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ENERGY-BASED TORSIONAL LOW-CYCLE FATIGUE ANALYSIS

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ABSTRACT A brief review of low cycle fatigue theories to evaluate the life expenditure of mechanical components is presented. The principal energy-based fatigue theories are discussedand applied to uniaxial and torsional loading conditions, on the basis of experimental results of tests performed by the authors and reported in previous papers.

INTRODUCTION

The paper concerns the applicability and the limitations of some fatigue theories which have been introduced for life prevision of structural componentsin low-cycle fatigue condition. The theories have been analyzed and a verification of these have been performed through laboratories tests on uniaxial and torsional specimens of different sizes, of high strenght structural steels. Indeed the main interest of the authors is the study of the torsional state of stress encountered in shafts of electrical machinery ,Curioni et al. (1983), Caligiana et al. (1984, 1985, 1986) Freddi et al. (1989), Jackson et al.(1979),Tanaka et al.(1983), EPRI (1984). Nevertheless, thinking on the possibility to extend the study to a contemporary bending and torsion loadings,it was considered useful to investigate on the push-pull and torsional material answers and to discuss on the fatigue criteria as preliminary approach to a real multiaxial condition.

ENERGY-BASED FATIGUE CRITERIA

The main advantage of the energy-based criteria is their capacity to represent the multiaxiality of the state of stress and strain in a simple and phisically correct way, Ellyin(1974,1985,1988), Feltner (1961). They describe at a macroscopic level, the non-linear and hysterethic behaviour of the material which is the result of complex and numerous microstuctural changes which are responsible of the damage. This assumption in no way imply that all the total strain energy or all the dissipated energy per cycle causes fatigue damage but simply try to establish a correlation between damage accumulation and one of these parameters. Such criteria developed by several authors for uniaxial, biaxial and multiaxial conditions differ one another in several points ; the most important point is the approach to the theory of plasticity: the formulation of the constitutive law of the material in integral or in differential form. Whereas laws of the first type relate stress to a history of strain in a form of a functional, the differential laws relate the increments of plastic strain and stress from a given instantaneous state. The criteria of first type are suitable for in-phase or proportional loading conditions, while the citeria of the second type are requested for out-of-phase or non-proportional loading conditions. Furtherly the energy-based criteria differ in the definition of the equivalent stress and strain, and in the hardening rule in the case of differential form of constitutive law, Mroz (1967).

Plastic energy approach with integral form constitutive law

Following Morrow (1965), Halford(1966) and Lefebvre (1981), the plastic energy per cycle is given by the expression:

$$W_p = k\,\frac{1-n'}{1+n'}\,\Delta\sigma_{eq}^{(1+n')/n'} \tag{1}$$

Here the equivalent stress and strain, according to Von Mises theory, are :

$$\sigma_{eq} = (\frac{3}{2}s_{ij}s_{ij})^{1/2} \quad \epsilon^p_{eq} = (\frac{2}{3}\epsilon^p_{ij}\epsilon^p_{ij})^{1/2} \tag{2}$$

where:

$$s_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{kk}\delta_{ij}$$

and the constitutine law is the following:

$$\Delta\epsilon^p_{eq} = k\Delta\sigma_{eq}^{1/n'} \tag{3}$$

The plastic energy per cycle was related to fatigue life , N_f, through a power-law, Lefebvre et al.(1981)

$$\Delta\epsilon^p_{eq}\,\Delta\sigma_{eq} = C\,N_f^c \tag{4}$$

where the equivalent plastic strain amplitude is also based on the von Mises yield function of plasticity. The parameter C and c are depending on the material and are function of the stress ratio. This functions can be evaluated from uniaxial fatigue data.
An extension to this approach is given by Lefebvre (1984) in which a method is suggested for calculating the area of the hysteresis loop in the case of a non Masing behaviour of the material. Ellyin (1985) {in Miller & Brownr (1986)}, extended the method to multiaxial fatigue.

