BIAXIAL CYCLIC PLASTIC BENDING

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INTRODUCTION.

Research in cyclic plastic bending was promoted by two major developments. Firstly the use of plastic methods of structural design with limit analysis and minimum weight design resulted in the acceptance that some elements in a structure would be subjected to plastic strains and that the loads on some structures would be cyclic, ie wind loads. As a result it was realised that structural members could fail by incremental collapse or ratchetting.

This phenomena has been observed and reported by many researchers often as the result of investigations into pressure vessel behaviour. Weil and Rapasky (1) showed that repeated thermal stresses caused progressive distortion. Coffin (2) in his work on fatigue under repeated loading of uniaxial specimens also observed incremental growth under combined steady and cyclic loads. Morton and Moffat (3) reported persistent cyclic strain increments in stainless steel pressure vessel components when repeated pressure loads exceeded the shakedown pressure. The shakedown problem has been extensively investigated and shakedown theorems developed by Neal (4) Leckie (5). It was shown that in many cases, although there might be initial plasticity, structural members would often shakedown under cyclic loads and eventually produce an elastic response. Parks (6) was probably the first to identify the various modes of deformation of components subjected to cyclic loading. Several problems of incremental collapse have been also examined by Edmunds and Beer (7) and Bree (8).

The other interest in cyclic bending arose when it was realised that to design for limited life was not necessary for many components and that many components were undergoing cyclic plastic strains in critical areas. During the start up and shut down of steam turbines plastic cyclic strains were shown to occur. However these cycles were often less than 1000 per year and therefore a 30 year life would only produce 30,000 cycles. Considerable research was undertaken to determine cyclic material properties. This work has been extensively covered by Manson et al (9) and Coffin (10).

The cyclic data was often obtained from push-pull tests, usually strain controlled. However this mode of testing was limited due to the buckling of specimens during the compression cycle even for small plastic strains. It was realised that a beam in bending could be considered as being made up of elemental fibres undergoing push-pull cycles. Therefore the fatigue life of a component in a tension-compression test would be the same as the outer fibre of a beam in cyclic bending at the same strain level. Failure in this sense refers to crack initiation in the outer fibre of the beam. It was thus possible to use the bending mode to generate quite large plastic cyclic strains. Conversely it was shown that the cyclic moment - curvature relationship could be predicted from cyclic push-pull data. Das (11) examined the problem of a strain concentration in bending by predicting the failure of a beam in 3 point bending using cyclic push-pull data. He then showed that the strains in the areas of the strain concentration could be predicted from the cyclic moment curvature relationship. Using these strains and a cumulative damage law he was able to predict the cycles to failure quite accurately.

UNSYMMETRICAL AND BIAXIAL PLASTIC BENDING.

These two forms of bending have many similar features. The analysis of unsymmetrical sections under the action of simple bending appears to have posed some difficulties. Even the standard text on the plastic theories of bending, do little more than mention the unsymmetrical problem. Clearly the more complex the shape of the cross-section the more difficult the problem becomes.

Heyman (12) studied the problem of unequal angle section cantilever and defined the "strong" and "weak" axis of bending. It can be appreciated that the analysis of a simple rectangular beam creates problems when the line of action of the bending force is at an angle to the principal axis of the cross section. There appears to be only two papers of any significance which investigate this problem and even in these the direction of the applied moment is assumed to be fixed.

Barrett (13) considered the plastic bending of a rectangular beam subjected to a pure bending moment acting in a plane other than one of symmetry. He showed that for a given angle of the plane of loading the neutral surface for a non elastic distribution of stress is rotated further than it would be if the stress distribution were elastic.

Brown (14) considered the unsymmetrical section as a general topic and established the concept of "centroidal locus". Brown gave no specific solutions but noted that the principal axis of elastic and plastic bending need not coincide. This was shown also to be the property of a section having only one axis symmetry, and that principal axes in plastic bending are not necessarily orthogonal.

The problem of biaxial cyclic plastic bending of cantilever lends itself to a form of finite element analysis. If we assume that plane sections remain plane during elastic and plastic bending, then the beam can be considered to be made up of elemental fibres undergoing push-pull cycles. If the cyclic material behaviour can be modelled mathematically then an attempt can be made to predict the cyclic load deflection characteristics of the cantilever in horizontal uni-directional cyclic bending, ie without a sustained follow-up load in the vertical direction. The analysis can then be extended to include the vertical load which then results in biaxial bending and incremental collapse. In order to assess the effects of a rotating neutral surface the cyclic material properties of each elemental fibre will need to be followed with each cycle and throughout each cycle.

CYCLIC MATERIAL PROPERTIES.

Cyclic material properties are usually characterised by the development of cyclic stress-strain hysteresis curves, and materials are observed to cyclically harden or soften, Fig (1). Annealed materials will usually show cyclic hardening characteristics while work hardened materials and others, hardened by heat-treatment, will cyclically soften. The cyclic strain hardening or softening is usually rapid in the early cycles and many materials then settle down to a steady state cyclic condition. Dugdale (15) has shown the same steady state hysteresis curve is developed for a particular strain level, irrespective of prior load history.

