected to multiaxial stresses, it is necessary to establish a 'failure criterion' for such stress systems, which expresses the dependence of failure on the combined action of stresses present, like the Tresca or von Mises criterion for metals.

For ceramic materials different criterions have been choosen, Fig. 4/6-10/

- Maximum Normal Stress Criterion
- Coulomb-Mohr Criterion
- Maximum Strain Energy Criterion
- Griffith Criterion
- Weibull Criterion.

The experimental work on multiaxial stressed ceramic specimens is not sufficiently extensive to give answer which criterion describes the data best /11-13/.

Another multiaxial stress approach uses the stress volume integration around a point /14,15/.

For the further considerations, it is assument that the failure probability of an element due to one principal stress is independent of the presence of other principal stresses. A second assumption is that the material is isotropic. It follows that the survival probability of an element subjected to three principal stresses is the product of the survival probabilities obtained by considering the element to be subjected to each of the three principal stresses in turn. Equation (10) therefore becomes

$$P_f = 1 - \exp\left[-\{\Gamma(1+\tfrac{1}{m})\}^m(\sigma_{nom}/\sigma_V)^m\frac{V}{V}\Sigma\right] \qquad (11)$$

where

$$\Sigma = \int_V \{(\frac{\sigma_1}{\sigma_{nom}H(\sigma_1)})^m+(\frac{\sigma_2}{\sigma_{nom}H(\sigma_2)})^m+(\frac{\sigma_3}{\sigma_{nom}H(\sigma_3)})^m\}dV/V$$

σ_{nom} is used for practical reasons.

This equation is first of all only valid at the beginning of external loading or for inert condtions, when no subcritical crack extension occurs.

For noninert conditions the strength degradation due to subcritical crack growth must be taken into account. This is done by substituting the unit tensile strength $\bar{\sigma}_V$ by equation (9) in unified form and using the time dependent principal stresses.

With equations (9) and (11) the statistical fatigue failure approach is formulated and can easily be connected with Finite Element Analysis for numerical Evaluation of multiaxial loaded ceramic components, Fig. 5. This treatment allows quantification of reliability.

5. Finite element analysis

For the finite element analysis of ceramic components linear elastic material behaviour is assumed since ceramics show practically no plastic deformations.

The problem must be defined in the real time scale, because the time values are used by static fatigue calculations in the ceramics lifetime processor. Time increments must be chosen suitably small for the periods, in which large fluctuations of external loading occur, in order to represent the response of the structure properly.

Since the regions subjected to the greatest tensile stresses will have a predominating influence on the value of P_f, it is important to make these elements small. The elements in the less critical, lower stressed regions may be larger.

By modelling a thin element layer on the outer surface of the component, the contribution of surface flaws to the failure probability can be taken into account.

In order to obtain the volumes V of the finite elements, it is necessary to carry out a dynamic analysis prior to the actual calculations and to evaluate the kinetic energy E_K, of each element

$$E_K = dVv^2/2 \qquad (12)$$

where d denotes the density and v the velocity.
With unit density of the material and unit velocity defined, the volume of the element can easily be found

$$V = 2E_K \qquad (13)$$

External loading of the component may consist of forces, moments and thermal loading applied at nodes or element faces. Residual stresses can be considered as initial conditions at the beginning of the analysis.

6. Conclusion

The numerical treatment of multiaxial static fatigue of ceramic components is theoretically well established. Different strength criterions and the strength function for static fatigue can be combined with the statistical strength description due to Weibull. Connecting the corresponding theoretical relationships with Finite Element Analysis gives the possibility to calculate the reliability of ceramic components. For validation of this treatment further experimental investigations are to be done.

