A VARIABLE AMPLITUDE MULTIAXIAL FATIGUE LIFE PREDICTION METHOD

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NOMENCLATURE

b	= fatigue strength exponent
c	= fatigue ductility exponent
D	= damage on a plane
E	= modulus of elasticity
G	= modulus of rigidity
n	= number of cycles in history
N_{fi}	= fatigue life for i^{th} cycle
$2N_f$	= reversals to failure
ε_f'	= fatigue ductility coefficient
$\Delta\varepsilon_1/2$	= maximum principal strain amplitude
$\Delta\varepsilon/2$	= tensile strain amplitude on an arbitrary plane
γ_f'	= fatigue shear ductility coefficient
$\hat{\gamma}$	= maximum shear strain amplitude
σ_f'	= fatigue strength coefficient
σ_y	= yield stress
σ_1	= maximum normal stress on maximum principal strain plane
σ_n	= maximum normal stress on $\hat{\gamma}$ plane
σ	= tensile stress acting on $\Delta\varepsilon/2$ plane
τ_f'	= fatigue shear strength coefficient

ABSTRACT

A methodology for the assessment of variable amplitude multiaxial fatigue damage is presented. The procedure, which is based upon an extension of critical

plane concepts, uses experimental strain gauge rosette data or analytical strain data as input. The corresponding stresses in the loading history are determined using a simple plane stress, non-proportional cyclic plasticity model. Damaging events occurring on an arbitrary plane are identified by rainflow counting the strain history acting on this plane. The stresses and strains corresponding to these events are used in appropriate critical plane multiaxial damage models. Damage is summed for the total loading history. This process continues until damage for all planes is evaluated. The plane experiencing the maximum damage is defined as the critical plane, and the fatigue life, in blocks to failure, is then determined using the damage calculations on this plane. Results from a computer model which implements this technique agree well with experimental results.

INTRODUCTION

Engineering components are often subjected to complicated states of stress and strain. Complex stress states--stress states in which the three principal stresses are non-proportional or whose directions change during a loading cycle--very often occur at geometric discontinuities such as notches or joint connections. In addition, the loading applied to the component may be of varying amplitude. Fatigue under these conditions, termed variable amplitude multiaxial fatigue, is an important design consideration for reliable operation and optimization of engineering components and structures. Currently, there exists a need for a method to estimate the fatigue life of a component subjected to this type of loading. The following paper presents a technique to do this.

BACKGROUND

Relative success has been obtained using the strain life approach for variable amplitude loading of notched components subjected to uniaxial fatigue. The strain life approach which is described in detail in such references as Refs. 1,2,3, is based upon the observation that in many components the response of the material in critical locations is strain or deformation controlled. Reference 4 is a compendium of ten papers that compares the results of life predictions to experimental uniaxial variable amplitude data.

It has only been in the recent past that multiaxial fatigue research has progressed to a point that the thought of extending uniaxial fatigue concepts to variable amplitude multiaxial fatigue loading has been seriously considered. Hoffman and Seeger [5] have outlined some of the tools needed and the current limitations that have existed in extending the local strain approach to multiaxial fatigue. Socie [6] has suggested a method based upon the strain life approach similar to the one presented here.

To predict the fatigue life of a component subjected to multiaxial loading using the strain life approach, a damage model is required. Articles that have reviewed and compared multiaxial damage theories have been published in the literature and several are given in Refs. 7-11. Critical plane approaches, first proposed by McDiarmid and further developed and extended by researchers such as Brown and Miller, Lohr and Ellison, as well as others, are founded upon a physical interpretation of the fatigue process. These approaches are based upon observations that cracks form and grow on critical planes. Critical plane models are able to account for such effects as out-of--phase hardening and the sensitivity of fatigue life to hydrostatic pressure. It has been shown, however, that the appropriate choice of a critical plane model must be representative of the dominant or controlling parameters consistent with observed damage [12,13]. This is discussed in the following section.

MULTIAXIAL FATIGUE DAMAGE

For a variable amplitude loading situation, the critical step in fatigue life prediction is relating the multiple stresses and strains and the variation of these to fatigue damage. The question is then: What constitutes or promotes damage in a variable amplitude loading situation?