One of the simplest relationships representing the stress-strain curve in the plastic region is the power law.

$$\sigma = \sigma_o \epsilon^n \qquad (1)$$

where σ_o and n are material constants which adequately describe many materials. The amount of strain hardening is represented by the index n.

In this investigation material behaviour will be assumed to be that described by equation (1) because the cyclic material stress strain curves, show very small elastic regions.

If we fit a best curve, using least squares technique, to each of the cyclic curves then we can evaluate σ_o and n for each curve. A plot of σ_o, n against ϵ will give graphical equivalent to equations.

TEST MATERIALS AND EQUIPMENT.

Two test materials were used in this investigation. A low carbon steel (EN32B) and an 18/8 stainless steel (T321).

Two types of specimen were used:-

(1) Square beams, 12mm x 12 mm.
(2) Circular beams, 11.5 mm diameter.

Both materials were supplied in the form of 12 mm x 12 mm square bar which were then machined to fit the test rig.

The test rig and test procedures is fully described elsewhere (15). Essentially, the bars were rigidly clamped at an end and a steady downward force P_y applied at the free end. A cyclic horizontal force P_z was then applied at the free end which was deflection controlled. Load cells and displacement transducers monitored loads and deflections which were used to control the cyclic processes.

ANALYTICAL APPROACH.

If the cross section of the beam is divided into a number of elemental fibres, Fig (2), then as the moments M_Y and M_z are applied, the stress and deformation of each elemental fibre can be identified and its position on the monotonic or cyclic stress-strain curve established.

The effect of unloading "elastic" elements as they are crossed or approached by the neutral surface is obviously zero because no Bauschinger effect is involved. However the unloading and reverse loading of elements which have been plastically deformed cannot be ignored. In fact they carry a reduced load which results in an increased load carried by other elements and hence increased stress.

Clearly the problem is of a complex nature, complicated by the fact that elements are unloading from different stress-strain states and therefore it is important that realistic material date is used and a record of individual elements' load history is kept.

If we assume that the material is such that it settles to steady state conditions after the initial plastic deformation then the material behaviour can be modelled using equation (1).

Now the resulting moment at any cross-section is given by

$$M = \sum_{\text{element} = 1}^{\text{element} = i} \sigma . v . \delta A \qquad -(2)$$

where i is the total number of elements making up the cross section.

and v is the perpendicular distance between the neutral surface and the centroid of the element.

The problem of establishing the position of the neutral surface for a rectangular cross-section is a complex one. Firstly because the rotation is a function of the material property and each element within a cross section is unloading from a different state of stress-strain curve. Therefore it could be regarded that each element has a different material property. Secondly because a cantilever beam could be totally elastic at one end and totally plastic at the other. A typical neutral surface could be positioned within the beam as shown in Fig (2).

It will be assumed that over a small elemental length the angle of rotation of the neutral surface is constant.

COMPUTER PROGRAM.

The analysis proposed clearly lends itself to a computer program, due to a vast amount of arithmetic operations and "book-keeping" involved.

The basic procedure adopted in the program is as follows:-

1. Apply P_y - constant vertical load.

2. Increment P_z from zero to Pmax.

3. Set strain value at the onset of plastic deformation.

4. Calculate geometry of element and hence its area, and co-ordinates with respect to the centroid.

5. Estimate initial strain distribution.

6. Evaluate stress using the relationship given in equation (1).

7. Calculate the internal bending moment.

8. Check if $M_{CALCULATED} = M_{APPLIED}$

   if this condition is satisfied then move to step 10.

9. Adjust estimated value of maximum strain and return to step 5.

10. Record all data associated with each element on Magnetic disk ie its load history and maximum strains.

Procedures 1 to 10 are repeated for all sections of the beam to evaluate strains. The method developed in the program usually require three to five iterations depending on the initial estimation to establish strains.

When P_z is increased in step 2, the material data is updated according to equation (1), and based on elemental load history. From the strains the curvature (K) is evaluated from the relationship $\epsilon = vK$ where v is the distance from the neutral surface and the deflections are then calculated.

RESULTS AND DISCUSSION.

Both materials cycled to a steady state condition after a few cycles in push-pull tests. The experimental data for the low carbon steel (EN32B) is shown in Fig (3), together with a best fit curve obtained from equation (1). Similar results obtained for the stainless steel are shown in Fig (4). These tests showed the dependence of σ_o and n on the cyclic strain range, and these constants were determined for the steady state cyclic condition. The variation in n is shown in Fig (5), and the variation in σ_o is shown in Fig (6). It can be seen that, for both materials, the constants rapidly approach constant values for cyclic strain greater than 1.2%.

Using this data the material properties of each elemental fibre shown in Fig (2) can be identified and the data incorporated into the Computer Program. The validity of this approach was tested by, firstly, calculating the cyclic load deflection characteristics for horizontal cyclic deflections only, ie without the vertical load which would cause incremental collapse. The results for the stainless square beam is shown in Fig (7), and for the circular beam in Fig (8). The results for the low carbon steel bars are shown in Fig (9) and Fig (10). The generally good agreement between experiment and theory is encouraging. The errors are felt to be consistent with our inability to model material behaviour more accurately with a simple power law.

