FATIGUE LIFE PREDICTION OF A NOTCHED SPECIMEN SUBJECT TO TORSION: THEORETICAL AND EXPERIMENTAL RESULTS

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ABSTRACT

A finite element program has been developed to examine the behaviour of axisymmetric structures subject to torsion. Output from finite element analysis is utilized as input to fatigue life prediction programs based on the local strain approach. Stress raisers are accounted for by Neuber's rule or by load-local strain curve. Torsional tests with constant or variable loading amplitude on notched specimens have been performed. Good correlation have been found between experimental results and theoretical predictions.

Symbols

- M_a torque amplitude;
- r_0 radius of the net section of the specimen of fig. 2.2;
- τ_a local shear stress amplitude on the specimen surface;
- γ_a local shear strain amplitude on the specimen surface;
- T_a nominal shear stress amplitude on the specimen surface;
- Γ_a nominal shear strain amplitude on the specimen surface;
- θ_a amplitude of the angle of twist per unit length of the specimen;
- N cycles to failure;
- G shear modulus;
- I_ρ polar moment of inertia;
- K_t theoretical stress concentration factor in torsion;
- K_τ effective stress concentration factor in torsion;
- K_γ effective stress concentration factor in torsion;
- K_f fatigue notch factor in torsion.

1. - INTRODUCTION

The influence of stress and strain concentrations on fatigue life strength of machine components has been studied since the end of the last century. The introduction of finite life design has focused the attention of researchers on low cycle fatigue. In this field strain becomes obviously the leading parameter [1]. The knowledge of the strain history experienced in some areas of stress concentration is a very useful tool to evaluate the life expense.

Aim of this work is to predict fatigue life of components subject to applied torque histories utilizing the local strain approach [2].

Low cycle fatigue tests have been performed on notched specimens in torsion. Tests have been carried out in load control with constant or variable torque amplitudes.

Specimens utilized simulate the stress and strain concentrations usually found in shafts connecting turbine to alternators in large electrical power plants. The samples have been analysed theoretically utilizing a finite element program laid out to study axisymmetric structures subject to antisymmetric loading.

The code has been tested in the elastic field comparing the results with data deriving by literature (Peterson). Numerical results have been summarized in torque-local strain amplitude curves.

The values obtained by the finite element analysis can be used as input for two life prediction programs, previously developed, based on Neuber's rule or on both load-local strain curve and Wetzel matrix methods.

Satisfactory comparison between theoretical life predictions and experimental results have been found.

2. - FINITE ELEMENT PROGRAM TO ANALYSE AN AXISYMMETRIC STRUCTURE SUBJECT TO TORSION

The theory corresponding to the finite element of axisymmetric structures subject to antiplane loading is utilized. Both elastic and plastic behaviour are considered. Results corresponding to a notched specimen subject to pure torsion are then exposed.

Referring to the geometry considered in fig. 2.1, it is assumed that plane sections remain plane after load being applied. It follows:

$$u = w = 0 \qquad v = v(\Theta)$$

Strain can be expressed as:

$$\underline{\varepsilon} = \begin{bmatrix} \gamma_{r\theta} \\ \\ \gamma_{z\theta} \end{bmatrix} = \begin{bmatrix} (\partial v / \partial r) - (v / r) \\ \\ (\partial v / \partial z) \end{bmatrix}$$

Stresses are:

$$\underline{\sigma} = \begin{bmatrix} \tau_{r\theta} \\ \\ \tau_{z\theta} \end{bmatrix} = \underline{\underline{D}}\,\underline{\varepsilon} = \frac{E}{2(1+\nu)} \begin{bmatrix} 1 & 0 \\ \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \gamma_{r\theta} \\ \\ \gamma_{z\theta} \end{bmatrix}$$

For an ispoparametric element with N nodes:

$$\underline{\varepsilon} = \begin{bmatrix} (\partial N_i / \partial r) - (N_i / r) & \ldots\ldots. \\ \\ (\partial N_i / \partial z) & \ldots\ldots. \end{bmatrix} \begin{bmatrix} v_1 \\ \vdots \\ v_N \end{bmatrix} = \underline{\underline{B}}\,\underline{\Delta}$$

where N_i are the shape functions of the element.
In the elastic field the stiffness matrix is expressed as [3]:

$$\underline{\underline{K}} = \iint \underline{\underline{B}}^T \underline{\underline{D}}\ \underline{\underline{B}}\ 2\pi r\, dr\, dz = \iint \underline{\underline{B}}^T G\ \underline{\underline{B}}\ 2\pi r \det \underline{\underline{J}}\ d\xi\, d\eta$$

where $r = \sum N_i r_i$.
In the plastic field, the stiffness matrix is found utilizing Prandtl-Reuss incremental theory. Equivalent stress (σ_{eq}) is evaluated according to the Von Mises criterion. Isotropic strain hardening is assumed. The incremental equilibrium equation is [4]:

$$\left(\iint \underline{\underline{B}}^T \underline{\underline{D}}_{eq}\ \underline{\underline{B}}\ dr\, dz \right) \underline{d\Delta} = \underline{R}(t+h) - \iint \underline{\underline{B}}^T \underline{\sigma}(t)\, dr\, dz$$

or:

