# Multiaxial Fatigue under Constant and Variable Amplitude Loading

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## Abstract

A new testing unit for combined bending and torsion for round bar specimens is described. Pure bending, pure torsion and in-phase combinations are possible with only one hydraulic actuator by means of levers and elastic sheets.
Fatigue tests were performed with smooth specimens of 30 Cr Ni Mo 8 steel at room temperature. The test programme includes several combinations of bending and torsion with constant amplitude and with a standard random load sequence of Gaussian type. The data from the fatigue tests under variable amplitude loading are compared with the prediction according to the Palmgren-Miner-rule.

## Introduction

Despite the fact that a large amount of research has been performed to determine the causes of damage, no essential improvement of the prediction capability of cumulative damage to fatigue life time for multiaxial loaded parts has been achieved. A large number of multiaxial experiments with constant amplitude loading have led only to models for calculating the fatigue endurance limit [1]. The possibilities for transforming results from experiments with constant amplitude loading to those with variable amplitude loading are insufficient. Therefore, suitable fatigue life calculations for variable amplitude loading are not yet known [2 - 9]. In a first step, fatigue behaviour of smooth specimens under combined, but in-phase bending and torsion has been investigated, and a comparison between fatigue behaviour under constant and variable amplitude loading has been performed. In a second step, which is not yet terminated, the influence of out-of-phase combinations and different frequencies will be worked out.

**Multiaxial Test Unit**

A special multiaxial test unit for in-phase combined loading by bending and torsion has been designed for a uniaxial fatigue test machine, which means that only one hydraulic actuator is necessary to perform both loadings (fig. 1). The specimen is not fixed in the load frame by bearings; instead, elastic sheets which also absorb the reacting deformations and avoid any tolerances and friction are employed. The main part of this test unit is a variable frame, called vario-frame, (fig. 2), with different fixing points for the elastic sheets to realize pure bending, pure torsion or any combinations of them by varying their levers. The round bar specimen is clamped elastically by a cone grip. The dynamic calibration has shown linear correlation between force and bending and torsion up to frequencies of about 20 Hz.

**Experimental Programme**

Fatigue tests were carried out on smooth round bar specimens. The mechnical properties were: tensile strength: 1017 $N/mm^2$ and yield stress: 812 $N/mm^2$. The material used in this investigation was 30 Cr Ni Mo 8 steel.
The experimental programme is summarized in figure 3. Three test series are of constant amplitude loading ($\sigma_a$, $\tau_a$, $\tau_a = 0{,}5\,\sigma_a$) and five test series of variable amplitude loading ( $\hat{\sigma}_a$, $\hat{\tau}_a$, $\hat{\tau}_a = 0{,}19 \cdot \hat{\sigma}_a$; $\hat{\tau}_a = 0{,}5 \cdot \hat{\sigma}_a$; $\hat{\tau}_a = 0{,}73 \cdot \hat{\sigma}_a$). The time function is a standard random load sequence of Gaussian type generated from a formula derived by Kowalewski [10] (fig. 4). The resulting standard load spectra are shown in figure 5.

**Test Results**

The results of the fatigue tests are shown in figures 6 to 9. The low inclinations of the S-N- curves and life-time-curves result from the unnotched specimens. When compared with the life-time predictions according to the Miner-rule, the life-time-curves are situated on the unconservative side. The factor depends on the level of the load and kind of loading and differs from about 5 by bending to about 1.5 by torsion.

Figure 9 summarizes all experiments performed in this investigation in one life-time-diagramme. It can be seen that the test results with constant amplitude loading and even the results with variable amplitude loading follow an elliptical curve very exactly. This is derived from the life time values for pure bending along the abscissa and those for pure torsion on the ordinate. The curves are not parallel.This results from the different inclinations of the S-N-curves and life-time-curves for bending and torsion. The respective inclinations for combined loading can be determined from these ellipses, for example between $\hat{N} = 10^6$ and $\hat{N} = 10^7$.

## Conclusions

The objektive of this work was to observe the fatigue behaviour of combined bending-torsion under variable amplitude loading. In comparison with uniaxal test series and constant amplitude test series, the following statements for a constant ratio $\hat{\tau}_a$ / $\hat{\sigma}_a$ are possible:

1. Relative-normal-stress-concept (fig. 10)
   If the S-N-curves for bending and for torsion are known, an estimation of the combined S-N-curves (fatigue strength and inclination) according to the ellipses in the fatigue life diagramme shown can be made. With the load spectrum of the classified time function, a damage accumulation calculation according to the Palgren-Miner-rule is possible; nevertheless a correction between $\hat{N}_{experiment}$ and $\hat{N}_{calculation}$ is necessary, as usual.