Strain energy approach with integral form constitutive law

Other theories suggest an unified approach at the low cycle and high cycle fatigue that account for mean stress.
An energy based multiaxial fatigue damage parameter is suggested by Leis (1977), on the hypothesis that damage is dependent on the internal total octahedral strain energy. The relation of the total octahedral energy is the following:

$$U_t = \xi(\overline{s}_m \Delta\epsilon^t_{eq} + \Delta\sigma_{eq}\Delta\epsilon^t_{eq}) = \Delta\sigma_{eq}\Delta\epsilon^t_{eq}) - A - B \qquad (5)$$

where:

$$\overline{s}_m = \frac{1}{3}\sigma_{kk}$$

$$A = \frac{\Delta\sigma^2}{2E_{eq}} \qquad B = \frac{2}{[2k]^{1/n'}} \frac{\Delta\sigma_{eq}^{1+\frac{1}{n'}}}{1+\frac{1}{n'}}$$

Ellyin (1988), Golos et al. (1987) remark that plastic strain energy plays an important role in the damage initiation process generating the piling up of dislocation along the slip plane, but the crack propagation is controlled by the variation of the J- integral that is a function of the cyclic strain energy density, concludind that the fatigue damage is better described by the cyclic strain energy . The relationship is the following :

$$\Delta W_t = kN_f^\alpha + C \qquad (6)$$

where the strain energy can be splitted in the elastic and plastic parts:

$$\Delta W_t = \Delta W_e + \Delta W_p$$

Also this parameter accounts for mean stress, as is clear from the elementary espression of W_e in the uniaxial case:

$$W_e = \frac{1}{2E}(\frac{\Delta\sigma}{2} + \sigma_m)^2$$

Plastic energy approach with differential form constitutive law

For a description of the stress-strain cycle in complex multiaxial conditions with non proportional loadings, an incremental theory of plasticity must be introduced that takes into account the path-dependence of the plastic deformation. For this reason the constitutive law is given in differential form. In many cases the principal directions of the alternating stress are not fixed but change orientation. Methods for handling such situations have been developed, Garud (1981). The plastic work is calculated by integrating the product of stress times plastic strain increment (the area of the hysteresis loop) for each of the six components of stress tensor. The sum of the six integrals is the plastic work per cycle. The fatigue relationship is:

$$W_p = AN_f^{\alpha} \tag{7}$$

where:

$$W_p = \int_{cycle} \sigma_{ij} d\epsilon_{ij}^{p}$$

In this case the constitutive law relates the increments of plastic strain with the increment of stress starting from a certain instantaneus state and the material response for a given loading history is obtained by integrating differential relations. According to this incremental theory, the response of a strain hardening material is a function of an initial yield condition, a hardening rule and a flow rule.

The Mroz hardening model, Mroz (1967) with the development and modification proposed by Garud (1981), has been used for implementing a code for computing the plastic energy per cycle, in the case of multiaxial conditions, starting from the uniaxial cyclic stress-strain relation for the material, approximated by a convenient number of segments, under strain controlled condition. In the present paper the calculation is limited to the case of push-pull and torsional loadings.

UNIAXIAL AND TORSIONAL LOADINGS

Push-pull and torsional fatigue tests have been performed. For these two fundamental cases integral and incremental laws have been used to describe the elasto-plastic behaviour of the material. Chemical composition, heat treatments and conventional mechanical properties of the steel (24 NiCrMoV 14-6) are listed in Tab. 1, Caligiana (1987): All the tests were conducted in strain control in the range 0.35% - 1.25% with a servohydraulic machine. Standard twelve millimeter diameter specimens were used for push-pull tests. Torsional apparatus,for torques from 2 to 25 kNm, developed in the Department's Laboratory,Caligiana (1985), fig.1 were utilized for hollow and solid , smooth and notched specimens of different magnitudes, Freddi et al. (1988).

TABLE 1 - Chemical composition, heat treatment and mechanical properties of the tested steel

Chemical Composition (%)

C	Si	Mn	P	S	Cr	Mo	Ni	V	Al	Cu
.28	.07	.23	.008	.004	1.63	.42	3.59	.09	.009	.06

Heat Treatment

Tempered	940 °C x 28 hrs.;	870 °C x 28 hrs.;	630 °C x 28 hrs.
Quality Tempered	845 °C x 19 hrs.;	630 °C x 28 hrs.;	
Stress-relieving	570 °C x 21 hrs.;	furnace cooled	(17.2 °C/hrs.)
	200 °C	air cooled	