Materials generally form one of two types of cracks--either cracks shear or tensile cracks--depending upon strain amplitude, material type and stress state [12]. Therefore, for a strain range and material that tends to develop tensile cracks, it is obvious that damage has some sort of dependence on normal strain, normal stress, or some combination of both. Alternatively, when shear cracking is observed, the damage is dependent upon shear terms which may be modified by some normal stress or strain terms. For example, the following tensile based damage parameter (the Smith-Watson-Topper [14] parameter applied to multiaxial loading)

$$\frac{\Delta\varepsilon_1}{2}\sigma_1 = \frac{\sigma_f'^2}{E}(2N_f)^{2b} + \sigma_f'\varepsilon_f'(2N_f)^{b+c} \qquad (1)$$

$\Delta\varepsilon_1/2$ = maximum principal strain amplitude

σ_1 = maximum normal stress on maximum principal strain plane

was significantly superior in correlating results of a material which developed damage and failed on the maximum tensile strain range plane [15]. Conversely, it was shown that use of a shear based parameter similar to that proposed by Fatemi and Socie [16]

$$\hat{\gamma}\left(1+\frac{\sigma_n}{\sigma_y}\right)=\gamma_f'(2N_f)^c+\frac{\tau_f'}{G}(2N_f)^b \qquad (2)$$

$\hat{\gamma}$ = maximum shear strain amplitude

σ_n = maximum normal stress on $\hat{\gamma}$ plane Δ

resulted in better correlation of fatigue lives for a material whose damage development was shear dominated.

Thus the appropriate damage parameters must be consistent with the observed damage. (The exact form of these models will undoubtedly change as more multiaxial fatigue test data becomes available. However, the need for the two damage models remains.) In addition, the variation of these components with respect to time and respect to each other must be accounted for. How to choose and interpret the damage model for variable amplitude loading then is a major question.

To begin to understand this, an example is provided. A simple case of variable amplitude multiaxial loading is presented in Fig. 1. An AISI 304 stainless steel (SS304) thin wall tube was subjected to the tension-torsion strain controlled history shown, termed a four-box loading path. (Material specifications and geometry details are given in Ref. [12].) The axial strain, ε_x, history and the shear strain, γ_{xy}, history for this non-proportional loading path are also shown in Fig. 1. The axial and torsional stress response and the stress-strain responses are shown in Fig. 2. Since SS304 has been observed to form tensile cracks for a wide range of strain amplitude and loading modes, a damage parameter based upon tensile stresses or strains, or some combination of these, is needed. However, the specific choice or interpretation of the critical plane damage parameter for variable amplitude loading requires an understanding how the applied loads combine to produce unique stresses and strains on each plane in the material.

For a general multiaxial loading path, the normal strain on any plane varies with time. The normal strain history can be computed by rotating the axial and shear strains to this plane for all points of time in the loading history. For example, the normal strain history for the plane rotated -20 degrees (20 degrees clockwise from the horizontal) is shown in Fig. 3. Similarly, the normal stresses on any plane can be computed by rotating the axial and shear stresses to this plane. The stress-strain response for the loading history on any plane can be thus obtained. Figure 4 shows the normal stress-strain response on various planes for the four-box loading path. (If the loading was completely uniaxial, the plane experiencing the maximum tensile stress or strain would be the 0 degree plane while in a completely reversed torsion test, the +45 and -45 degree planes experience the maximum tensile stress or strain.)

As can be seen in Fig. 4, if one was to define the critical plane as the plane experiencing the largest range of tensile strain, then both the planes oriented at +30 and -30 from the horizontal would experience the same damage. However, the maximum damage (cracking) occurred on a plane between -30 and -40 degrees as shown in Fig. 5. Only after looking at the stress-strain response on these +30 and -30 degree planes can one explain the preferential cracking on the -30 degree plane. This plane experiences a maximum normal or peak stress 10 percent larger than the +30 degree plane. Therefore, the damage parameter must include some combination of normal stress and strain.