2.) Relative-life-time-concept (fig. 11)

   If the life-time-curves for bending and for torsion for the random load sequence are known (or can be well determined), an estimation of the combined life-time-curves is possible in the same manner as shown in the relative-normal-stress-concept for the combined S-N-curves. The ellipse formula with a life time for example, for $\hat{N} = 10^7$ and $\hat{N} = 10^6$ for $\hat{\tau}_a$ and $\hat{\sigma}_a$, allows calculation of life-time-curves for any bending-torsion combinations.

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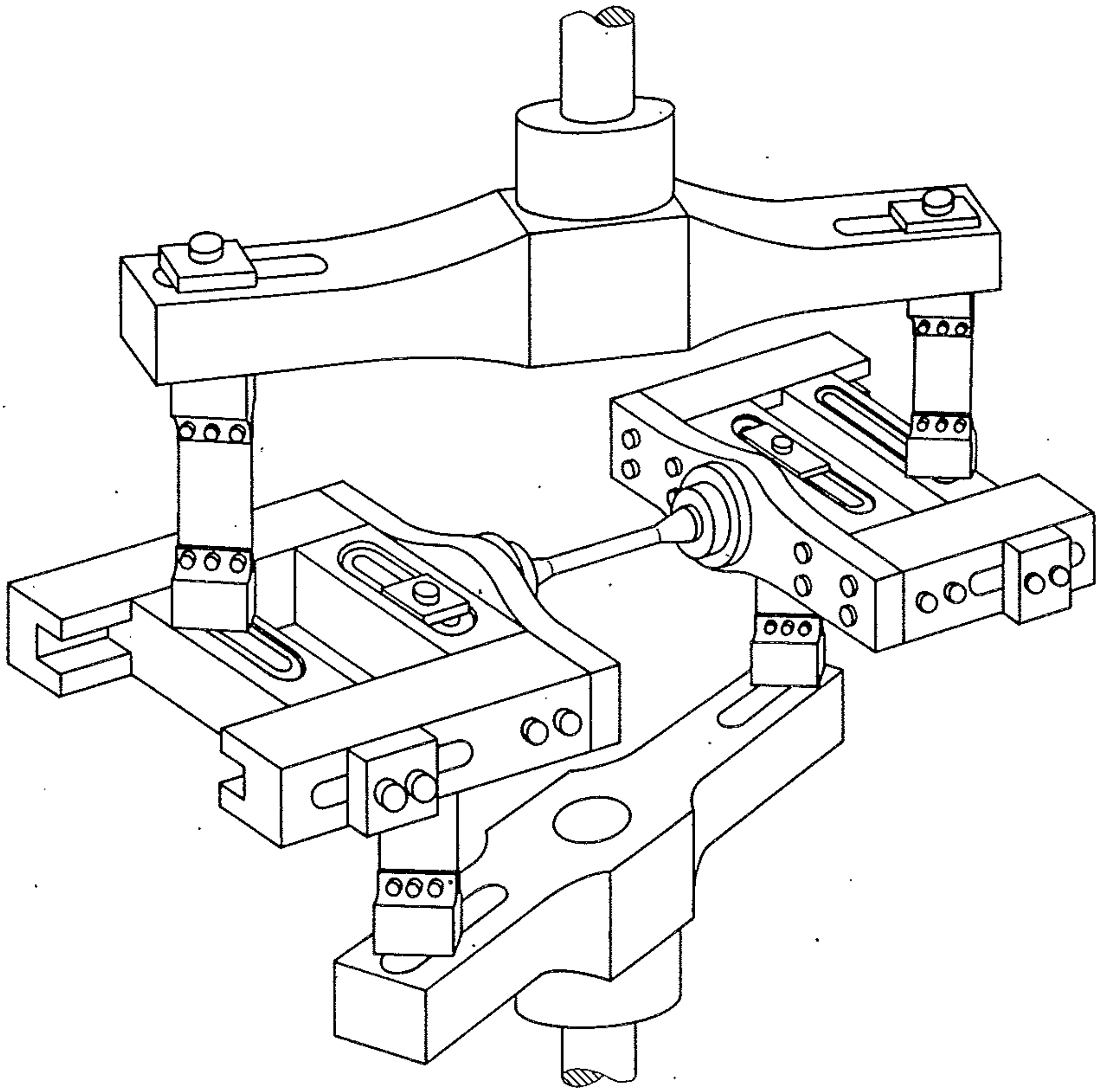

Fig. 1: Multiaxial test unit for in-phase combined loading by bending and torsion