$$\underline{\underline{K}}_T(t)\ \underline{d\Delta} = \underline{R}(t+h) - \underline{F}_i(t) \tag{2.1}$$

where:

$$\underline{\underline{D}}_{eq} = \underline{\underline{D}} - \frac{\underline{\underline{D}}\ \underline{q}\ \underline{q}^T\ \underline{\underline{D}}}{\underline{p}^T\ \underline{q} + \underline{q}^T\ \underline{\underline{D}}\ \underline{q}}$$

$$\underline{p} = E_p \frac{\underline{\sigma}}{\sigma_{ys}} = \frac{E_p}{\sigma_{ys}} \begin{bmatrix} \tau_{r\theta} \\ \tau_{z\theta} \end{bmatrix}$$

$$\underline{q} = (3/2) \frac{\underline{\sigma}'}{\sigma_{eq}} = \frac{\sqrt{3}}{\sigma_{eq}} \begin{bmatrix} \tau_{r\theta} \\ \tau_{z\theta} \end{bmatrix}$$

$$E_p = \frac{E\,E_T}{E - E_T}$$

E_T is the tangent Young modulus of the discretized (trilinear) stress-strain curve, σ_{ys} is the yield strength, R (t+h) and $F_i(t)$ represent the external and internal loadings.

The non linear equation (2.1) can be solved according to the Newton-Raphson iterativemethod.

An "ad hoc" finite element program running on VAX 780 has been written according to the above mentioned procedure. The code has been tested utilizing the specimen geometry of fig. 2.2. The mesh of 70 isoparametric elements of 8 nodes is shown in fig. 2.3. Fig. 2.4 shows the material curve utilized as input to the program. Torque-local strain curve is reported for the specimen of fig. 2.2. The corresponding curve for a smooth cylindrical specimen with the same net section is shown as a comparison (fig.2.5).

3. - VERIFICATION OF THE RELIABILITY OF NEUBER'S RULE

If Neuber's rule is assumed to be valid, it results for the actual problem:

$$K_\tau K_\gamma = K_t^2 \tag{3.1}$$

where:

$$K_\tau = (\tau_a / T_a) \quad K_\gamma = (\gamma_a / \Gamma_a)$$

where:

$\tau_a = G\gamma_{ae}$ is the local shear stress amplitude
$T_a = G\Gamma_{ae}$ is the nominal shear stress amplitude

and:

γ_{ae} is the local elastic strain amplitude
Γ_{ae} is the nominal elastic strain amplitude

Equation (3.1) may be rewritten:

$$K_t^2 = (\gamma_{ae}/\Gamma_{ae})(\gamma_a/\Gamma_a) \tag{3.2}$$

The finite element program gives, as output, shear strain (γ) values as a function of torque values (M). It is straigthforward to compare the curve $M = f(\gamma)$ corresponding to the notched specimen (fig. 2.2) and the curve $M = g(\gamma)$ corresponding to the smooth cylindrical specimen with cross section equal to the net section of the notched one. Results are shown in fig. 3.1. Values of K_t^2 obtained according to equation (3.2) are reported. In the elastic range it should result $K_t^2 = (1,47)^2 = 2,16$, while the computed value is about 1,85. It must be considered, however, that evaluations have been made in Gauss points without extrapolating the results to the surface. This can be misleading expecially for the notched specimen.

4.1 - EXPERIMENTAL TESTS AND MATERIAL UTILIZED

Low cycle fatigue tests have been performed in torsion on notched specimens (fig. 2.2) characterized by a theoretical stress concentration factor (K_t) of 1,47. Experiences have been made in load control. Seventeen different torque histories have been considered. Seven of them have constant torque amplitude; they have been applied until specimen failure (it is considered as failure a fixed percentage decrease in the load amplitude). Four histories are constituted of four blocks of decreasing torque amplitude; each block has a preset constant load amplitude and the whole history is applied several times until failure. The last six histories are composed of only two blocks of decreasing torque amplitude; the second block is applied until failure.
An Instron series 8000 hydraulic testing machine has been utilized together with a testing rig able to convert alternate linear motion of the actuator to alternate torsion [5].
The rotor steel utilized have been previously characterized through push-pull and torsion low cycle fatigue tests (chemical composition and mechanical properties are reported in [6]).

4.2 - LIFE PREDICTION NUMERICAL PROGRAMS

Two numerical programs previously developed [5] have been considered; they predict fatigue life of components subject to periodical loading histories utilizing the local strain approach [2].