One approach that has been used successfully for very short repeating load histories has been the SWT parameter given in Eq. (1). For a variable amplitude loading situation though, the interpretation of this parameter must be slightly modified. An easy example to emphasize this is the case of one cycle of tension followed by many smaller cycles of torsion. It is intuitively obvious that the plane experiencing the largest alternation of tensile strain will not be the failure plane. Rather, the many thousands of torsion cycles will cause the damage, and the critical plane will be the plane most damaged by the torsion cycles.

Therefore, for a variable amplitude loading situation, the critical plane may be defined as the plane experiencing the maximum damage rather than the maximum strain range. In a variable amplitude situation, damage is summed throughout the loading history using an appropriate damage parameter consistent with observed damage. Damage on all planes is evaluated and the critical plane is determined from the maximum value of the summed damage. From this, the fatigue life may be calculated. This procedure is explained in detail in the following discussion of the variable amplitude multiaxial fatigue life prediction method.

VARIABLE AMPLITUDE MULTIAXIAL FATIGUE LIFE PREDICTION MODEL

A technique to predict the fatigue life of a component subjected to variable amplitude multiaxial loading has been developed. A computer code based upon this approach has been implemented. The technique and description of the model and computer code is first reviewed below with some of the complexities and subtle details discussed. Results from the computer model are then compared to the experimental results of the four-box loading path.

The basic outline of the computer program is presented schematically in Fig. 6. As shown, strain data, either from a strain gauge rosette or from a known or approximated history (such as obtained from a finite element analysis) is used as input. Since rainflow counting is used for cycle counting, the strain gauge input data must be inspected for peaks. Unlike uniaxial fatigue procedures, however, where only one channel of strain must be inspected, in a multiaxial situation using strain gage rosette data, three channels of strain must be monitored for peaks. Any time one channel reaches a peak, the three values of strain must be simultaneously stored. This maintains the phase relationship between the strain peaks for all points in time.

From the strain histories, stress histories must be measured or calculated. For experiments conducted on thin walled tubes in the laboratory, the stresses may be measured. In most situations, however, the stresses must be calculated. Since fatigue problems occur in critical locations where the stress exceeds the yield stress, a non-proportional cyclic plasticity model must be used. Fortunately, fatigue cracks often occur on the surface of the component where the stress state is a plane stress situation ($\sigma_z = 0$, $\tau_{xz} = 0$, and $\tau_{yz} = 0$). This simplifies the calculations.

Incremental plasticity models have been developed by researchers [17-24] and have been reviewed and compared [25,26]. Generally these models employ a von Mises or Tresca yield function and use the normality flow rule. The hardening rules used include isotropic, kinematic, or combinations of both. Models which have been observed to have the best success in predicting stress response compared to experimental observations use the Mroz kinematic hardening rule.

Researchers dealing with non-proportional multiaxial plasticity models have continued to develop sophisticated models that attempt to reproduce the detailed material response including transient hardening effects due to non-proportional loading. However, for applications to variable amplitude block loading a cyclic stable material response will be assumed, just as in the case of uniaxial loading.

The plasticity model used is a two-surface model with a fixed limit surface, similar to that proposed by Lamba [22]. Lamba developed a model for tension-torsion loading. However, in this case, the model must be generalized to include a full plane stress situation. The model uses a von Mises yield criteria with Mroz hardening and the normality flow rule. These calculations result in the stress histories for σ_x, σ_y, and τ_{xy}, and the strain history, ε_z.

The assumption made in the application of the strain-life approach for variable amplitude uniaxial fatigue life predictions is that the total strain history of the component may be represented by a repeating block of cycles. This assumption is also made in the method proposed here for multiaxial fatigue. This justifies neglecting the transient material behavior since all transient response will occur in the first several loading blocks. However, because the stress-strain response of a material is path dependent, set-up cycles must be appended to the beginning of the strain history. After the first loading block, the stress and strain response is no longer the same as that applied to a component which has experienced no loading. Therefore, it is assumed that the material will remember the largest cycle in the previous block and the stress-strain response in the current block will be thus affected.