References

/1/ Weibull, W.
A Statistical Theory of the Strength, Ingeniörsvetenskapsakademien, Handl. Nr. 151 (1939)

/2/ Griffith, J.A.
Philosophical Transactions of the Royal Society of London Series A, 221 (1920) 163-198

/3/ Tradinik, W.; Kromp, K.; Pabst, R. F.
A Combination of Statistics and Subcritical Crack Extension Mat. Sc. Eng., 56 (1982) 39-46

/4/ Lauf, S.; Pabst, R. F.
Strength-Probability-Time Diagrams and the Strength Function - A Critical Examination, W. Bunk, H. Hausner (Editors), Ceramic Materials and Components for Engines, Verlag DKG (1986) 961-977

/5/ Stanley, P.; Fessler, H.; Sivill, A. D.
An Engineer's Approach to the Prediction of Failure Probability of Brittle Components, Proc. Brit. Cer. Soc., D. J. Godfrey (Ed.), 22 (1973) 453-487

/6/ Batdorf, S. B.
Fundamentals of the Statistical Theory of Fracture. R. C. Bradt, D.P.H. Hasselman, F.F. Lange, (Editors), Fracture Mechanics of Ceramics, 3 (1978) 1-30

/7/ Petrovic, J. J.; Stout, M. G.
Multiaxial Loading Fracture of $Al_2 O_3$ Tubes: II, Weibull Theory and Analysis, J. Amer. Ceram. Soc. 67 (1984) 1, 18-23

/8/ Matsuo, Y.
Statistical Fracture Theory for Multiaxail Stress States Using Weibull's Three-Parameter Function, Eng. Fract. Mech. 14 (1981) 527-538

/9/ Sih, G. C.; Mac Donald, B.
Fracture Mechanics Applied to Engineering Problems-Strain Energy Density Fracture Criterion, Eng. Fract. Mech. 6 (1974) 361-386

/10/ Jayatilaka, A. de S.
Fracture of Engineering Brittle Materials, Applied Science Publishers Ltd., London 1979

/11/ Broutman, L. J.; Krishnakumar, S. M.; Mallik, P. K.
Effects of Combined Stresses on Fracture of Alumina and Graphite, J. Amer. Ceram. Soc. 53 (1970), 12, 649-654

/12/ Tappin, G.; Davidge, R. W.; Mc Laren, J. R.
The Strength of Ceramics Under Biaxial Stresses, R. C. Bradt; D.P.H. Hasselman; F.F. Lange (Editors), Fracture Mechanics of Ceramics, 3 (1978) 435-449

/13/ Stout, M. G.; Petrovic, J. J.
Multiaxial Loading Fracture of Al_2O_3, Tubes: I, Experiments, J. Amer. Ceram. Soc. 67 (1984) 1, 14-18

/14/ Georgiadis, C.
Probability of Failure Models in Finite Element Analysis of Brittle Materials, Comp. & Struct. 18 (1984) 3, 537-549

/15/ Mörgenthaler, K. D.
Reliability Analysis of Ceramic turbine wheels, Fortschrittsberichte DKG 1 (1985) 3, 37-49