They assume:
a) a counting method to reduce complex loading histories to a simple superposition of constant amplitude closed loops;
b) a damage accumulation criterion to evaluate the life reduction due to the simultaneous application of the previously identified loops;
c) analytical models for cyclic and fatigue curves of the material;
d) a method to account for stress and strain local concentrations.

a) Both programs utilize a "rain-flow" counting method modified to account for the "memory effect" of the material; the peculiarity of "rain-flow" is that it considers each peak-valley range only once, it assumes each minor peak-valley range as an interruption of the larger ones and it identifies as cycles those pairs of peaks and valleys corresponding to closed loops in the stress-strain plot relative to local areas of strain concentrations [2,7].

b) Miner's rule is chosen as the damage accumulation criteria; the approximations which affect the fatigue results often do not justify the use of more sophisticated methods that can override the shortcomings of Miner's rule only by introducing much more involved calculations or by determining further experimental parameters.

c) An extension of Ramberg-Osgood's equation has been utilized as a mathematical model for the shear stress-shear strain curve:

$$\gamma_a = (\tau_a/G) + (\tau_a/H')^{(1/n')}$$

The following equation has been employed for the fatigue curve in torsion (which is the analogous of the expressions developed by Manson, Coffin and Basquin in uniaxial low cycle fatigue [1]):

$$\gamma_a = A\,(2N)^{\alpha} + B\,(2N)^{\beta} \qquad (4.1)$$

d) Program FATICA 3 considers an extension to torsion [8, 9] of the Neuber's rule [10]:

$$\tau_a\,\gamma_a = K_t^2\,T_a\,\Gamma_a$$

to account for stress and strain raisers.

Program FATICA 4 utilizes a torque-local strain curve to analyse the stress and strain concentration zones in a component (§ 4.3).

4.2.1 - Numerical program based on Neuber's rule (FATICA 3)

Nominal stress history is analysed, peak to peak, through the "rain-flow" algorithm starting by the largest peak or valley; a subroutine simultaneously predicts the local stress-strain hysteresis history.

The first peak (or valley) results by solving the system between analytical expressions of the stress-strain curve and Neuber's hyperbola:

$$\tau_a\,\gamma_a = K_t^2\,T_a\,\Gamma_a$$
$$\gamma_a = (\tau_a/G) + (\tau_a/H')^{(1/n')}$$

where:

$$\Gamma_a = (T_a/G) + (T_a/H')^{(1/n')}$$

If the hypothesis of homothety between cyclic and hysteresis curves is assumed [11], all the remaining peaks (or valleys) can be obtained by the solution of the system:

$$\Delta\tau\,\Delta\gamma = K_t^2\,\Delta T\,\Delta\Gamma$$
$$\Delta\gamma/2 = (\Delta\tau/2G) + (\Delta\tau/2H')^{(1/n')}$$

The strain amplitude and the mean stress are obtained for each closed loop identified by "rain-flow" counting method; total damage results as the summation of the partial damage of each cycle (eq. 4.1) according to Miner's criterion. "Rain-flow" algorithm has been modified to account for the "memory effect" of the material [12].

4.2.2 - Nominal stress determination

Stress concentration factor is defined as the ratio between local notch stress and nominal stress. Nominal stress is defined in torsion (elastic field) as:

$$T_a = 2M_a/\pi r_o^3$$

by considering the net section of the specimen.
If the stress amplitude is too large even the nominal stress becomes larger than the yield strength. Nevertheless the following equation developed by Nadai can be utilized to obtain a significant nominal stress:

$$T_a = (1/2\pi r_o^3)\,[\theta_a\,(dM_a/d\theta_a) + 3\,M_a] \qquad (4.2)$$

If the amplitude of the angle of twist per unit length (θ_a) is expressed as a function of the torque amplitude (M_a) by means of:

$$\theta_a = M_a/GI_p + (M_a/H_o)^{(1/n_o)} \qquad (4.3)$$

the equation (4.2) can be rewritten:

$$T_a = (M_a/2\pi r_o^3)\,\{3 + \theta_a/[M_a/GI_p + (1/n_o)\,(M_a/H_o)^{(1/n_o)}]\} \qquad (4.4)$$

Expression (4.4) allows the nominal stress to be obtained directly by the torque amplitude.
Values of H_0 and n_0 can be evaluated by tests on smooth specimens with cross sections equal to the net sections of the notched samples, in a very similar way to that followed for the determination of the cyclic stress strain curves [2]. In this case the M_a-θ_a curve is not available experimentally but it can be obtained by finite element, starting from the torque-shear strain of smooth specimens (fig. 2.5), or theoretically, utilizing the expression also due to Nadai:

$$M_a = (2\pi/\theta_a^3) \int f(\gamma)\,\gamma^2\,d\gamma$$

which allows the determination of the M_a - θ_a curve by the knowledge of the τ_a - γ_a curve relative to the steel considered [6].

4.3 - Program utilizing the load-strain curve (FATICA 4)

The program utilizes directly the torque-local strain curve. The latter can be obtained by several methods.

a) Experimental method

The curve results as the locus of the tips of the stable hysteresis loops corresponding to different torque value in the M_a - γ_a plot. The method needs the local strain amplitude being measured accurately. This is not always feasible.

b) Finite element

The M_a - γ_a curve can be easily derived by the finite element analysis of the component.

The mathematical approximation of the load-local strain curve is obtained. The curve is discretized and the coordinates of the extreme points of the resulting elements are memorized in arrays inside the life prediction program. The first peak of the hysteresis loops in the torque-local strain amplitude plane (M_a - γ_a) can be identified by the value of the first peak of the applied torque history. All the remaining peaks or valleys can be obtained by the knowledge of the applied torque peaks utilizing the already mentioned hypothesis of homothety between hysteresis and cyclic load-local strain curves. An "availability" index is associated to each element in which the load-local strain curve is divided. Fractions or multiples of the elements are considered and treated according to the method of availability matrix introduced by Wetzel [2].