When vertical loads are applied the neutral surface will rotate as the horizontal loads are applied at the end of the cantilever. This results in some elemental fibres unloading and being reloaded in the opposite direction, with a subsequent Bauschinger effect. It was noted that there were slight differences in the cyclic load-deflection characteristics in the forward and reverse loading mode. The general principles adopted are shown in Fig (2), where particular elemental fibres are identified together with its position within the stress-strain hysteresis curve. The cyclic load deflection characteristics for mild steel (low carbon) and stainless steel square beams are shown in Fig (11) and Fig (12). It can again be seen that the theoretical predictions are good and the differences are consistent with our inability to mode material behaviour more accurately.

CONCLUSIONS.

The work has shown that accurate material data is a pre-requisite to any analysis of cyclic plastic bending and that accurate mathematical modelling of this data is also essential. The non-radial loading problem presents many more difficulties than radial loading because the movement of the neutral surface causes individual elements to unload and re-load during a cycle of bending.

The iterative "finite element" method allowed material properties to be updated with each increment of strain and the position of the neutral surface to be located. The characteristics of the cyclic stress-strain curves, with a negligible elastic component on stress reversal, leads to accurate modelling with relatively simple equations.

However, if there is an extensive elastic region then more complex equations are required, otherwise gross errors in the prediction of loads and deflection occur.

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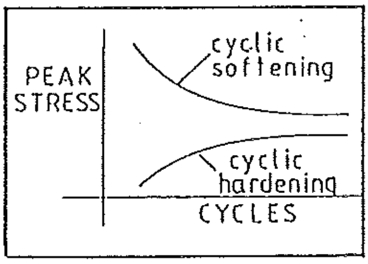
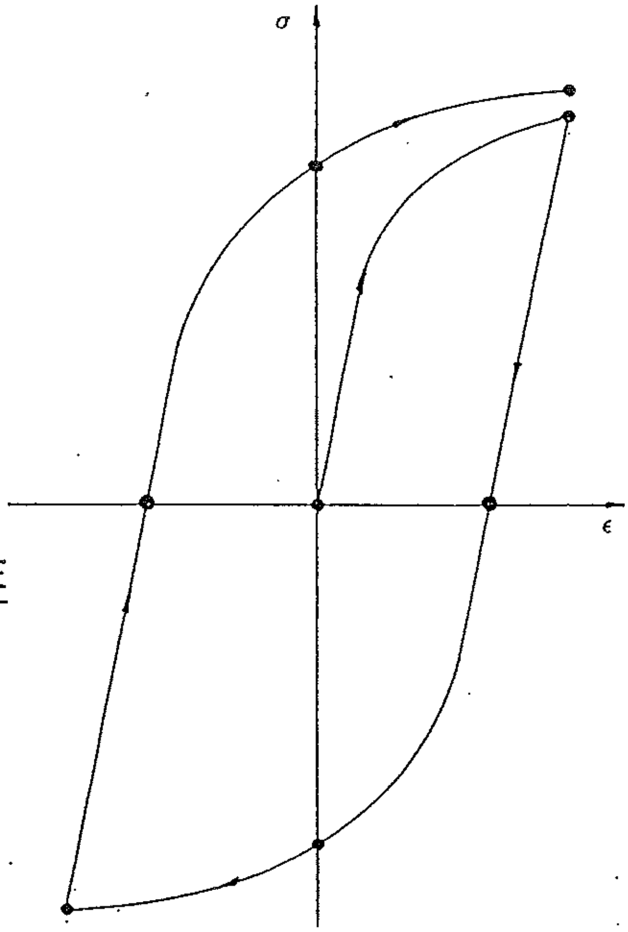
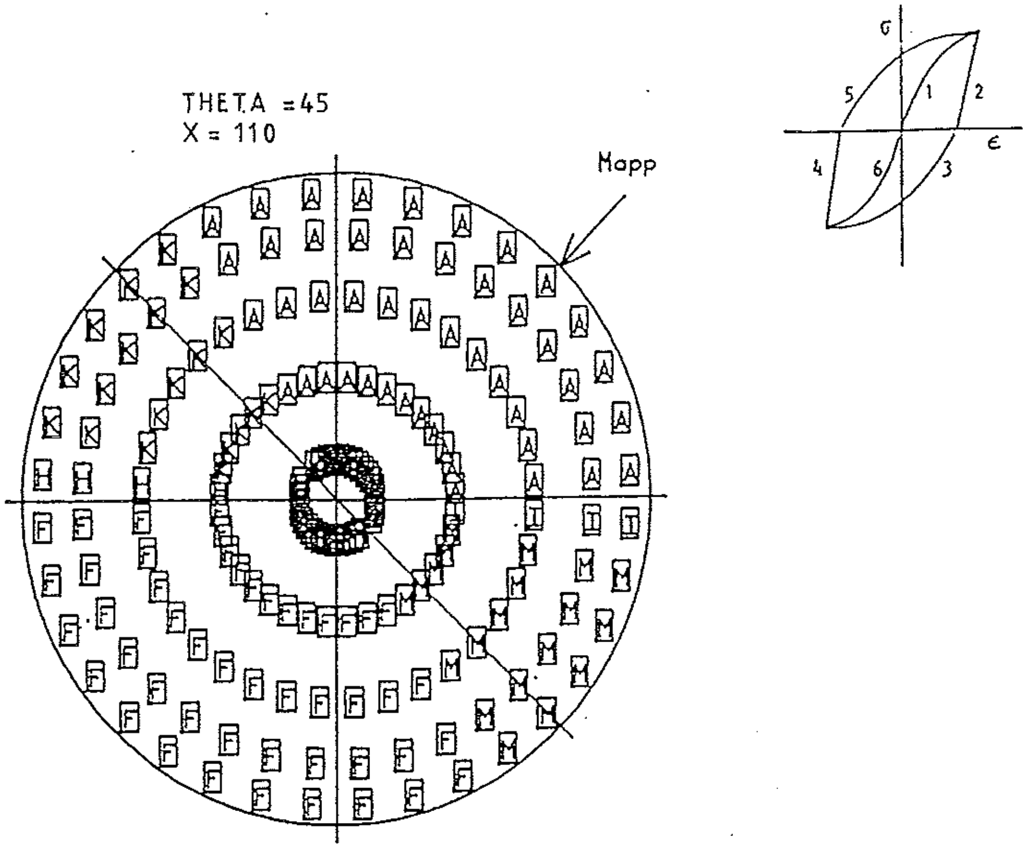