Once the stress and strain histories are known, the critical plane damage models are used to predict fatigue life. However, the question arises: What is the critical plane? In some instances, the critical plane may be known a priori. For example, in the SAE notched shaft, Fash [27] found that in many cases cracks initiated and damage developed in the plane of the notch. However, in the general case, the critical plane is not known beforehand. For the situation where the critical plane is not previously known, the critical plane is defined as the plane experiencing the maximum damage. This requires that damage on all planes be computed as follows.

In this proposed technique, the stresses and strains on an arbitrary plane are determined. Damaging events--the appropriate cycles of strain--are identified on this plane. For example, for a tensile crack dominated material, the tensile strain, $\Delta\varepsilon/2$, on the plane is rainflow counted while for a shear crack dominated material, cycles of shear strain are rainflow counted. (An efficient rainflow counting scheme must be used that does not require the maximum peak to be determined before counting begins. Consequently, a one-pass algorithm like the one proposed in Ref. [28] is used.) Once a cycle is identified the damage parameter is determined. For example, using a tensile damage parameter similar to that used in Eq. (1), the damage parameter is the product

$$\frac{\Delta\varepsilon}{2}\sigma$$

where σ is the peak tensile stress during the current cycle of strain on this plane. Fatigue life corresponding to the magnitude of this damage parameter can then be determined from the uniaxial material properties used in an appropriate damage model such as Eq. (1). This is done for all cycles in the history for this plane. Using Miner's rule

$$D = \sum_{i=1}^{n} \frac{1}{N_{fi}}$$

where n is the number of cycles and N_{fi} is the fatigue life for the i-th cycle, a damage value, D, can be determined for this plane. This process continues until the damage corresponding to all possible planes is evaluated. The fatigue life of the component subjected to multiaxial variable amplitude loading, in blocks to failure, is then determined from the plane experiencing the maximum damage.

RESULTS AND DISCUSSION

This method was used to evaluate the fatigue life of the SS304 thin walled tube subjected to the loading history shown in Fig. 1. The measured stresses and strains were used as input with the results of the analysis shown in Fig. 7. As can be seen, the plane predicted to experience the maximum damage (minimum fatigue life) is oriented at -20 degrees from the horizontal resulting in a fatigue life of 150,000 cycles with the damage on the -20 and -30 degrees planes almost equal. These results are in good agreement with the actual fatigue life of 85,000 cycles with the failure crack oriented at an angle between -30 and -40 degrees.

The method seems to work well for variable-amplitude, low-cycle fatigue tests conducted in the laboratory. The critical test of this method will be the ability of the model to predict fatigue lives of actual components. Tests are currently be conducted and the model will be evaluated using this data. It is evident, however, that a multiaxial method must be used for fatigue life predictions for components subjected to multiaxial variable amplitude loading. Attempting to employ uniaxial techniques to multiaxial situations will result in errors and inaccuracies due to the multiaxial stress-strain response as discussed below.

An interesting example of the importance of considering the mutiaxiality effects of the stress and strain response is shown in Fig. 4. In this figure, the strain history on the -20 degree plane is shown. If a uniaxial analysis technique had been employed using this strain history as input, the stress history predicted would have been very different from the stress-strain curve for the -20 degree plane shown in Fig. 4. The unusual subcycles which are "hung on the outside" of the major hysteresis loop would not have been predicted using the uniaxial analysis technique. Although in this case the subcycles do very little damage, in a randomly varying load situation where the subcycles do a significant portion of the damage, this error in the location of the subcycles may significantly affect life predictions.

The variable amplitude multiaxial fatigue prediction method presented here combines the successful strain life variable amplitude techniques developed for uniaxial fatigue with procedures used to account for the multiaxial effect of the stress and strain. Multiaxial damage models that incorporate the parameters consistent with observed damage in the material must be used. The damage models currently used in the technique presented here are typical of those needed, however, further refinement of these damage models may be made. These refined models could then be employed in the variable amplitude life prediction technique outlined in this paper.

CONCLUSIONS

Damage caused by multiaxial stresses and strains must be summed on all planes for a randomly varying load history. Fatigue life predictions can then be made by relating the maximum damage to fatigue life on the critical plane.

A variable amplitude multiaxial fatigue life prediction method has been developed. Results from a computer model implementing this procedure agree well with laboratory test results.