The resulting strain history peaks and valleys are analysed in a similar manner to identify the stress-strain hysteresis loops utilizing discretized local stress-local strain (τ_a - γ_a) curve. The closed loops are individuated (in term of strain ranges) through a "rain-flow" counting method utilizing the already mentioned availability method [2]. Cumulative damage is again obtained by application of Miner's rule.

4.4 EXPERIMENTAL RESULTS AND DISCUSSION

Experimental results against theoretical predictions for the histories described in & 4.1 and for the component of fig. 2.2 are summarized in the graphs of fig. 4.1 and 4.2.

The predictions deriving by the application of program FATICA 3 are even too conservative especially in the low cycle fatigue region. This is in agreement with data of other researchers concerning previsions based on Neuber's rule [1, 2].

Program FATICA 4 is more realistic; theoretical predictions are nearer to the experimental results (with a factor of less then three for the ratio between them) but unfortunately they are not conservative.

If the agreement could be more deeply assessed the use of program FATICA 4 together with a somehow defined safety factor could be the best way to face the problem of life predictions.

In conclusion FATICA 3 and FATICA 4 seem to predict lower and upper bounds for the number of cycles the actual component can sustain.

5. - CONCLUSIONS

The comparison between M_a - γ_a curves relative to notched and smooth specimen shows that Neuber's rule is not completely verified expecially at high values of the applied loading (fig. 3.2). This can, however, be due to the coarse mesh utilized in the finite element analysis.

Nevertheless the application of Neuber's rule (FATICA 3) leads to conservative predictions of fatigue life for the component of fig. 2.2.

The use of the fatigue notch factor ($K_f \cong 1{,}41$) instead of the theoretical stress concentration factor ($K_t \cong 1{,}47$) in the equation 4.2 slightly improves numerical predictions of experimental fatigue lives (fig. 4.3) owing to the closeness between the values of K_f and K_t.

The more realistic predictions of FATICA 4 could be interesting but they are strongly dependent on the rigth determination of the analytical expression of the M_a - γ_a curve. Two methods to obtain it is indicated in this paper, each one with its own shortcomings. In the experimental approach, local strains are not easy to measure for the actual component; the finite element method needs a fine mesh near the strain concentration zones to be realized.
Future work could be the verification of the influence on the results of the alternative application of the Henky or Prandtl-Reuss equations on the results of finite elements analysis. Experimental tests utilizing long periodical histories simulating the real random torsional fatigue cycles sustained by turbine-alternator shafts are in progress; the comparison with theoretical predictions could be useful to better understand the worthiness of the different programs for fatigue life predictions.

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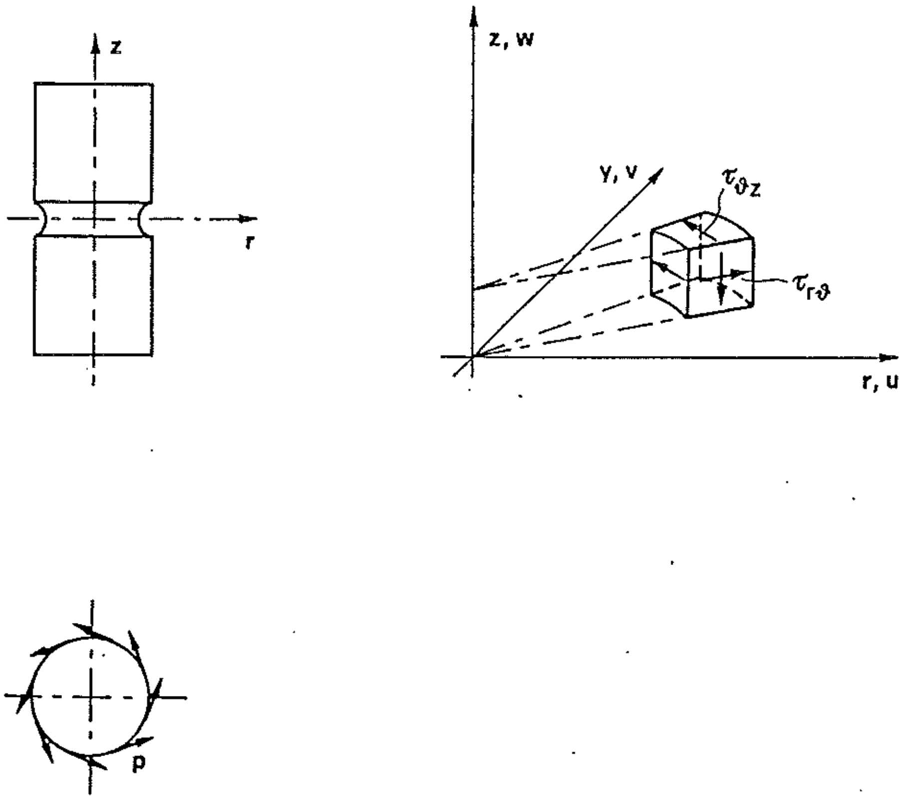


Fig. 2.1 - Model for the specimen in torsion.