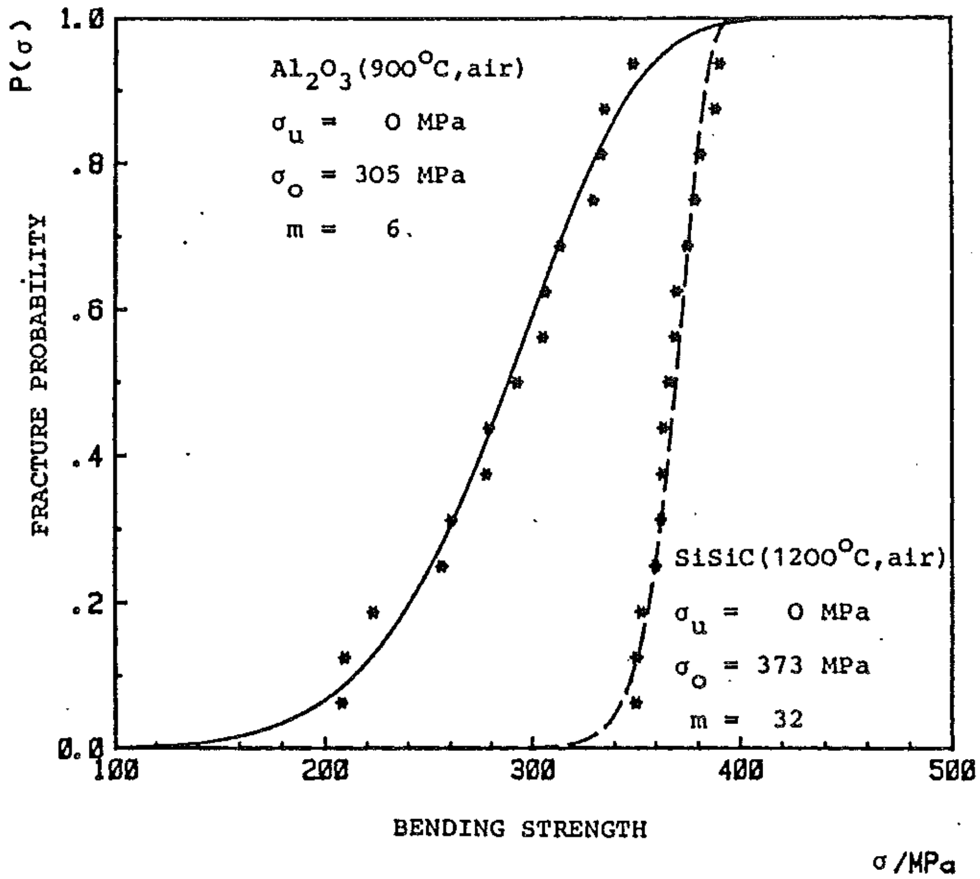

Fig. 1 Fracture probability versus bending strength

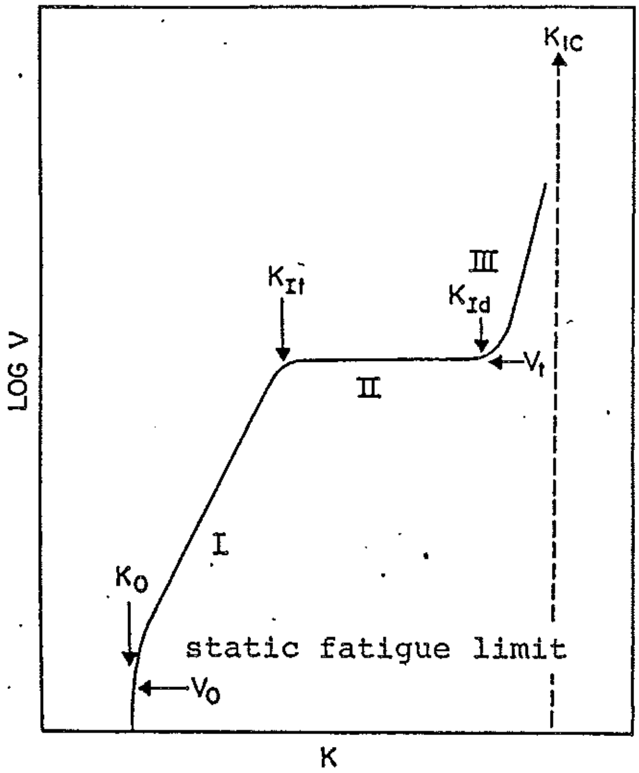

Fig. 2 Schematic relationship between crack velocity v and stress intensity factor K_I