Mechanical Properties

Tensile Strenght (MPa)	.02 Yield Strenght (MPa)	Elong. (%)	Red. (%)	KV (J)	FATT (°C)
820	700	20	70	136-193	-12/-14

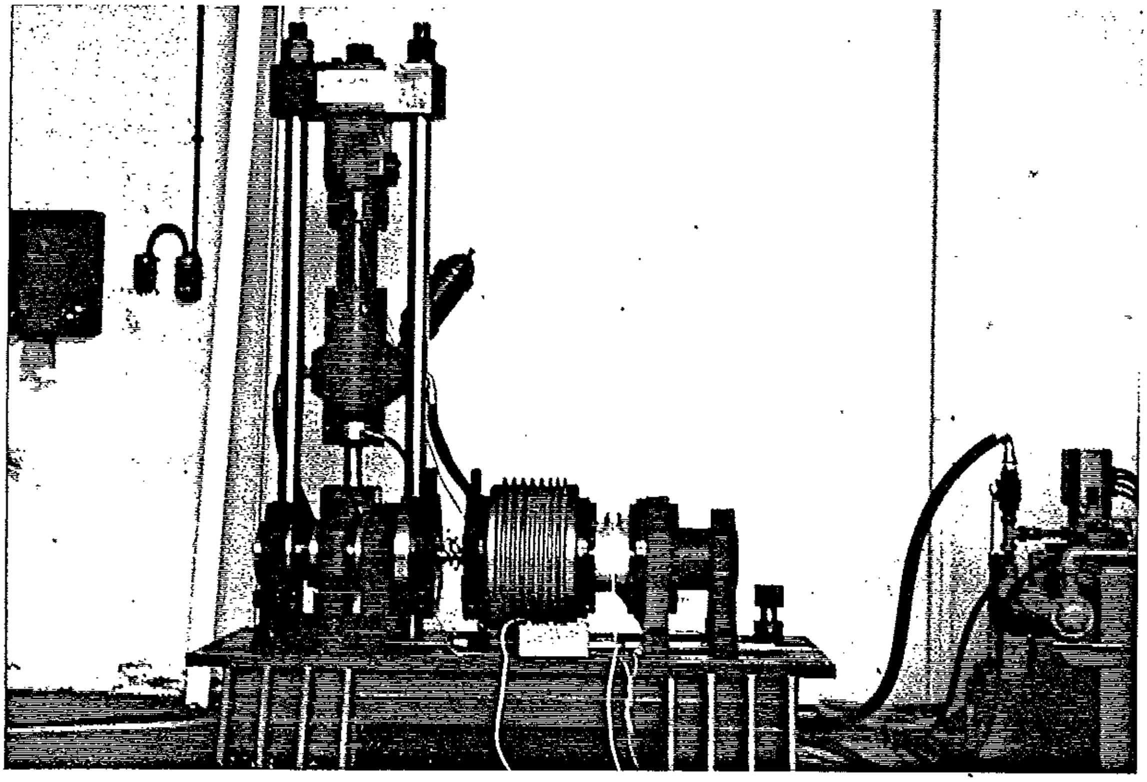

Fig.1 - Torsional apparatus for torques up to 25 kNm

A critical point of all of the energy criteria is the experimental evidence that the total plastic or total strain energy accumulated up to failure is not constant but increases with the number of cycles at failure , see e.g. fig.2.