HYSTERESIS CURVE

DIMENSIONS

FIG. 1



A	1
B	2
C	3
D	4
E	5
F	6
H	1 - 6
I	6 - 1
J	1 - 2
K	1 - 2 - 3
L	6 - 4
M	6 - 4 - 5
N	3 - 4
P	3 - 4 - 5
R	5 - 2
S	5 - 2 - 3
W	2 - 3
X	4 - 5

FIG.2

CYCLIC STRESS/STRAIN MAT'L. M.S. EN32B - STRAIN +/-1.25%

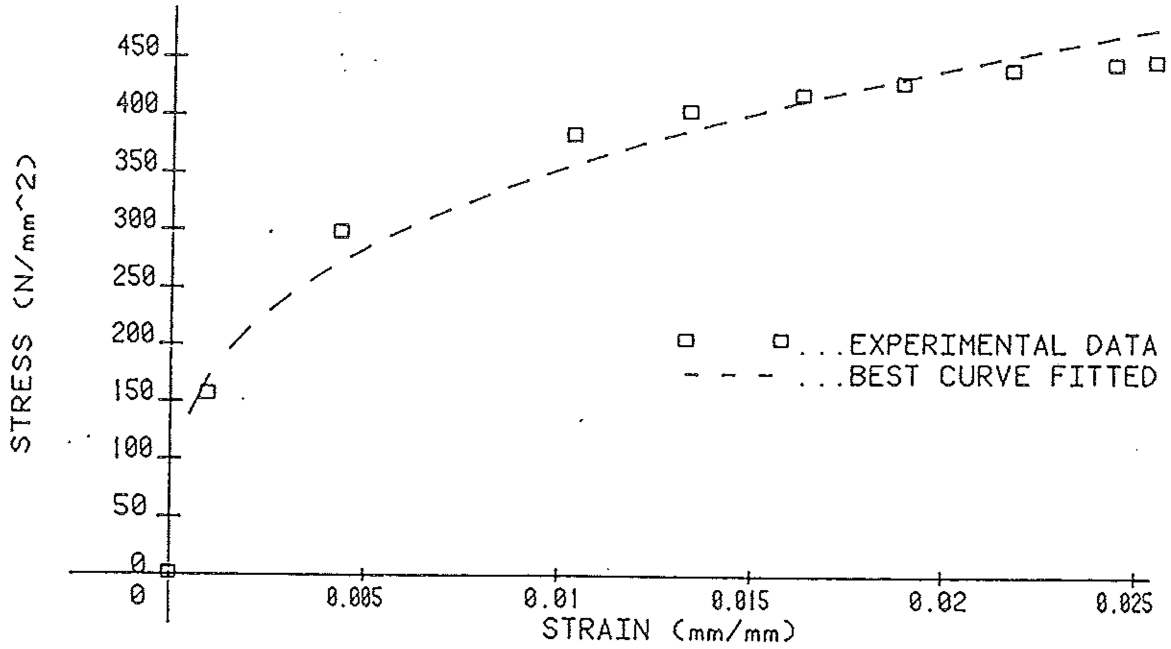


FIG.3

CYCLIC STRESS/STRAIN - MATERIAL S.S.321 - STRAIN +/-2%

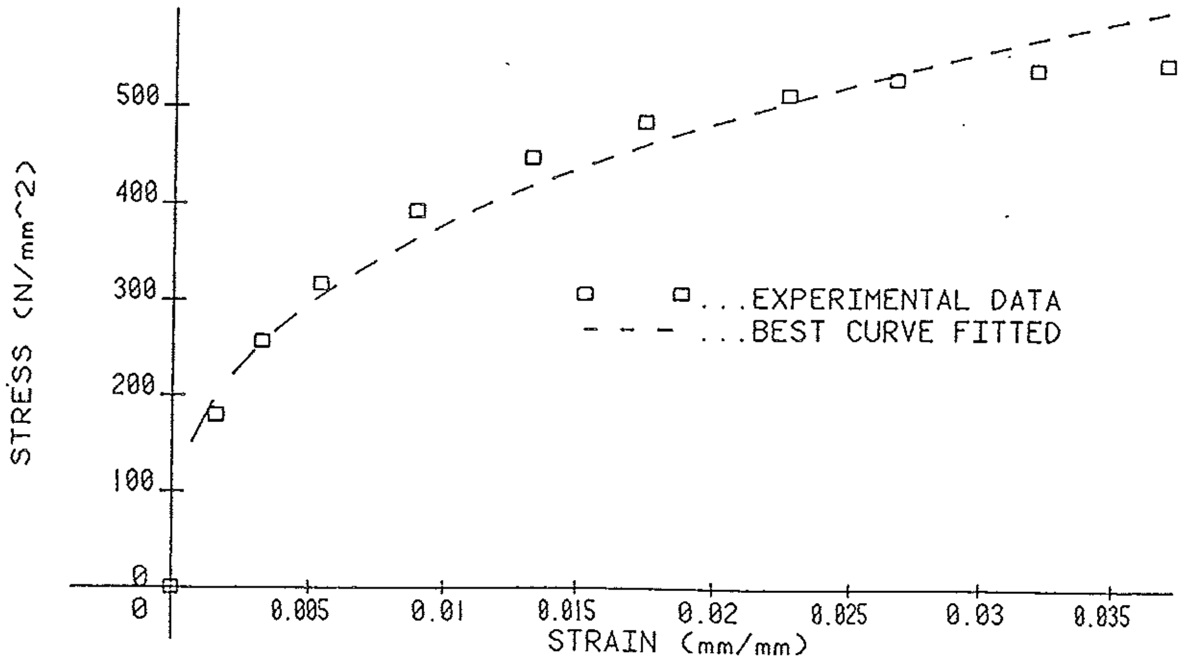


FIG.4

VARIATION OF MATERIAL CONSTANT n WITH STRAIN