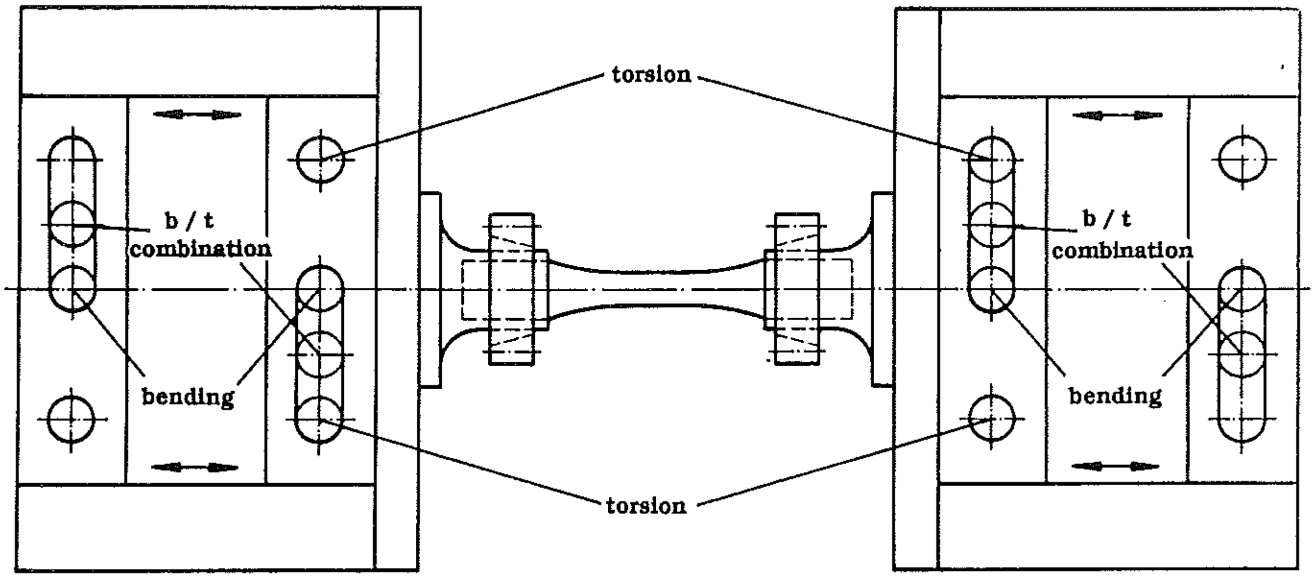


Fig. 2: Vario-frame for bending, torsion and combined loading

| Experiments | VR Nr. | $\hat{\sigma}_a$ | $\hat{\tau}_a$ | | | $\sigma_a$ | $\tau_a$ |
|---|---|---|---|---|---|---|---|
| Random loading | 1.1 | + | 0 | Constant amplitude loading | | | |
| life-time | 1.2 | 0 | + | | | | |
| | 1.3 | + | 0,19 $\hat{\sigma}_a$ | number of cycles | 2.1 | + | 0 |
| $\hat{N} => 10^6$ | 1.4 | + | 0,50 $\hat{\sigma}_a$ | $N = 5 \cdot 10^4$ | 2.2 | 0 | + |
| | 1.5 | + | 0,73 $\hat{\sigma}_a$ | to $5 \cdot 10^6$ | 2.3 | + | 0,5 $\sigma_a$ |

