

EXPERIMENTAL ANALYSIS OF BIAXIAL LOW CYCLE FATIGUE  
WITH MEAN STRAIN EFFECT

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ABSTRACT

Biaxial low cycle fatigue of SAE 1045 carbon steel is experimental investigated. Because most of the operated machine elements are subjected to biaxial stress state rather than uniaxial stress state, thus the study of biaxial and multiaxial fatigue properties of materials is needed. The biaxial fatigue experiments are performed by Instron's axial-torsional servo-hydraulic testing system. Four sets of biaxial strain ratio  $\rho = \Delta\gamma/\Delta\epsilon = 0, 1/2, 2$  and  $\infty$  are selected. According to the Tresca's equivalent strain criterion, the basic fatigue curves of all four sets of biaxial strain ratio can be described as straight lines in the double-log coordinates.

Biaxial fatigue with mean strain effect is also investigated. Three sets of biaxial strain ratio (1/2, 2 and  $\infty$ ) are chosen to be tested. Both tensile and compressive mean strain are considered in each set of biaxial strain ratio. From the testing results, it is seen that the mean stress of each case is decreased to a negligible value when the cycle ratio  $n/N_f$  equal 0.1, and the fatigue life of the materials is almost the same as in the basic fatigue experiments (without mean strain). Moreover, the biaxial fatigue life with and without mean strain effect can be predicted by the basic uniaxial low cycle fatigue data in reasonable agreements.

## INTRODUCTION

Most of the operated machine components and structural members are subjected to repeated loading, these loading may cause repeated stress and strain in materials. Fatigue failures are generally occurred when materials subjected to those repeated loadings. According to the statistical data presented by the American Society of Materials [1], over 80% of failure in machine elements are caused by fatigue. Thus fatigue is a very important effect in machine design.

Fatigue failures are still perhaps the commonest mode of failure among engineering components and structures despite the generation of a wealth of test data on the subject over many decades of research. However most of these data have been obtained from laboratory experiments involving uniaxial loading conditions which are seldom present in practical engineering situations. For example, components and structures found in power and chemical plants, such as pressure vessels and piping systems, aircraft structures, turbine blades, and drive shafts are subjected to multiaxial stress conditions during cyclic loading. Many components are exposed to varying degrees of multiaxial strain especially at notches or geometric discontinuities. In order to apply these limited fatigue data to more complex stress conditions, attempts have been made to correlate multiaxial fatigue loading to an equivalent uniaxial fatigue loading condition as suggested in some design codes.

Multiaxial fatigue is a subject of concern to both engineers and research scientists. In the eventuality of failure, fatigue lifetime is determined in the majority of cases by the applied multiaxial stress-strain state, whether generated by multiple loading or the component geometry itself as mentioned. In this paper, the biaxial low cycle fatigue of SAE 1045 carbon steel is experimental investigated. Mean strain effect is also considered in this study.

Experiments are performed by Instron's axial-torsional servohydraulic testing system, all the experiments are controlled by axial and torsional strains, four sets of biaxial strain ratio ( $0 = \Delta\gamma / \Delta\epsilon = 0, 1/2, 2$  and  $\infty$ ) are selected in the paper. The most

general transformations of biaxial strain state to an equivalent uniaxial strain state are the von Mises criterion and Tresca's criteria (maximum shear strain), in this paper, the Tresca's transformation is used in all cases.

The concept of low cycle fatigue was first proposed in the early 1960s, the uniaxial loading condition are considered in most of the presented papers. In the past twenty years, basic experimental studies on biaxial low cycle fatigue were discussed [2-6], most of the researchers interested in the study of fatigue life under the condition of both equivalent strain range  $\Delta\epsilon_{eq}$  and biaxial strain ratio  $\rho$  as constants. The discussion in the mean strain effect is very few. In the present work, two types of experiments are performed, the first one is to determine the basic  $\log \Delta\epsilon_{eq} - \log N_f$  curve with four different biaxial strain ratios, and the second one is to perform the low cycle fatigue tests with mean strain effect under three different biaxial strain ratios (except  $\rho = 0$ ). The comparisons between both (with and without mean strain) are presented.

#### BRIEF REVIEW ON LOW CYCLE FATIGUE

In 1962 Manson and Coffin [7,8] first proposed the concept of low cycle fatigue, the plastic strain range is used to describe the fatigue life, the Manson-Coffin relationship is proposed as

$$\Delta\epsilon_p = C N_f^m \quad (1)$$

where  $\Delta\epsilon_p$  is the plastic strain range;  $N_f$  is the fatigue life;  $C$  and  $m$  are material constants. If equation (1) is combined with Basquin's equation which proposed in 1910 [9], the relationship becomes

$$\Delta\epsilon_T = p N_f^q + r N_f^s \quad (2)$$

where  $\Delta\epsilon_T$  is the total strain range;  $p$ ,  $q$ ,  $r$  and  $s$  are material constants. The basic low cycle fatigue curve of most metallic metals can be generally described by equation (1) or (2), the

material constants are determined by curve fitting from the experimental results.

In the biaxial low cycle fatigue test, the concept of equivalent strain is commonly used. Equation (1) or (2) can be used in biaxial low cycle fatigue if the uniaxial strain range is replaced by the equivalent strain range. Most of the transformations used are von Mises or Tresca's criteria [10-12], which depends upon the material itself. There are no any criterion can be used in all kind of materials.