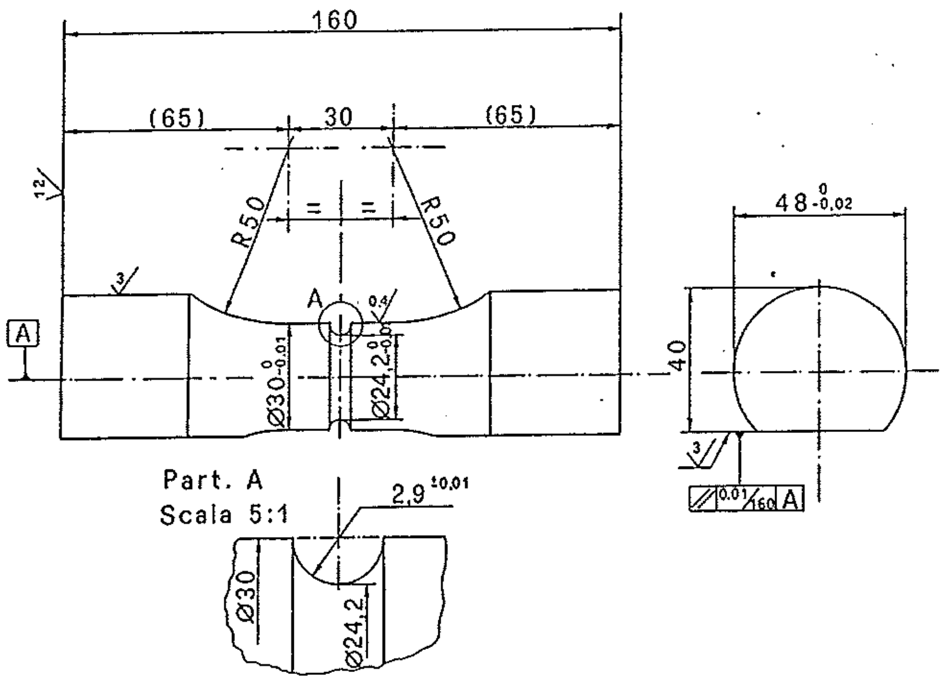


Fig. 2.2 - Torsional specimen $K_t = 1,47$.

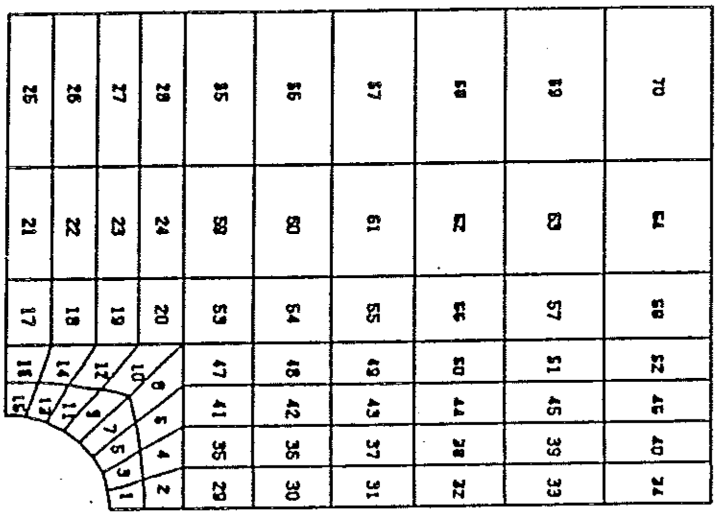


Fig. 2.3 - Discretization of the specimen (finite element program).

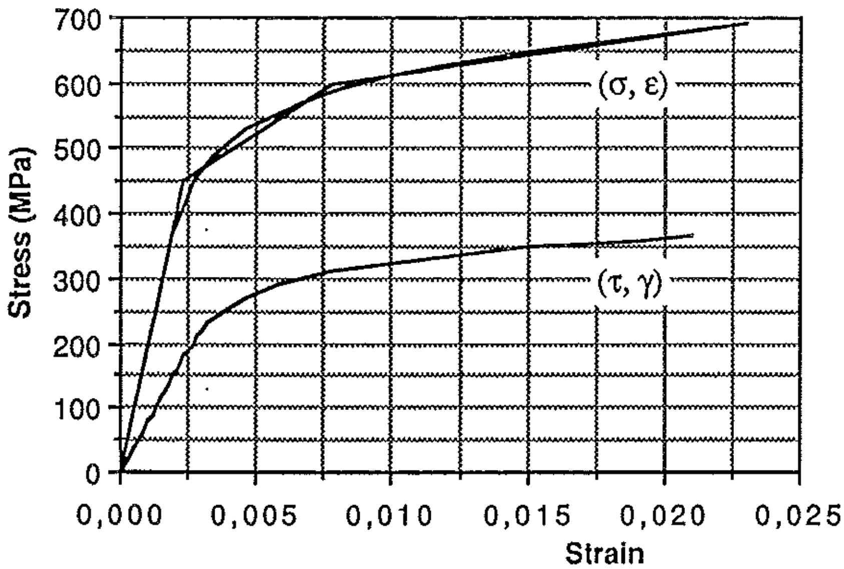


Fig. 2.4 - Model for the stress-strain curve of the material.