Fig. 3: Experimental programme for constant and variable amplitude loading

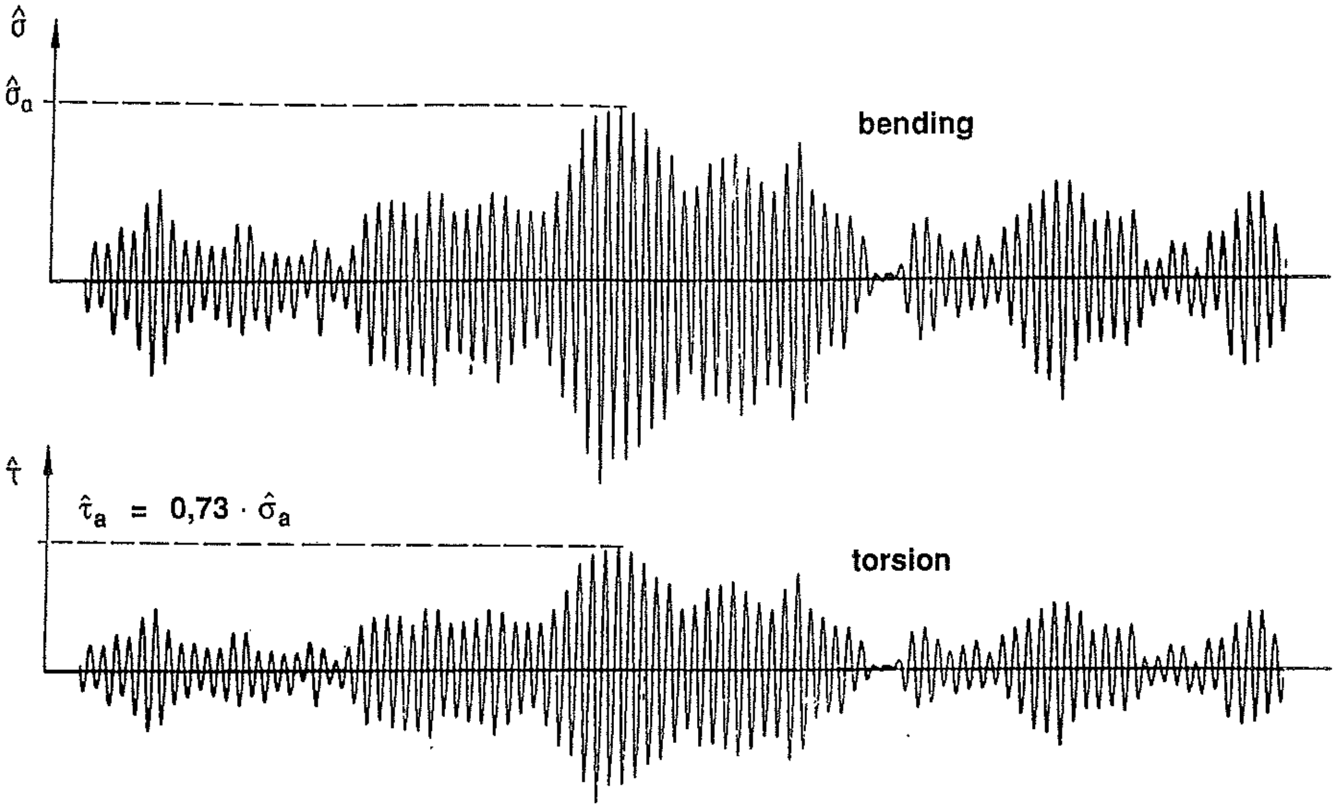


Fig. 4: Standard random load sequence of Gaussian type generated for $\hat{\tau} = 0{,}73\,\hat{\sigma}_a$

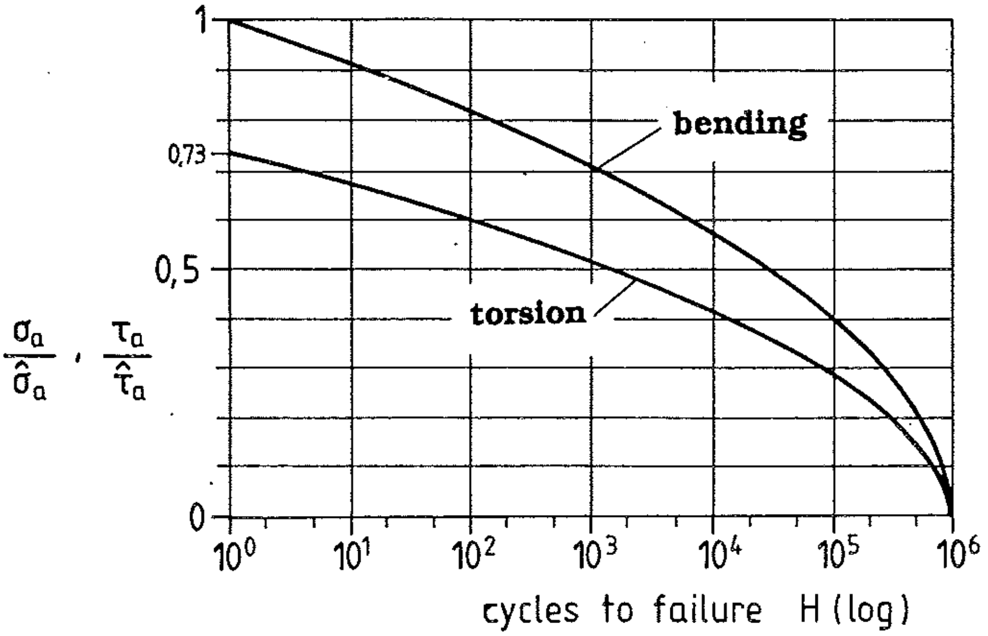


Fig. 5: Standard load spectra for bending and torsion for $\hat{\tau}_a = 0{,}73\,\hat{\sigma}_a$