The mean strain effect in low cycle fatigue were discussed in [13-21], all those papers considered the uniaxial loading condition only. Most of those papers treated the mean strain as a prestrain, and the cumulative fatigue damage law is combined to determine the fatigue life. From those experimental results, the fatigue life is influenced by the remaining mean stress, that means if the mean stress is relaxed to zero in the tests, the fatigue life is almost the same as that in the basic curve. In this paper, the biaxial mean strain effect is considered in each set of biaxial strain ratio, the results are compared with the basic biaxial and uniaxial low cycle fatigue experiments.

## EXPERIMENTAL RESULTS AND DISCUSSIONS

SAE 1045 carbon steel is selected as the test material in this study. The specimen size is shown in figure 1. All the specimens are normalized by stress relief heat treatment (850°C 1 hour and air cooled) before test. Instron's 1322 axial-torsional servohydraulic testign system and its biaxial strain extensometer are used for all tests. All the tests are performed at room temperature condition in air.

In the present work, four sets of biaxial strain ratio  $\rho = 0, 1/2, 2$  and  $\infty$  are selected in the basic low cycle fatigue test. The axial strain ranges are chosen between 0 to 3.5% and the torsional strain ranges are chosen between 0 to 6.5%. In each set of biaxial strain ratio, three or four different strain ranges are selected to be tested, and at least 3 specimens are tested in each condition. The definition of fatigue life is defined as the corresponding cycle when the loading is dropped to

90% of its steady loading (macrocrack occurred). In this study, there are two number of cycles corresponding to 90% of axial and torsional steady loadings respectively, but the difference is negligible, thus the failure cycle in this paper is defined as the average of these two corresponding cycles. For the equivalent strain transformation, Tresca's criterion is used in this paper. The equivalent strain range is defined as

$$\Delta\epsilon_{e,q} = \sqrt{9/4 \Delta\epsilon^2 + \Delta\gamma^2} \quad (3)$$

where  $\Delta\epsilon$  is the axial strain range and  $\Delta\gamma$  is the torsional strain range.

The experimental results show that all the data in  $\log \Delta\epsilon_{e,q}$  -  $\log N_f$  coordinates can be described as straight lines, that means equation (2) is suitable to describe the basic low cycle fatigue behavior of this material (use single term only, set  $r = s = 0$ ). The material constants are determined by curve fitting with least square method. The relationships between  $\Delta\epsilon_{e,q}$  and  $N_f$  for four sets of biaxial strain ratio are as follows

$$\phi = 0 : \Delta\epsilon_{e,q} = 0.3164 N_f^{-0.270} \quad (4)$$

$$\phi = 1/2 : \Delta\epsilon_{e,q} = 0.3287 N_f^{-0.276} \quad (5)$$

$$\phi = 2 : \Delta\epsilon_{e,q} = 0.5585 N_f^{-0.337} \quad (6)$$

$$\phi = \infty : \Delta\epsilon_{e,q} = 0.6384 N_f^{-0.365} \quad (7)$$

$$\text{all data} : \Delta\epsilon_{e,q} = 0.4610 N_f^{-0.316} \quad (8)$$

In which equation (8) is fitted by all experimental data (the tests under four different sets of biaxial strain ratio are all used), this result is shown in figure 2, and the other results was presented in [22].

The effect of biaxial mean strain in fatigue life is also discussed in this paper. In the uniaxial high cycle fatigue range, tensile mean stress decreases the fatigue life based on the acceleration of fatigue crack, and compressive mean stress slightly increases the fatigue life based on the residual stress occurred in the crack tip. In low cycle fatigue range, the influence of mean strain is inapparent in uniaxial case [13-21],

the fatigue life do not influenced by mean strain effect is the most general conclusion in those papers, but Ellyin [20] and Socie [21] pointed out the influence of fatigue life is due to the remaining mean stress, if the mean stress due to mean strain effect relaxed to zero rapidly, then the fatigue life is almost the same as that in the basic fatigue curve. In the biaxial case, the experimental data with biaxial mean strain is deficient. In this paper, the low cycle fatigue test with biaxial mean strain is performed in three different sets of biaxial strain ratio,  $\rho = 1/2, 2$  and  $\infty$ , the equivalent mean strain in each case is taken in both tensile and compressive conditions,  $(\epsilon_{eq})_m = 3\%, -1.5\%$  and  $9\%$ , and the corresponding equivalent strain ranges are chosen between  $2.1\%$  to  $7\%$  in each set of biaxial strain ratio.