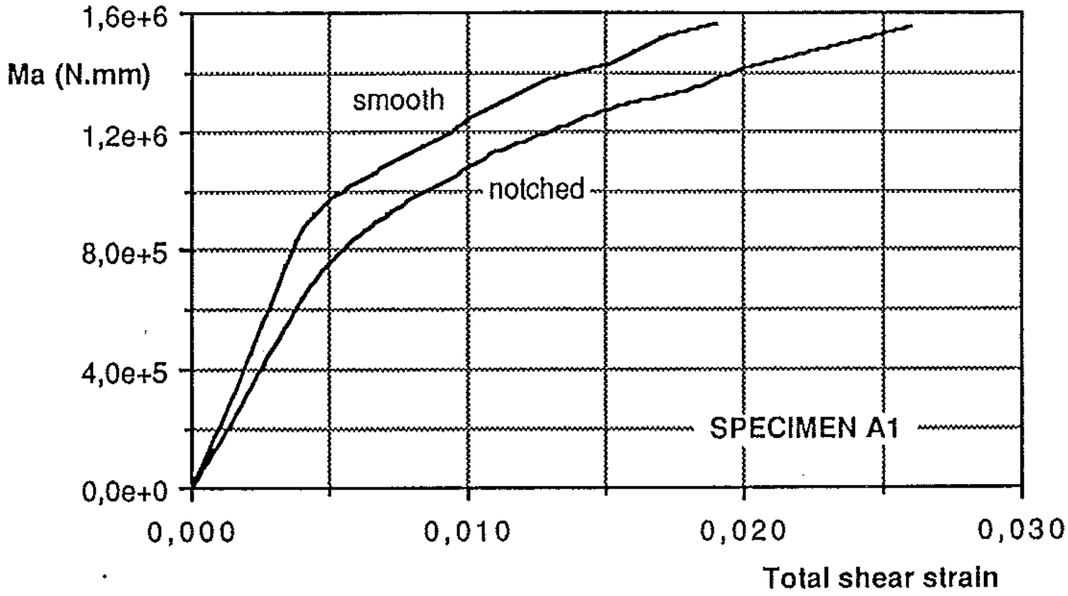


Fig. 2.5 - Torque-local strain curve (comparison between notched and smooth specimens).

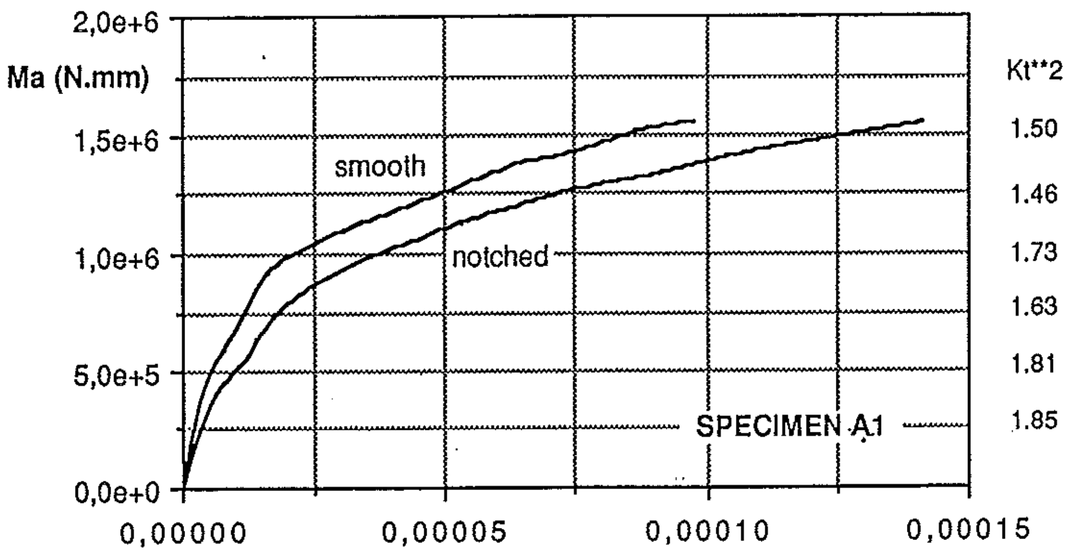


Fig. 3.1 - Calculation of K_t^2 for the specimen of fig. 2.2.