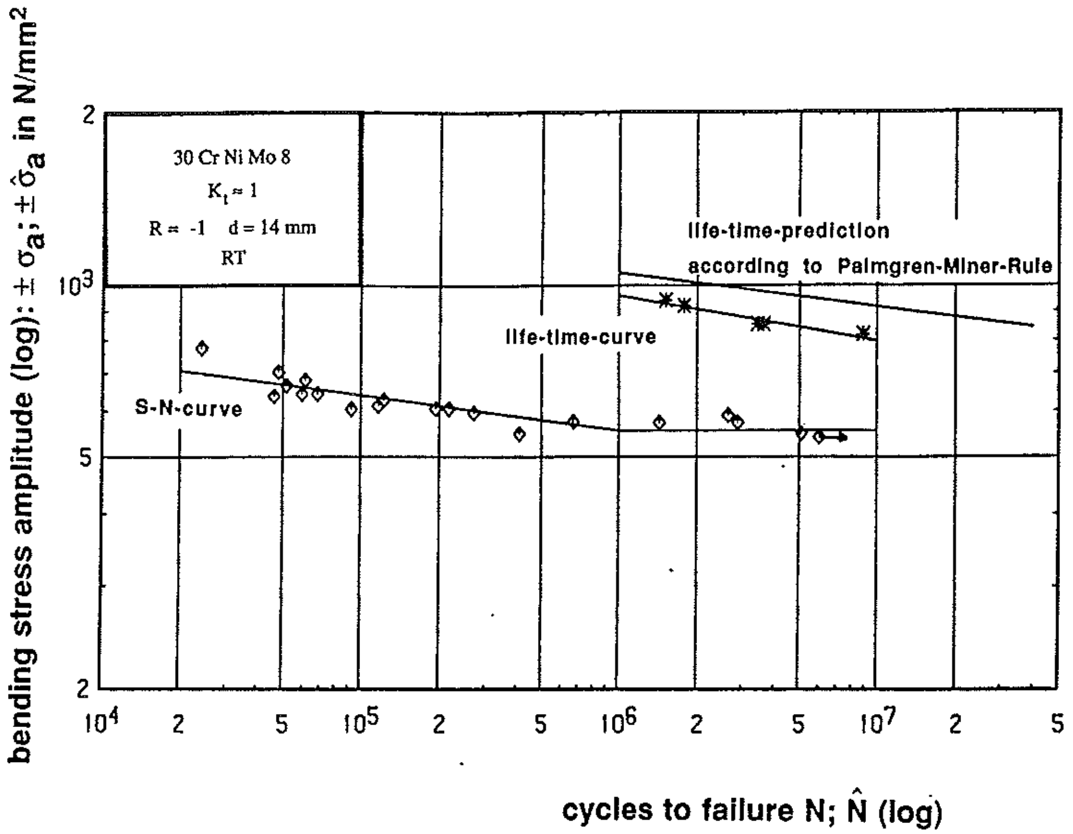


Fig. 6: S-N-curve, life-time-curve and life-time prediction according to Palmgren-Miner-rule for bending

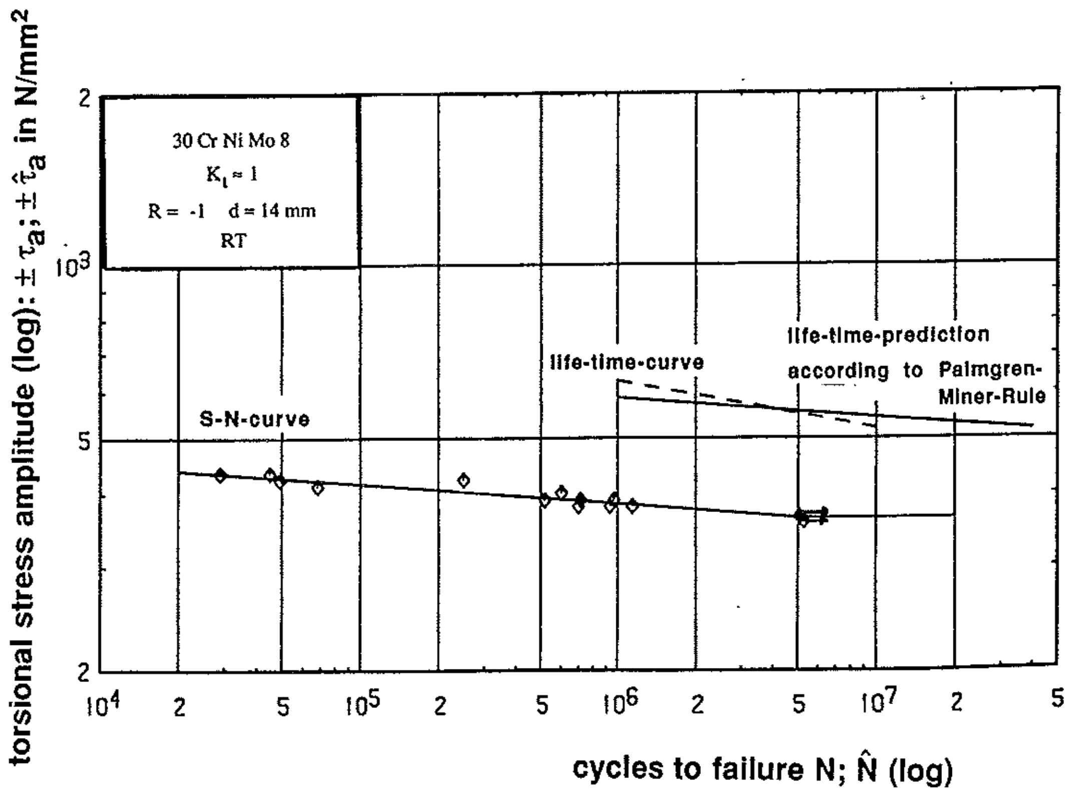


Fig. 7: S-N-curve, life-time-curve and life-time prediction according to Palmgren-Miner-rule for torsion