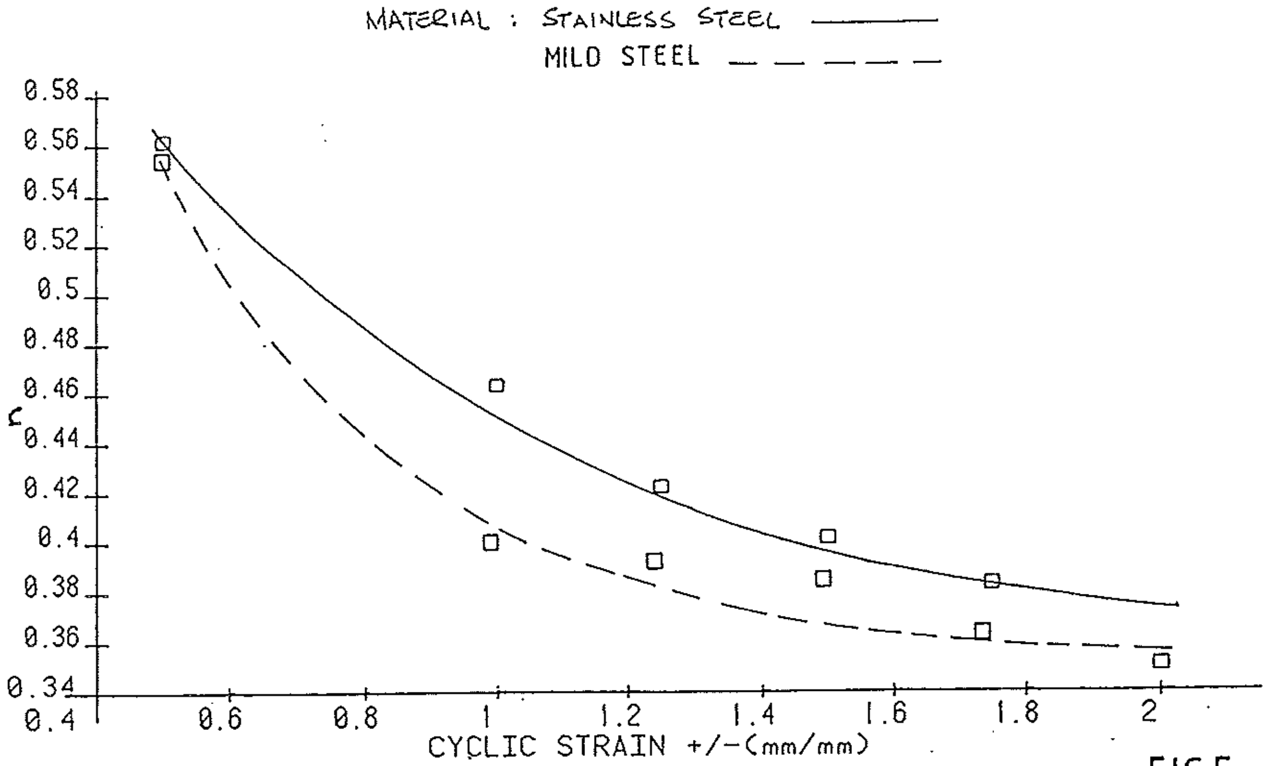


FIG.5

VARIATION OF MATERIAL CONSTANT σ_o WITH STRAIN

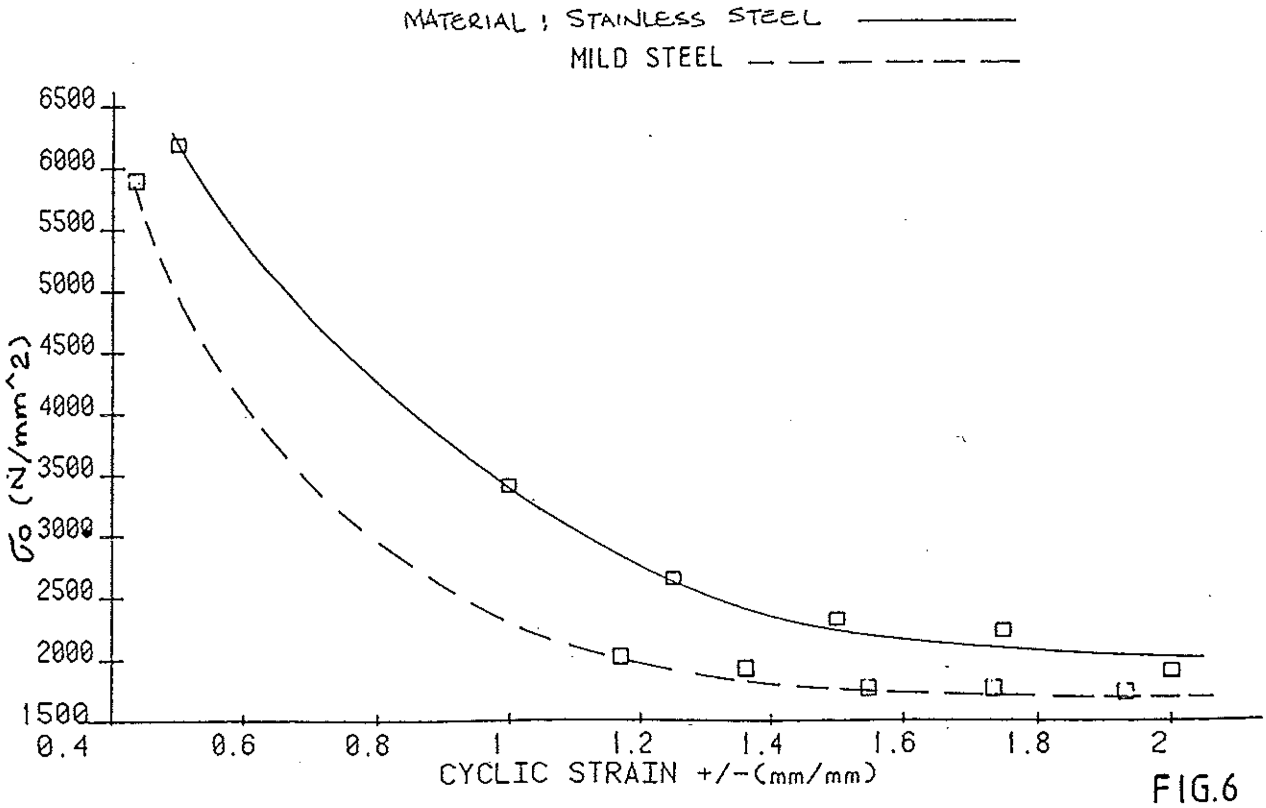


FIG.6

CYCLIC LOAD / DEFLECTION CHARACTERISTICS (Py=000)