Experimental results are listed in tables 1 to 3, in each set of biaxial strain ratio, three values of mean strain are used, including two tensile mean strains and one compressive mean strain. Both the equivalent mean strain and equivalent strain range are also calculated by the Tresca's criterion as stated in equation (3), for the same value of mean strain, the components of axial and torsional mean strain are difference for different biaxial strain ratio. The experimental data plotted in  $\log \Delta \epsilon_{eq} - \log N_f$  coordinates are shown in figures 3 to 5, the results show that the data in all cases can be represented as straight lines in the double log coordinates. In figures 3 to 5, three different basic low cycle fatigue curves are also drawn for the comparison with the experimental data in each case. The first one is the basic fatigue curve corresponding to the same biaxial strain ratio with the tested data, the second one is the basic fatigue curve of  $\rho = 0$  (uniaxial case), and the final one is the basic fatigue curve fitted from all basic low cycle fatigue experimental data. It is seen that all three groups of basic fatigue curves fit the experimental data in good agreements. The better one in each case is the basic fatigue curve with the same biaxial strain ratio. From the results of comparison, it is seen that the mean strain effect does not influence the fatigue life apparently in all cases. That means these results can be predicted by the basic fatigue curve of the same biaxial strain ratio. Moreover, in the case of shortage in biaxial fatigue data, the uniaxial basic fatigue data can also be used for the predictions when combined with the Tresca's criterion for the equivalent strain

transformation. It is seen that from the results shown in figures 3b, 4b and 5b, the reasonable agreements can be accepted .

The relaxation of mean stress during the tests are shown in figures 6 and 7. Figure 6 shows that the mean stress is relaxed with the number of cycles, and figure 7 shows that the mean stress is relaxed with the cycle ratio, where the cycle ratio is defined as  $n/N_f$ . It is seen from the figures that the mean stress relaxed with the number of cycle or the cycle ratio increased. The mean stress during mean strain effect relaxates rapidly in the first few ten cycles, and it relaxates to a small value when the number of cycle equal  $0.1 N_f$  in most of the all cases. This phenomena are of the same with [20,21] as that in uniaxial case, and the fatigue life does not have an apparent influence when compared with the basic low cycle fatigue data.

Ohji [16] presented the following expression to predict the mean strain effect in uniaxial case

$$\Delta \epsilon = \frac{2 (1-R) \epsilon_F}{[4 (1-R) \cdot N_f + (1+R) \cdot a]^{1/a}} \quad (9)$$

where  $R$  is the strain ratio,  $R = \epsilon_{min} / \epsilon_{max}$ ,  $\epsilon_F$  is the fatigue strength corresponding to  $N_f = 1/4$ , and  $a = -1/q$ , in which  $q$  is the slope of the basic fatigue curve in double log coordinates. Figure 8 shows that the influence of mean strain at a constant  $N_f$ . It is seen that when  $(\epsilon_{e,q})_m$  less then  $0.4 \epsilon_F$ , this effect can be neglected, and from equation (9), it can be calculated that the account of mean strain effect is needed to be considered when  $(\epsilon_{e,q})_m / \epsilon_F$  greater then 0.6.

## CONCLUSIONS

From the experimental results discussed, general conclusions of this study are as follows

1. In low cycle fatigue experiments, the mean stress due to mean strain effect is relaxed to a small value (approach to zero) when the cycle ratio  $n/N_f$  equal 0.1.
2. The influence of fatigue life due to mean strain effect

is inapparent, the fatigue life with mean strain effect can be predicted from the basic fatigue curve of the same biaxial strain ratio. That means the most important factor in fatigue life is the equivalent strain range but not the equivalent mean strain.

3. In the case of shortage in biaxial low cycle fatigue data, the uniaxial basic low cycle fatigue data can be used to predict the mean strain effect in different biaxial strain ratio in a reasonable agreement. In this case, the Trseca's criterion for the equivalent strain range and equivalent mean strain transformations is needed to be used.

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Table 1 Experimental Data of Low Cycle Fatigue with Mean Strain Effect ( $\sigma = 1/2$ )

$\epsilon_m$ (%)	$\Delta \epsilon_{eq}$ (%)	$N_f$			Ave. $N_f$
3	4.56	1655	1429	1537	1540
3	3.79	2846	3150	3417	3138
3	3.04	6757	6140	5054	5597
-1.5	4.73	1563	1478	1530	1524
-1.5	3.65	3792	3072	3986	3617
-1.5	2.74	10360	10613	9987	10320
9	4.56	1581	1438	1520	1513
9	3.04	6678	5344	5666	5896
9	2.58	9866	9064	12503	10478

Table 2 Experimental data of Low Cycle Fatigue with Mean Strain Effect ( $\sigma = 2$ )

$\epsilon_m$ (%)	$\Delta \epsilon_{eq}$ (%)	$N_f$			Ave. $N_f$
3	5.73	990	887	935	937
3	4.58	1544	1567	1601	1571
3	3.27	4041	4204	4120	4122
3	2.46	9357	9804	9552	9571
-1.5	4.26	2221	2272	1969	2154
-1.5	3.60	3159	2651	3612	3140
-1.5	2.95	5155	7884	6376	6472
9	6.55	632	682	646	653
9	4.91	1413	1430	1396	1413
9	3.27	3784	5458	3328	4190

Table 3 Experimental data of Low Cycle Fatigue with Mean Strain Effect ( $\sigma = \infty$ )

$\epsilon_m$ (%)	$\Delta \epsilon_{eq}$ (%)	$N_f$			Ave. $N_f$
3	6.93	493	446	457	465
3	3.64	1945	2311	2339	2195
3	2.08	11833	11767	10864	11488
-1.5	6.93	612	453	383	483
-1.5	5.20	978	1000	1056	1011
-1.5	2.94	3919	3103	3738	3586
9	6.93	410	505	358	424
9	5.20	799	1158	1140	1032
9	3.46	2269	2736	2875	2627