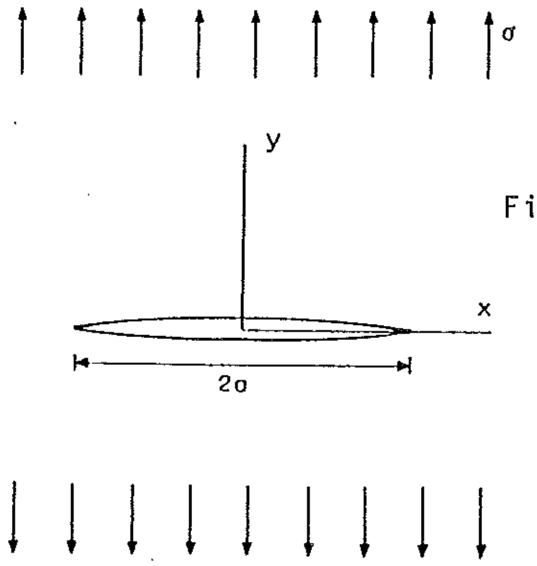

Fig. 3 Mode I crack

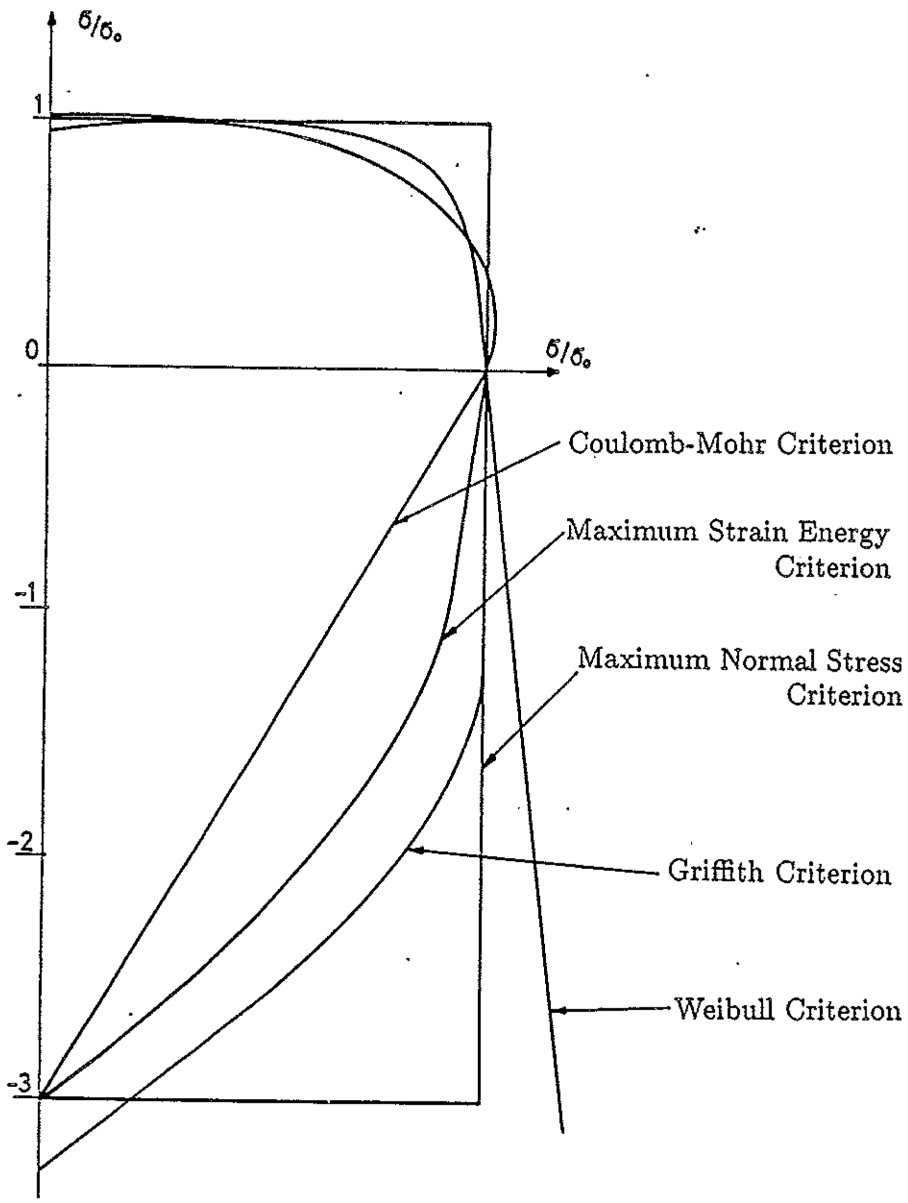

Fig. 4 Failure criteria