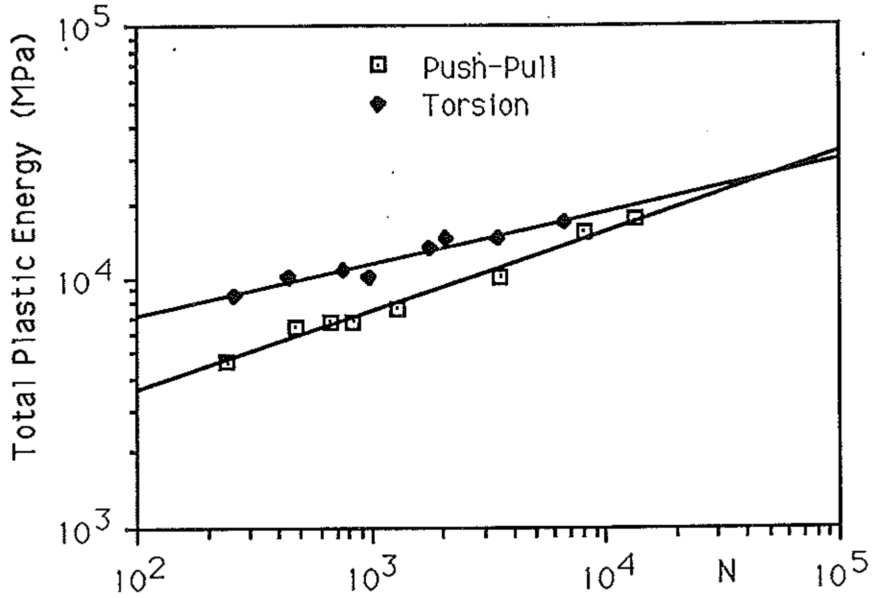

Fig.2 - Total Plastic Energy absorbed at failure

This means that not the whole dissipated energy generates damage but the part of absorbed energy per cycle which is closely correlated to the damage is proportionally greater in the high strain cycles than in the low strain cycles up to the limit case of the monotonic loading in which all the absorbed energy of one half cycle is absorbed at fracture. This observation induced some researchers to find a parameter which, better than all the dissipated energy or all strain energy, could better represent damage. These efforts unfortunatly are limited to the uniaxial case, as Hatanaka et al. (1980) who subdivided the plastic strain into an effective component and ineffective component which is the range of plastic strain for infinite life, or Romanov (1980) , who started from a Novozhilov theory, Kadashevitch & Novozhilov (1958) and from accurate measurements of the Bauschinger effect,for dividing the hysteresis cycle area in an uneffective and in an effective energy, direct responsible of the damage accumulation. For this way the plastic energy per cycle was calculated for point at the external surface of the push-pull and the hollow torsional specimens, Caligiana et al.(1986). Following the Garud method the same calculation was performed on the base of incremental theory of plasticity. Fig.3 shows a comparison for the push-pull case between calculations done with different work-hardening models and experimental results. Fig. 4, presents similar comparison for the torsional case.

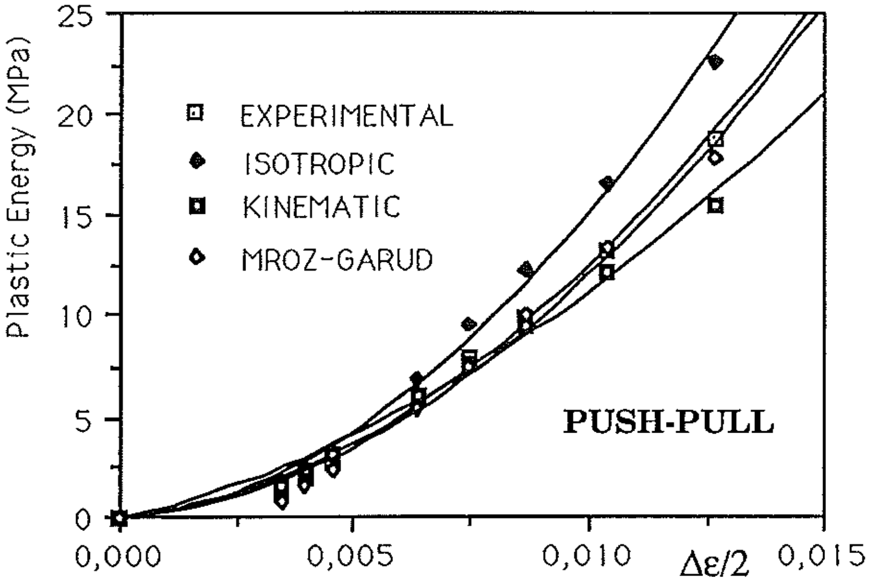

Fig. 3 - Plastic strain energy at fracture versus strain amplitute(push-pull)