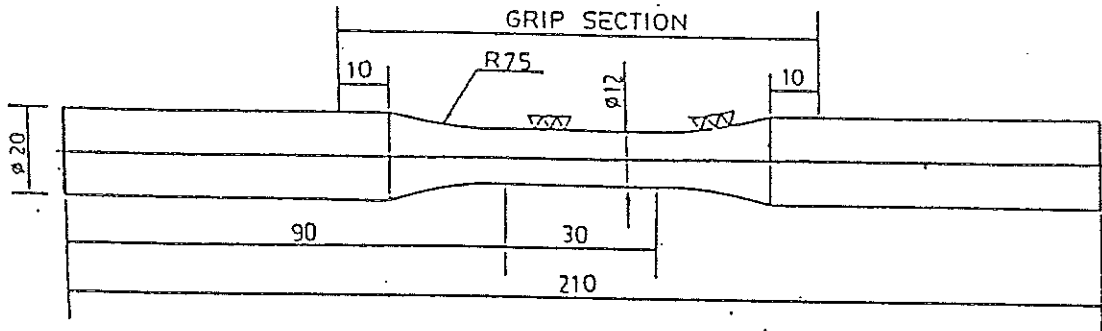


Figure 1 Specimen Size (unit : mm)

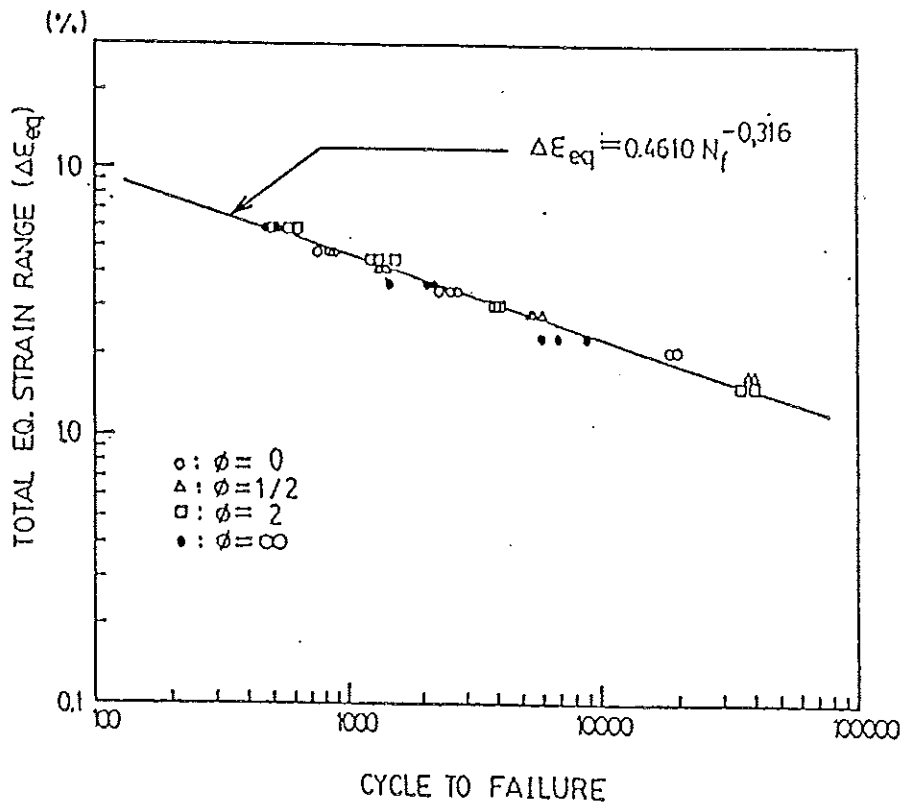


Figure 2 Basic Biaxial Low Cycle Data

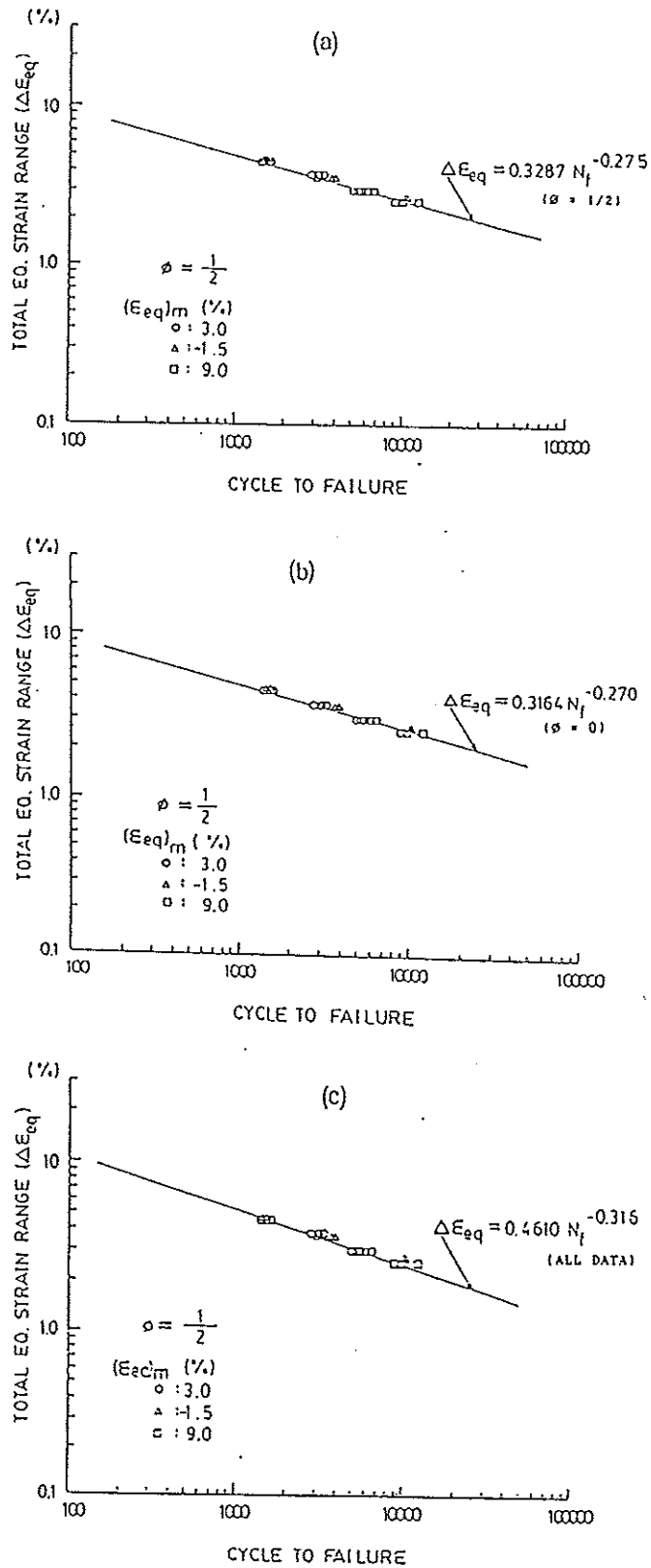


Figure 3 Comparison of Low Cycle Fatigue Data with Mean Strain Effects ( $\phi = 1/2$ ) to Basic Fatigue Curves : (a) with  $\phi = 1/2$  curve, (b) with  $\phi = 0$  curve, (c) with all data curve.