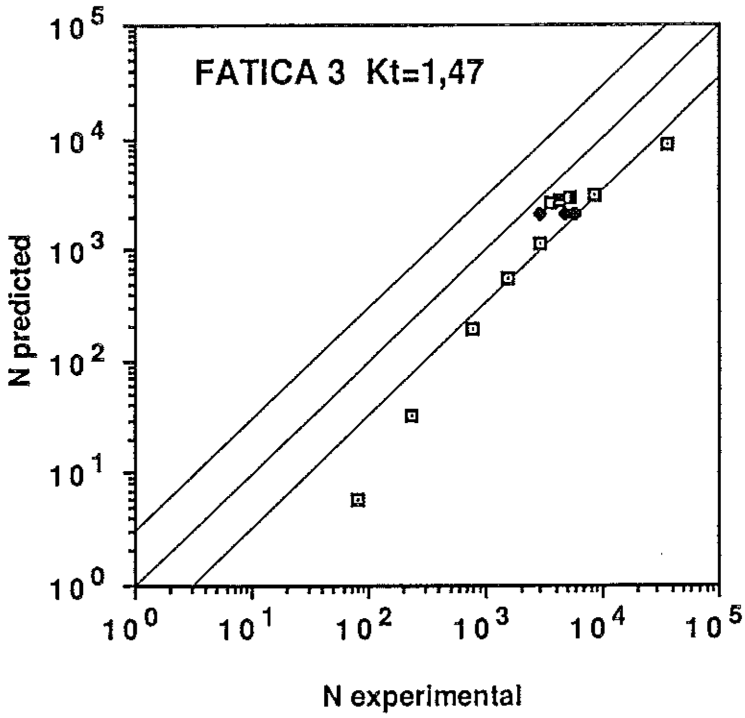


Fig. 4.1 - Comparison between experimental results and numerical predictions of fatigue lives (FATICA 3; K_t = 1,47).

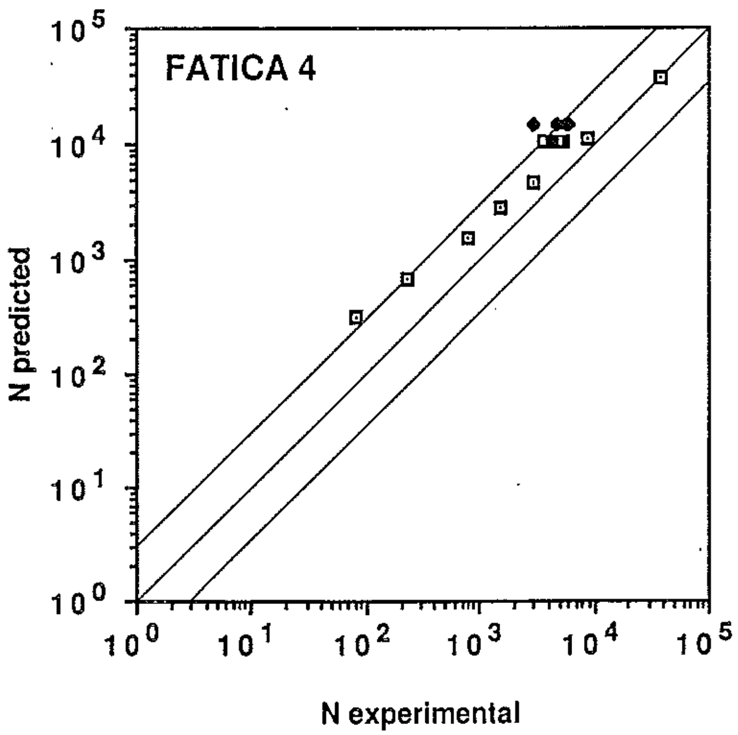


Fig. 4.2 - Comparison between experimental results and numerical predictions of fatigue lives (FATICA 4).

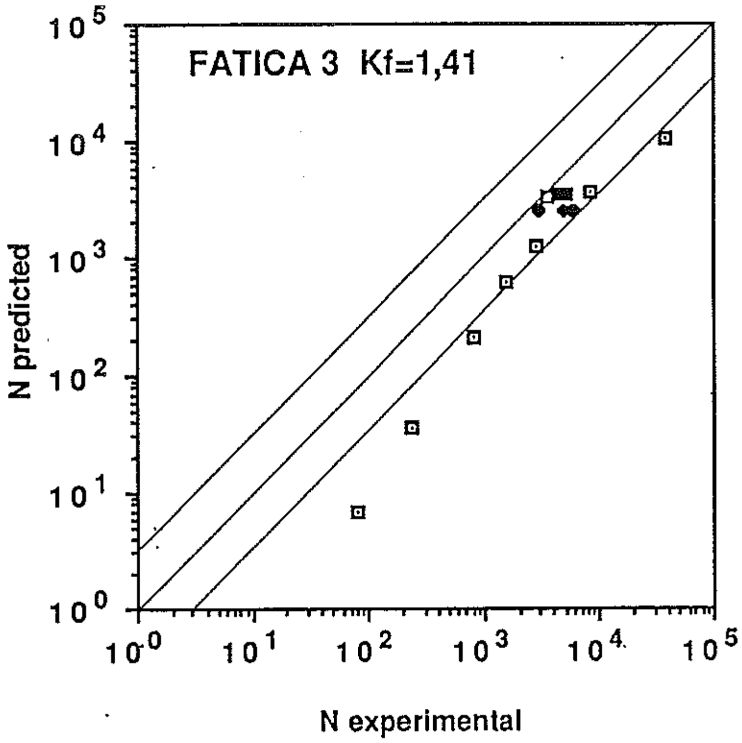
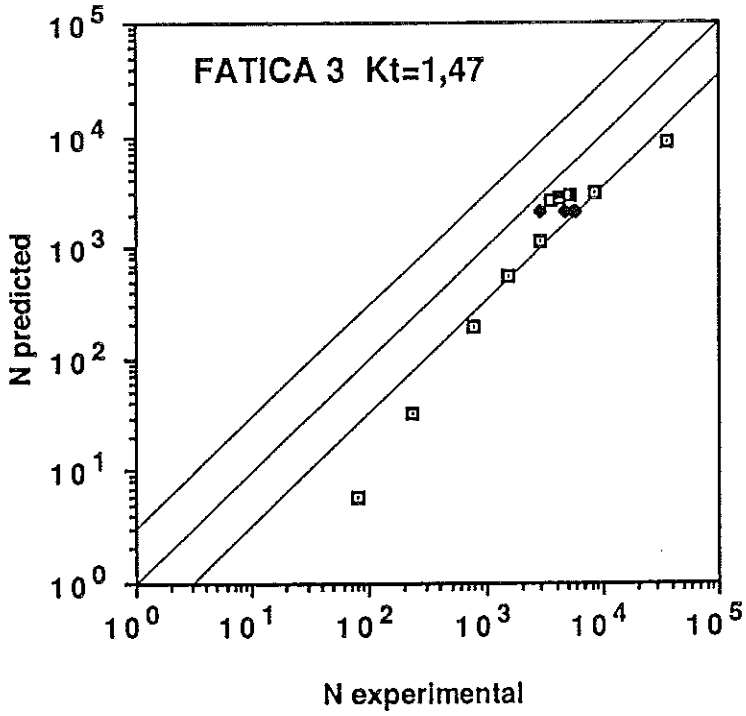