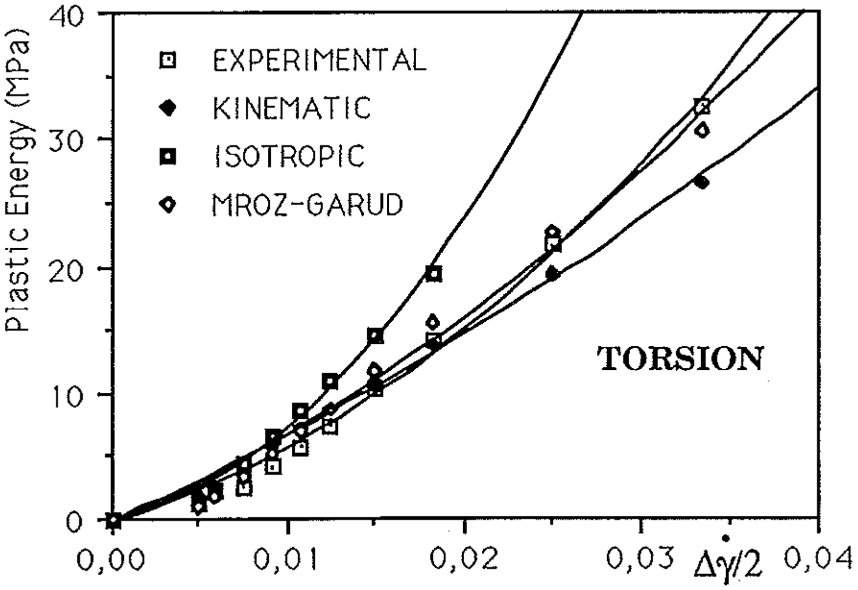

Fig.4- Plastic strain energy at fracture versus shear strain ampliyude (torsion)

Determination of the fatigue curves

Fatigue curves have been calculated for all the discussed theories. In the case of application of the theory (4) the fatigue curves are shown in fig.5 and the constant are :$C = 860\ MPa$,$c = -0.69$ for push-pull and $C = 2556\ MPa$,$c = -0.79$ for torsion:

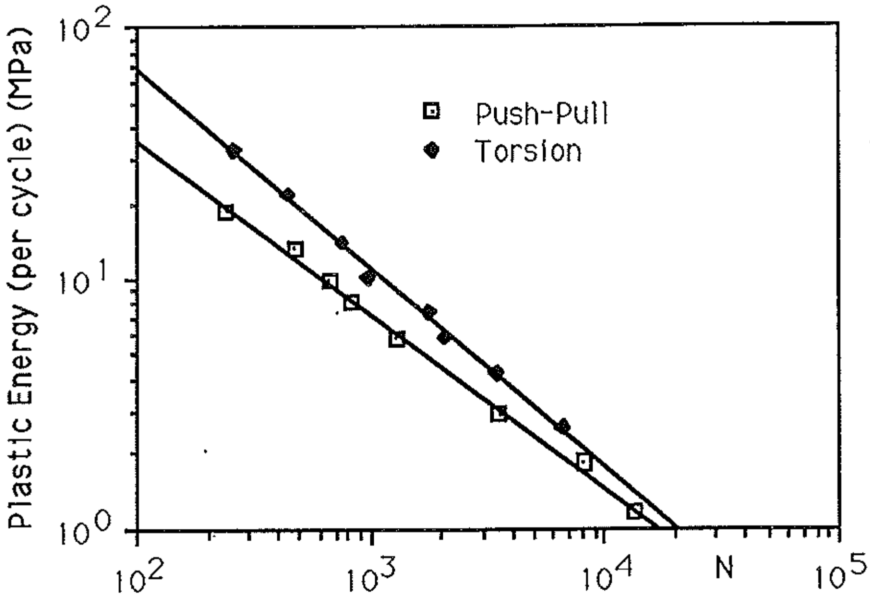

Fig. 5- Fatigue curves (plastic energy versus cycle number)

According to the Leis theory U_t is plotted versus N_f with an espression similar to (4):$U_t = DN_f^d$. For the push-pull case and for torsional case figs 6 and 7 show similar form of the fatigue curves and the constants are: $D = 270\ MPa$ $d = -0.44$ for push-pull and $D = 642\ MPa$,$d = -0.52$ for torsion:

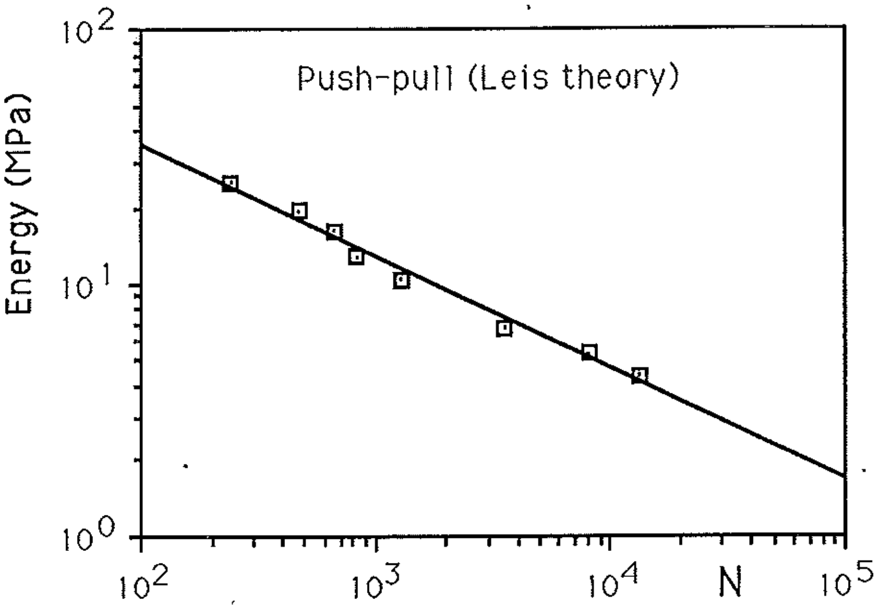

Fig. 6 - Total octahedral energy for push-pull

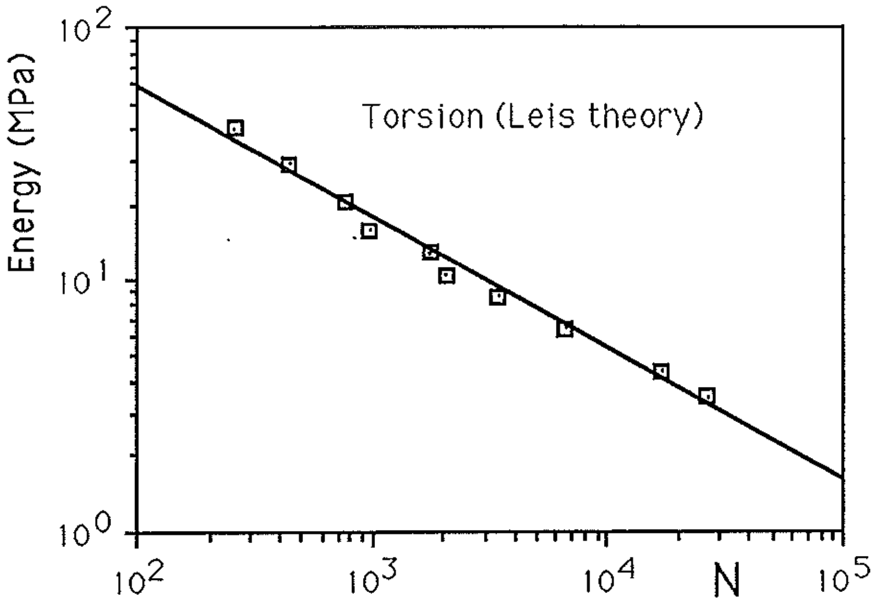

Fig.7 Total octahedral energy for torsion

CONCLUSIONS

Several energy-based criteria for fatigue life prevision have been shown and applied to experimental results obtained for uniaxial and torsion conditions. The Mroz hardening model and the Garud numerical method together with isotropic and kinematic models have been applied to the calculation of the plastic strain energy per cycle for different strain conditions and compared with tests. The results obtained from the Mroz model are in a very good agreement with experimental data, while the isoparametric overextimates and the kinematic model underextimates the plastic energy. Fatigue curves for push-pull and torsion loadings calculated on the basis of integral form constitutive law and differential form constitutive low have been proposed. Two theories (Leis and Ellyin) suggest parameter sensitive to hydrostatic pressure. Neverthless the multitude of fatigue failure criteria is an indication of a leak of an universal theory which can describes the material answer to a complex stress state.

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