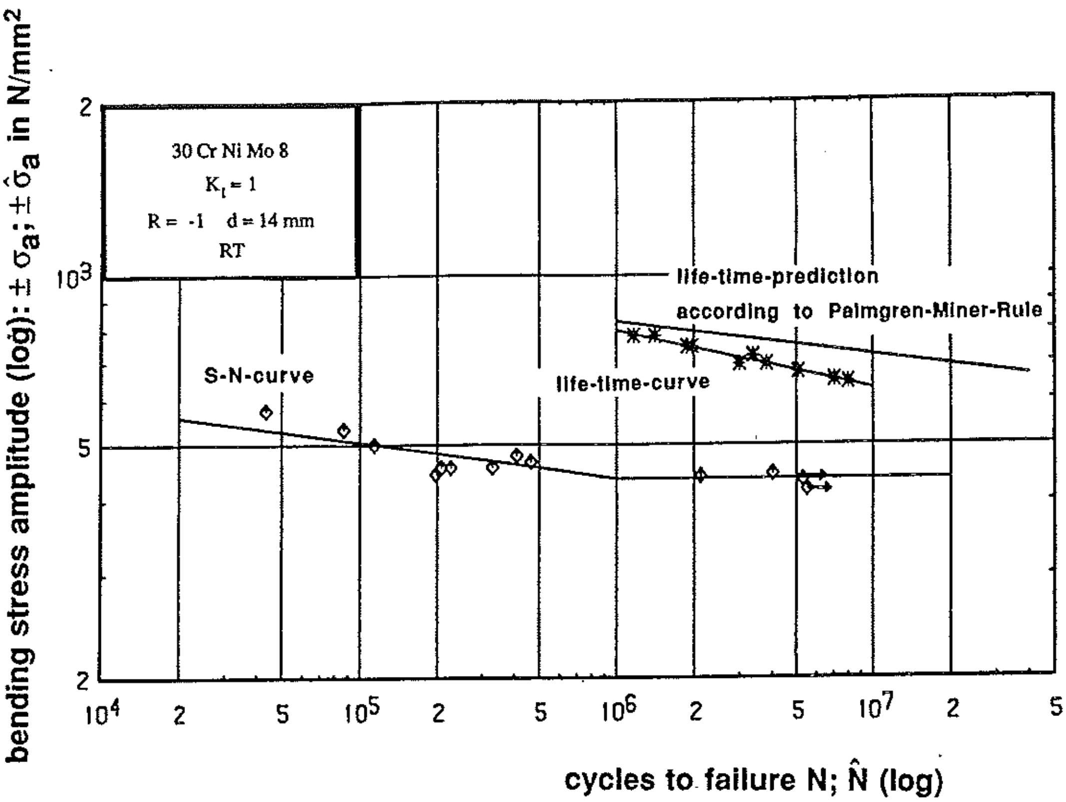


Fig. 8: S-N-curve, life-time-curve and life-time prediction according to Palmgren-Miner-rule for combined loading $\hat{\tau}_a = 0{,}5\,\hat{\sigma}_a$