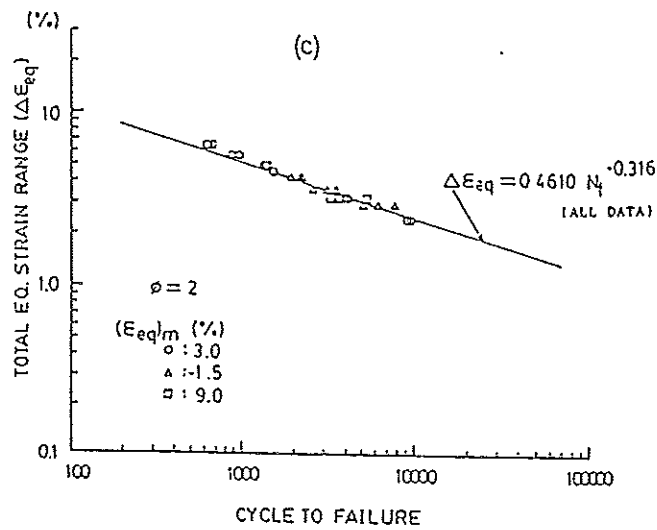
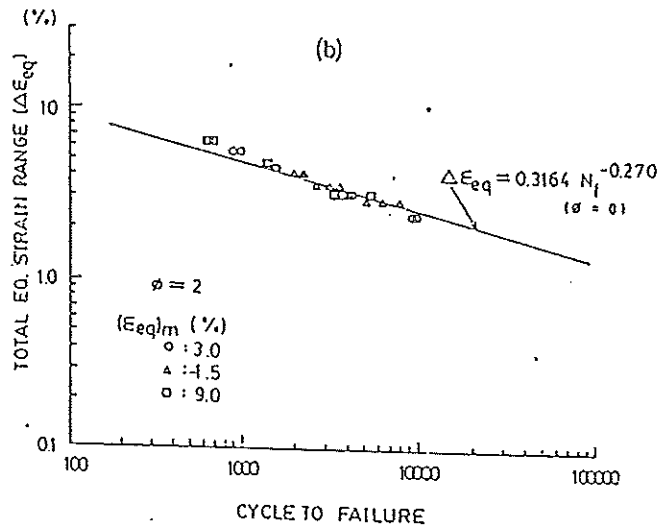
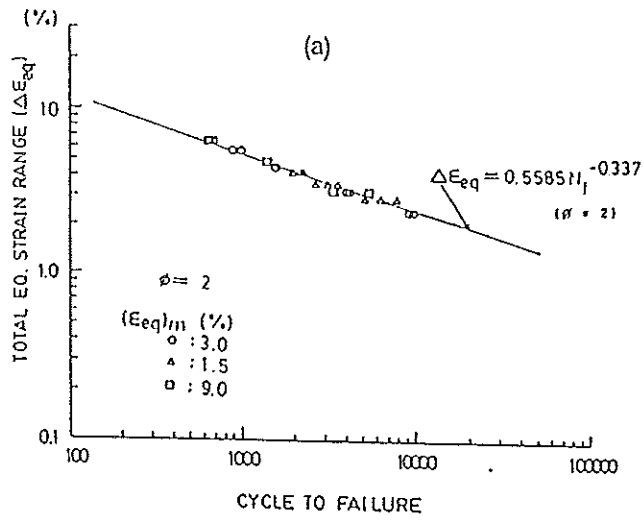


Figure 4 Comparison of Low Cycle Fatigue Data with Mean Strain Effects ( $\phi = 2$ ) to Basic Fatigue Curves : (a) with  $\phi = 2$  curve, (b) with  $\phi = 0$  curve, (c) with all data curve.