for ceramic materials

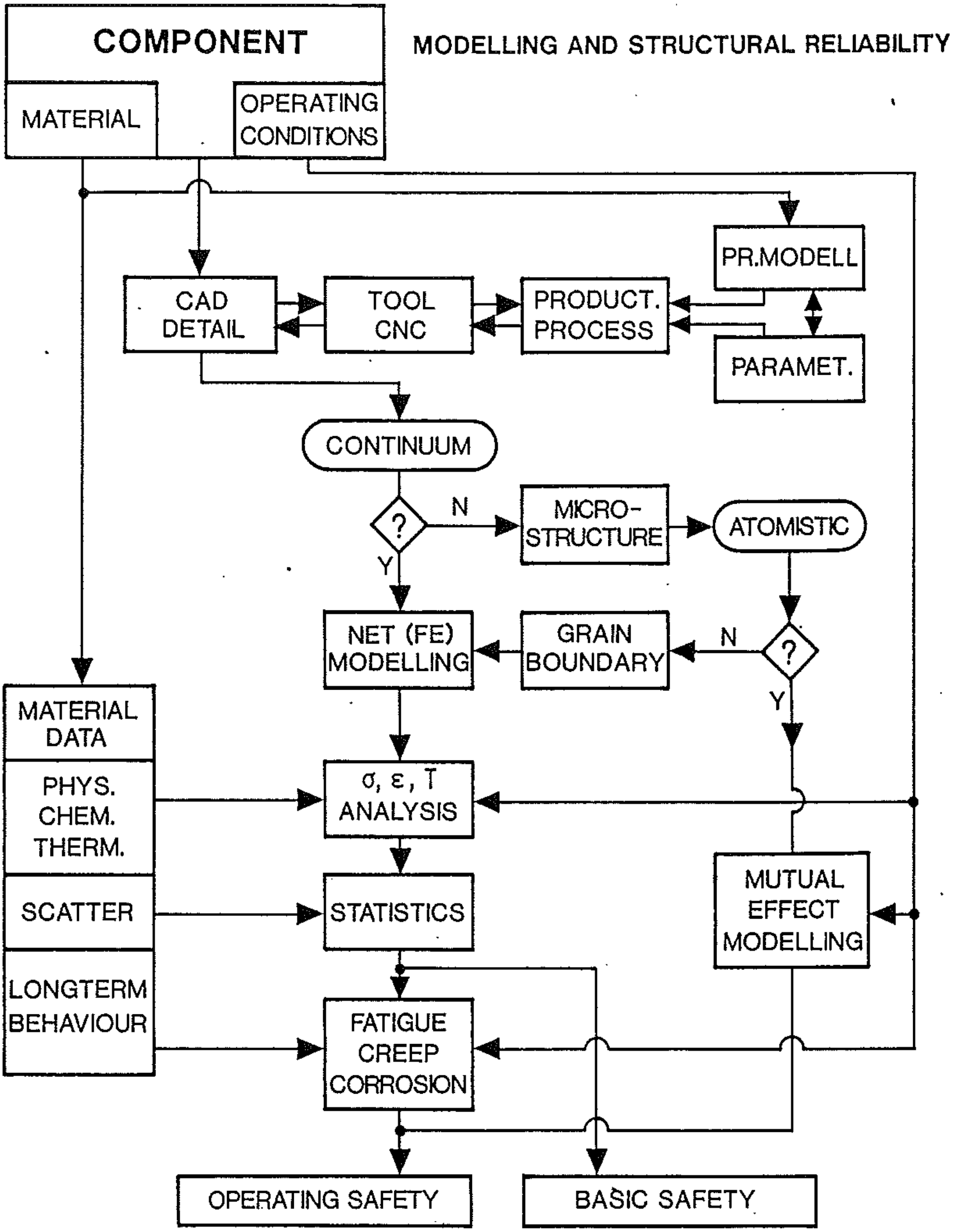

Fig. 5 Flow chart 'Component modelling and structural reliability'