STAINLESS STEEL - SQUARE BEAM

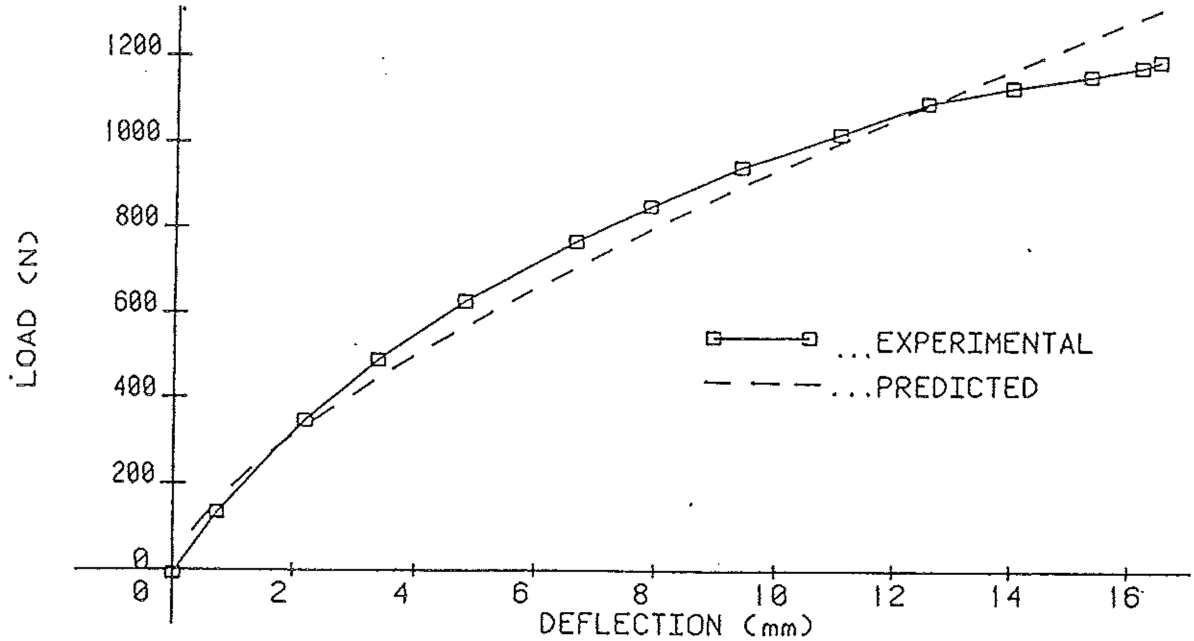


FIG.7.

CYCLIC FORCE / DEFLECTION CHARACTERISTICS (Py=000)

STAINLESS STEEL - CIRCULAR BEAM

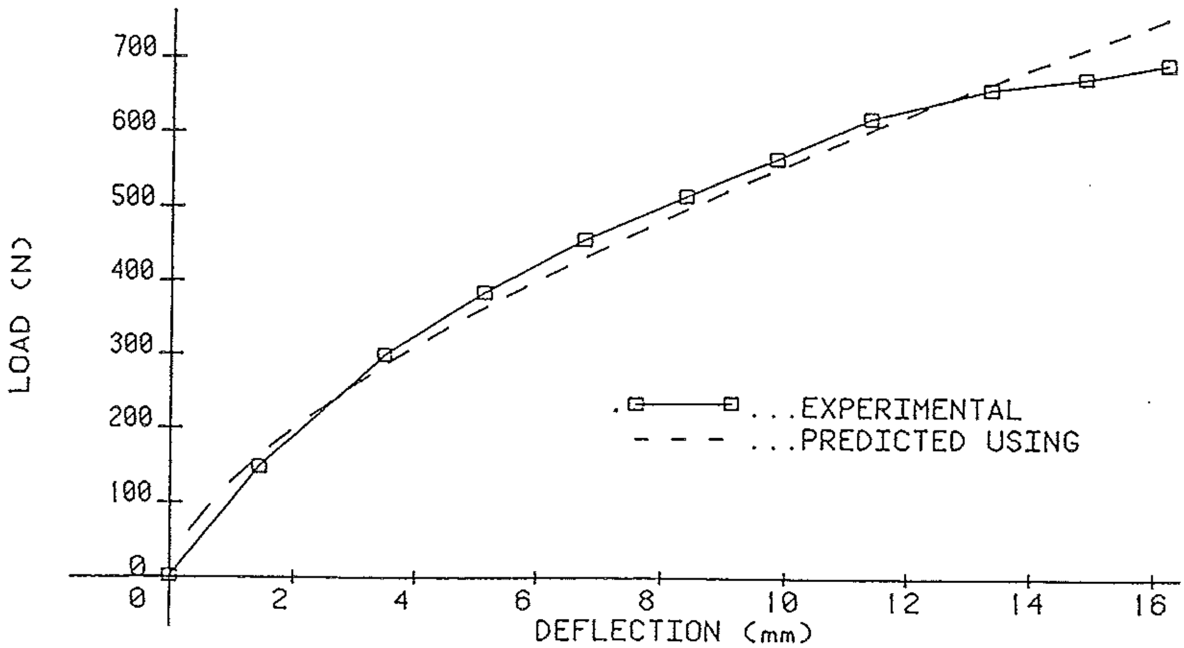


FIG.8

CYCLIC LOAD / DEFLECTION CHARACTERISTICS (Py=000)