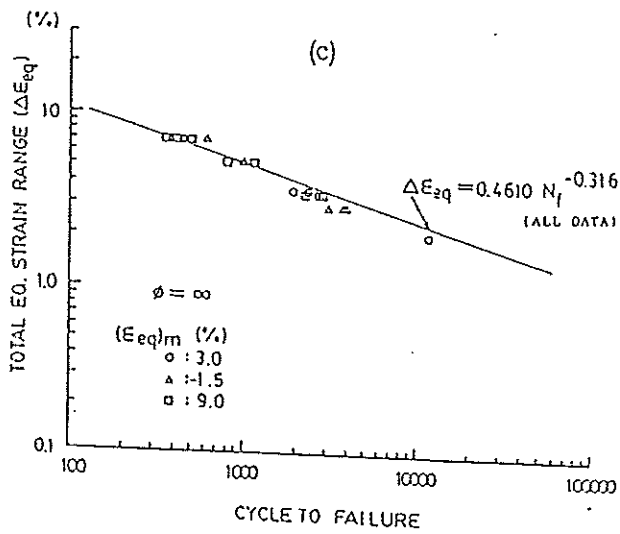
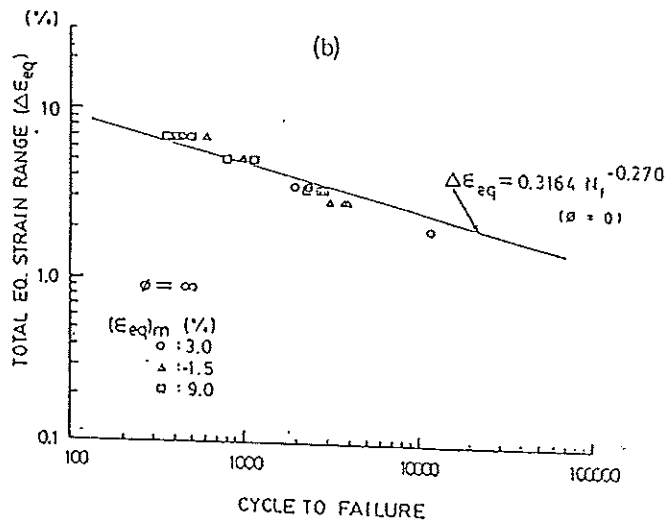
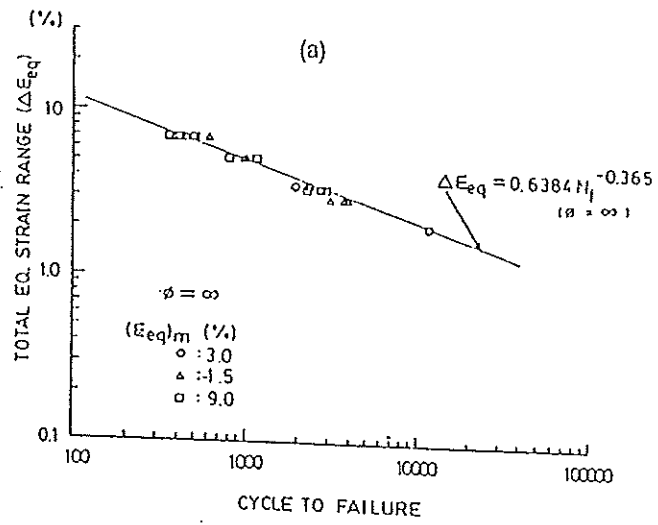
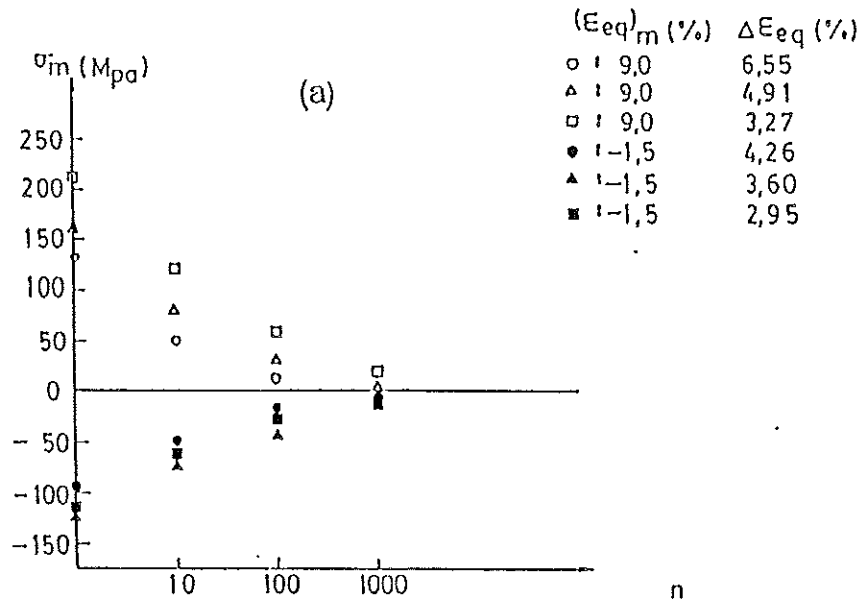