Fig. 4.3 - Comparison between experimental results and numerical predictions of fatigue lives (FATICA 3; K_f = 1,41).



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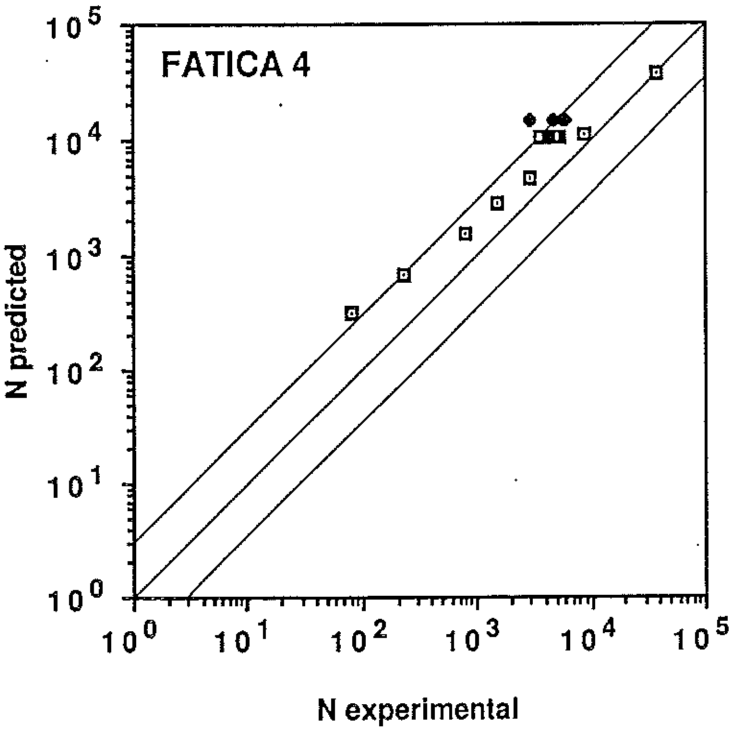

Fig. 4.2 - Comparison between experimental results and numerical predictions of fatigue lives (FATICA 4).

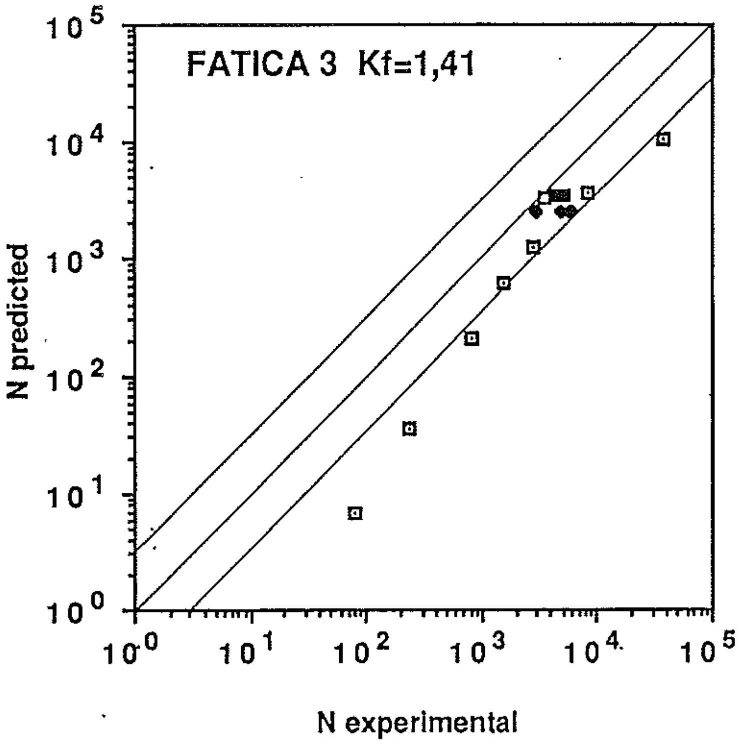

Fig. 4.3 - Comparison between experimental results and numerical predictions of fatigue lives (FATICA 3; K_f = 1,41).