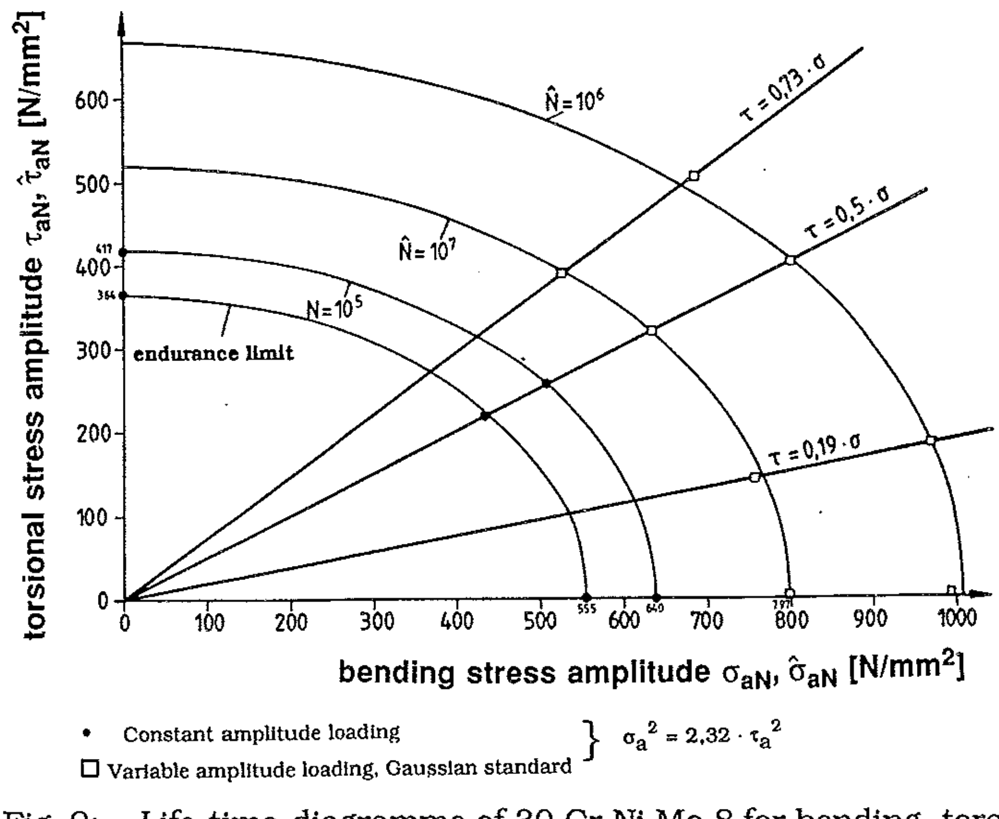


Fig. 9: Life-time-diagramme of 30 Cr Ni Mo 8 for bending, torsion and combined loading