Figure 5 Comparison of Low Cycle Fatigue Data with Mean Strain Effects ( $\phi = \infty$ ) to Basic Fatigue Curves : (a) with  $\phi = \infty$  curve, (b) with  $\phi = 0$  curve, (c) with all data curve.

MEAN STRESS RELAXATION

$\phi = 2$



MEAN STRESS RELAXATION

$\phi = \frac{1}{2}$

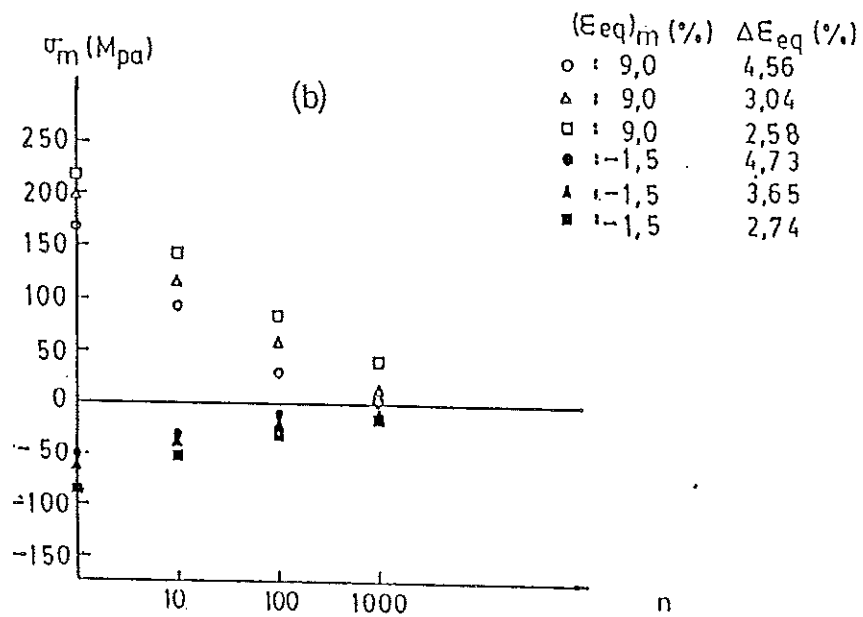