MILD STEEL : SQUARE BEAM

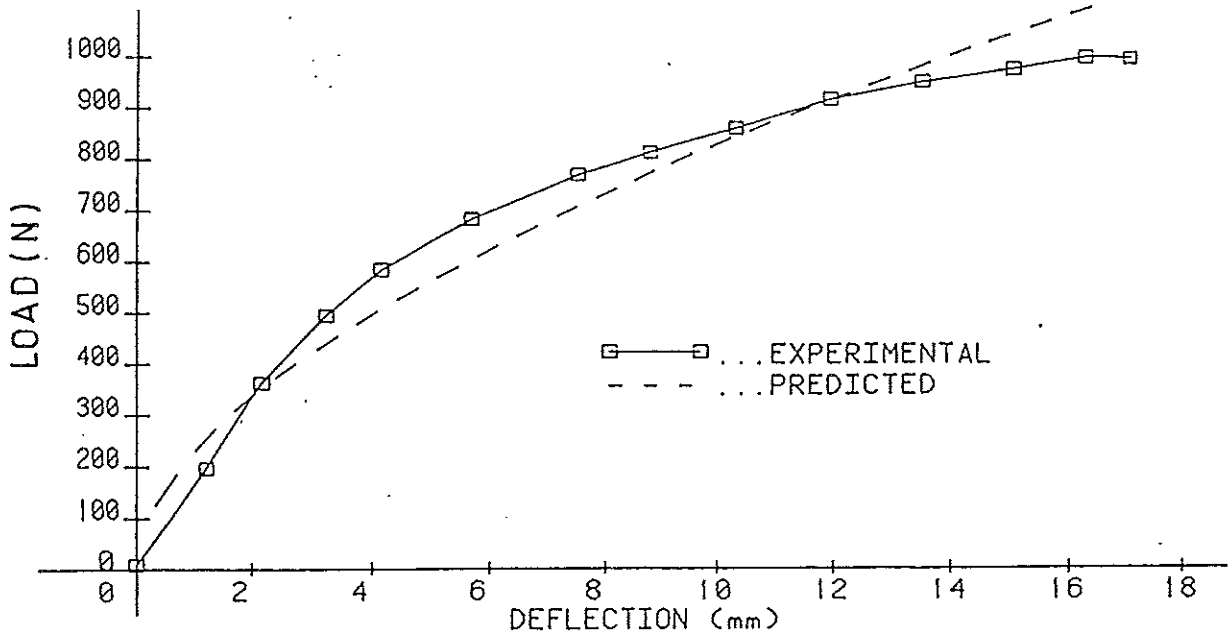


FIG.9

CYCLIC LOAD / DEFLECTION CHARACTERISTICS (Py=000)

MILD STEEL : CIRCULAR BEAM

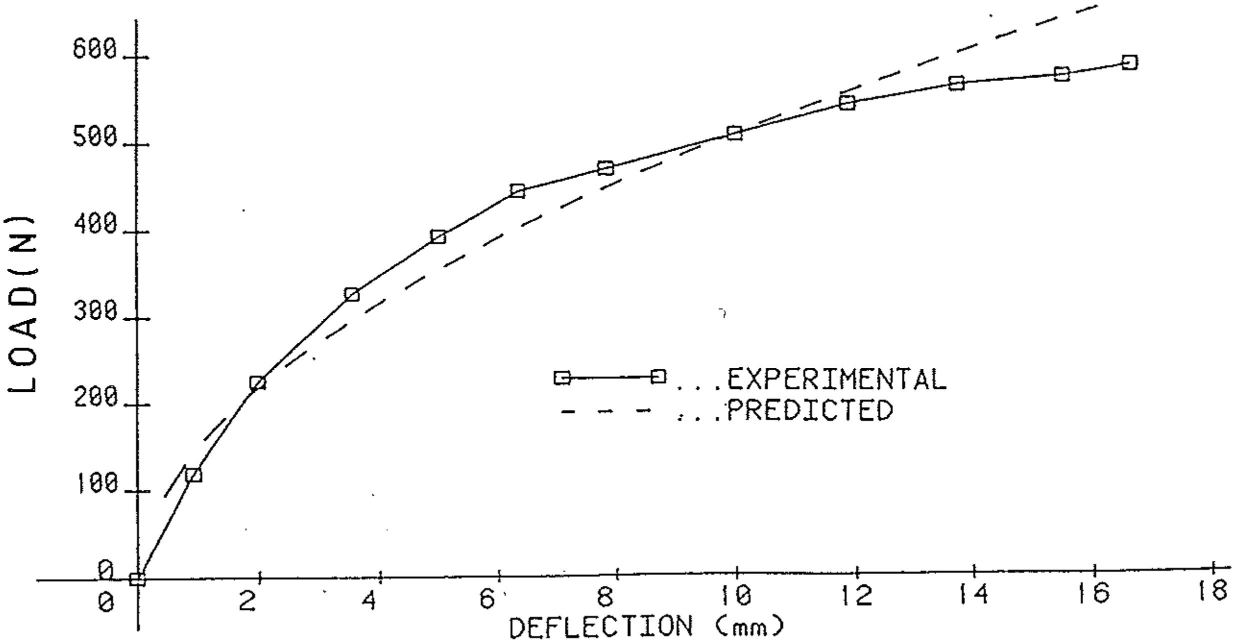


FIG.10