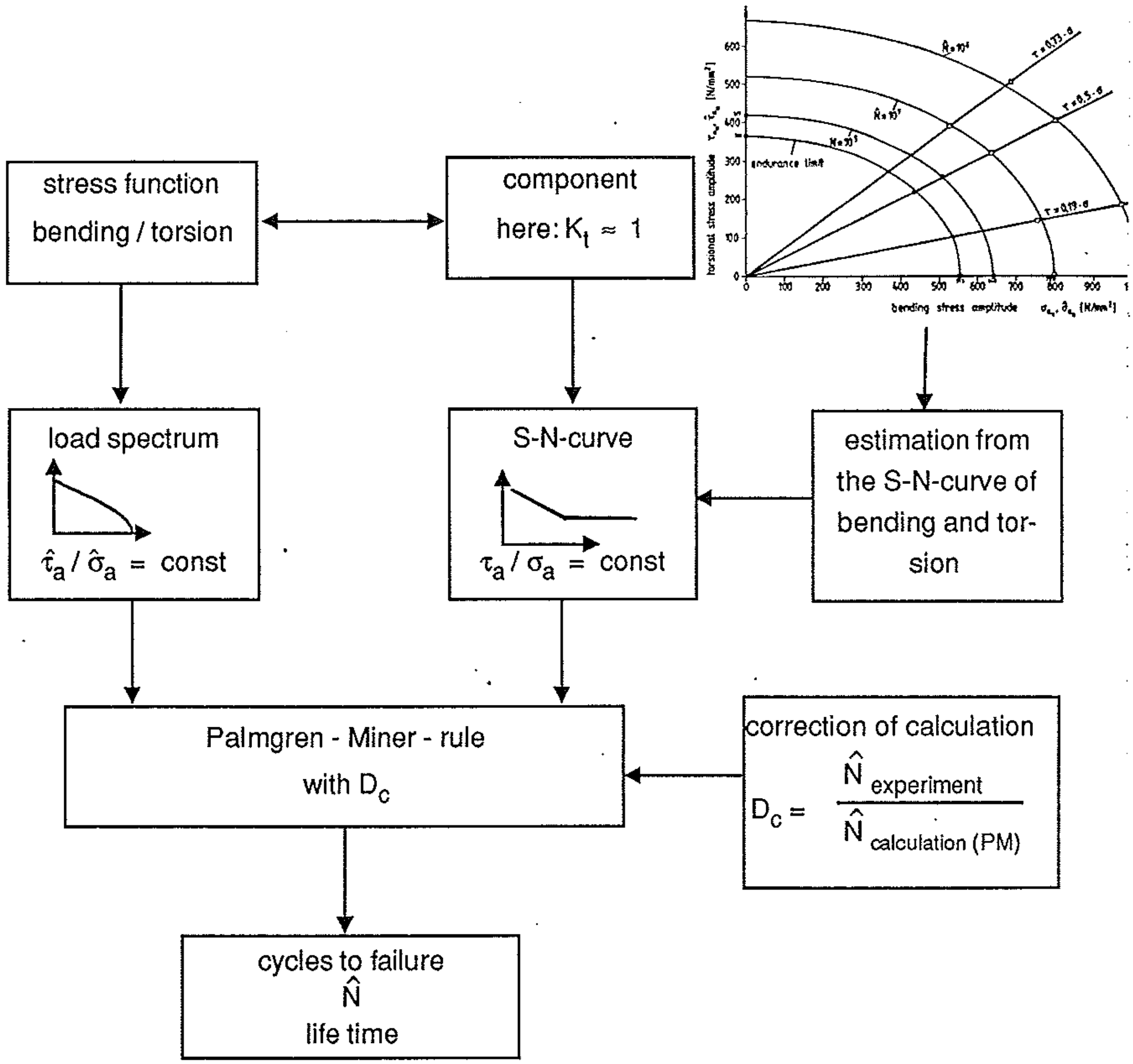


Fig. 10: Relative-normal-stress-concept for a constant ratio of $\hat{\tau}_a / \hat{\sigma}_a$

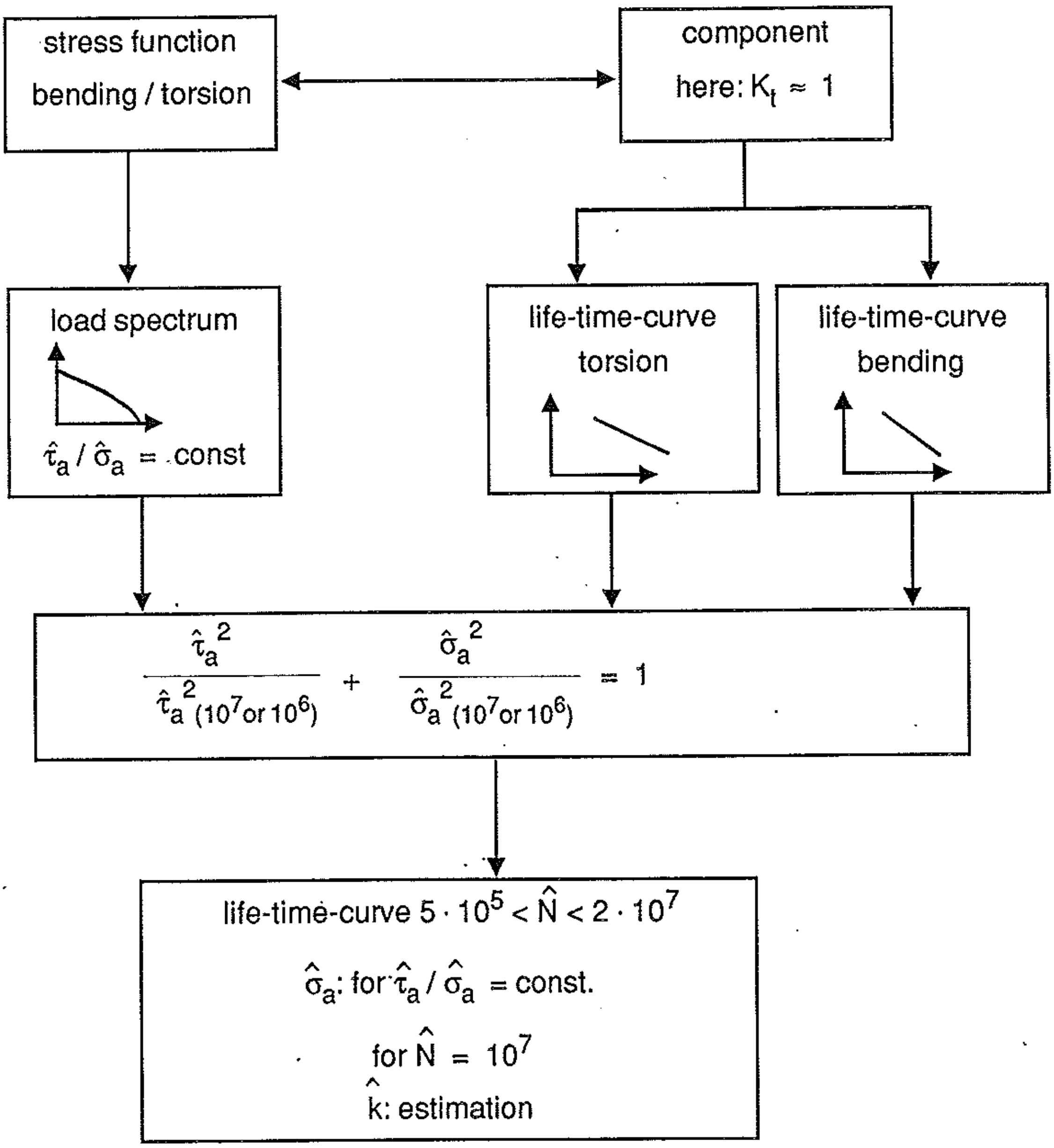


Fig. 11: Relative-life-time-concept for a constant ratio of $\hat{\tau}_a / \hat{\sigma}_a$