Figure 6 Mean Stress Relaxation with Number of Cycles



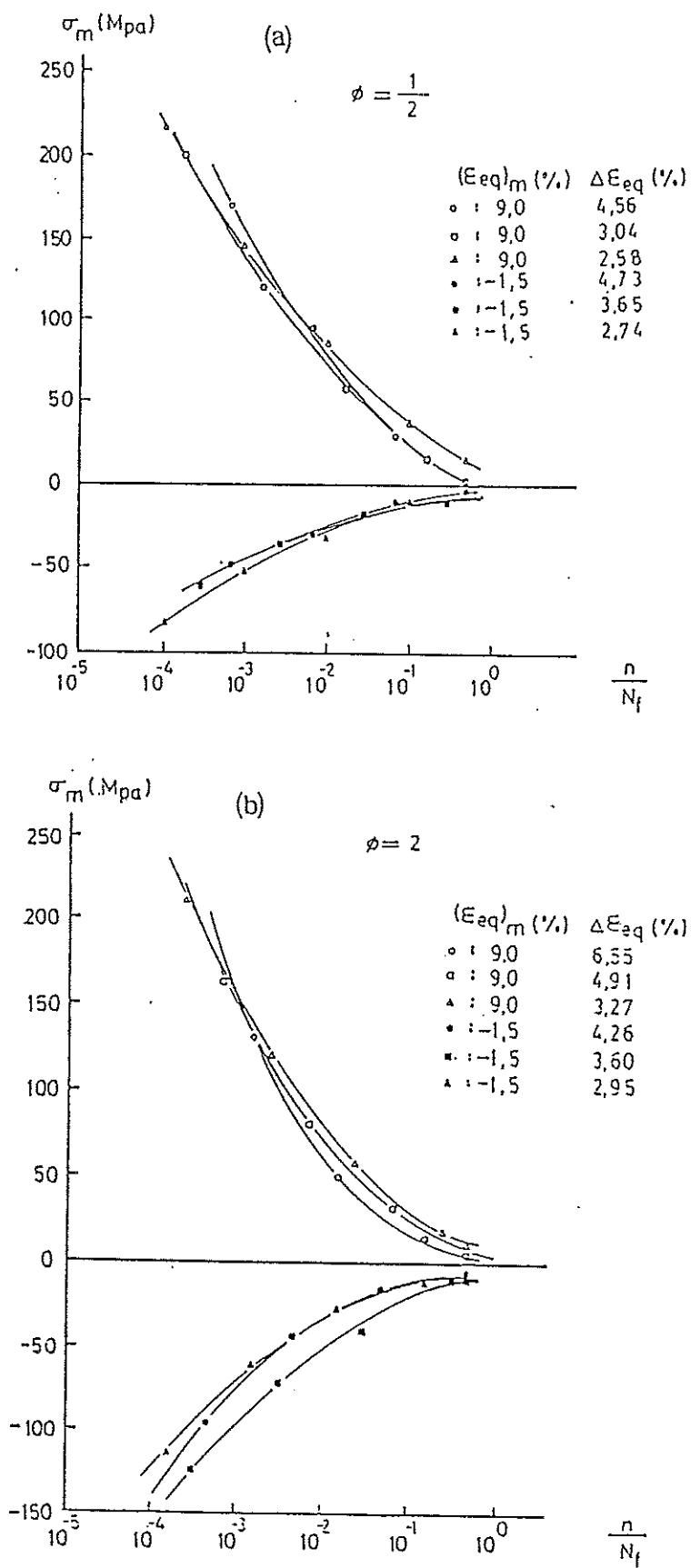


Figure 7 Mean Stress Relaxation with Cycle Ratio  $n/N_f$

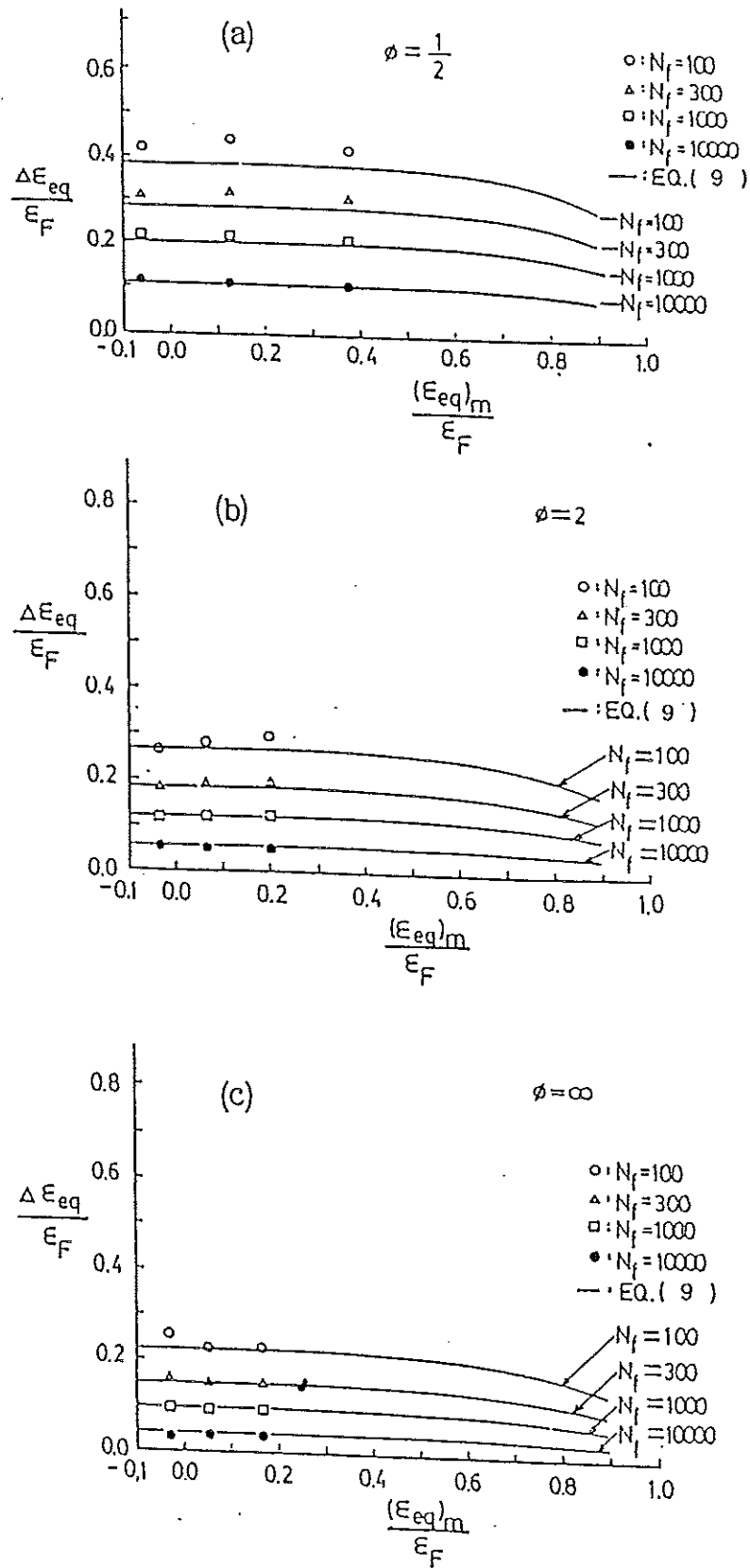


Figure 8 Mean Strain Effects on the Biaxial Fatigue Strength: (a)  $\phi = 1/2$ , (b)  $\phi = 2$ , (c)  $\phi = \infty$