ACKNOWLEDGMENTS

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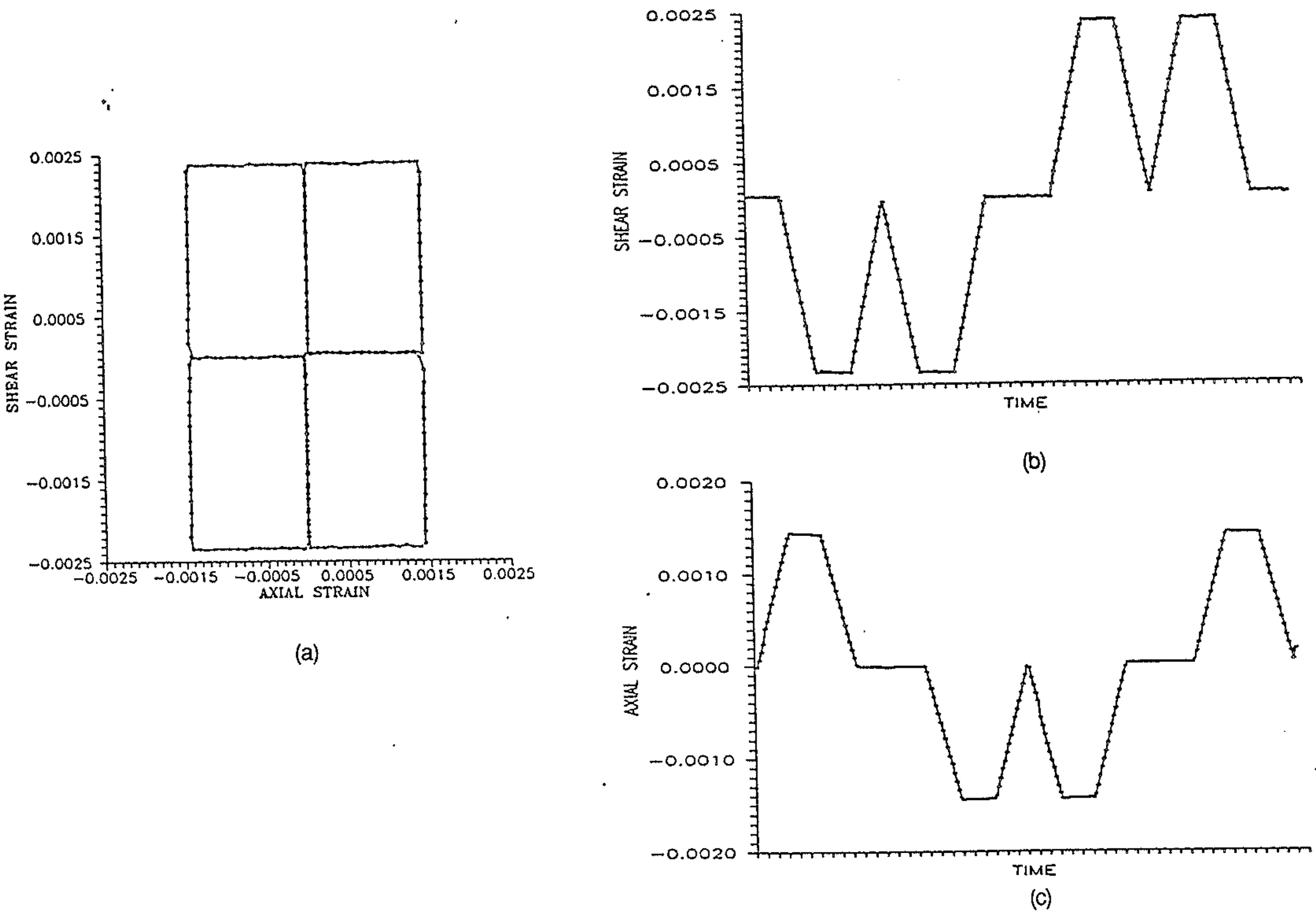

Figure 1 (a) Tension-Torsion Strain Controlled Four Box Loading Path (b) Shear Strain History (c) Axial Strain History

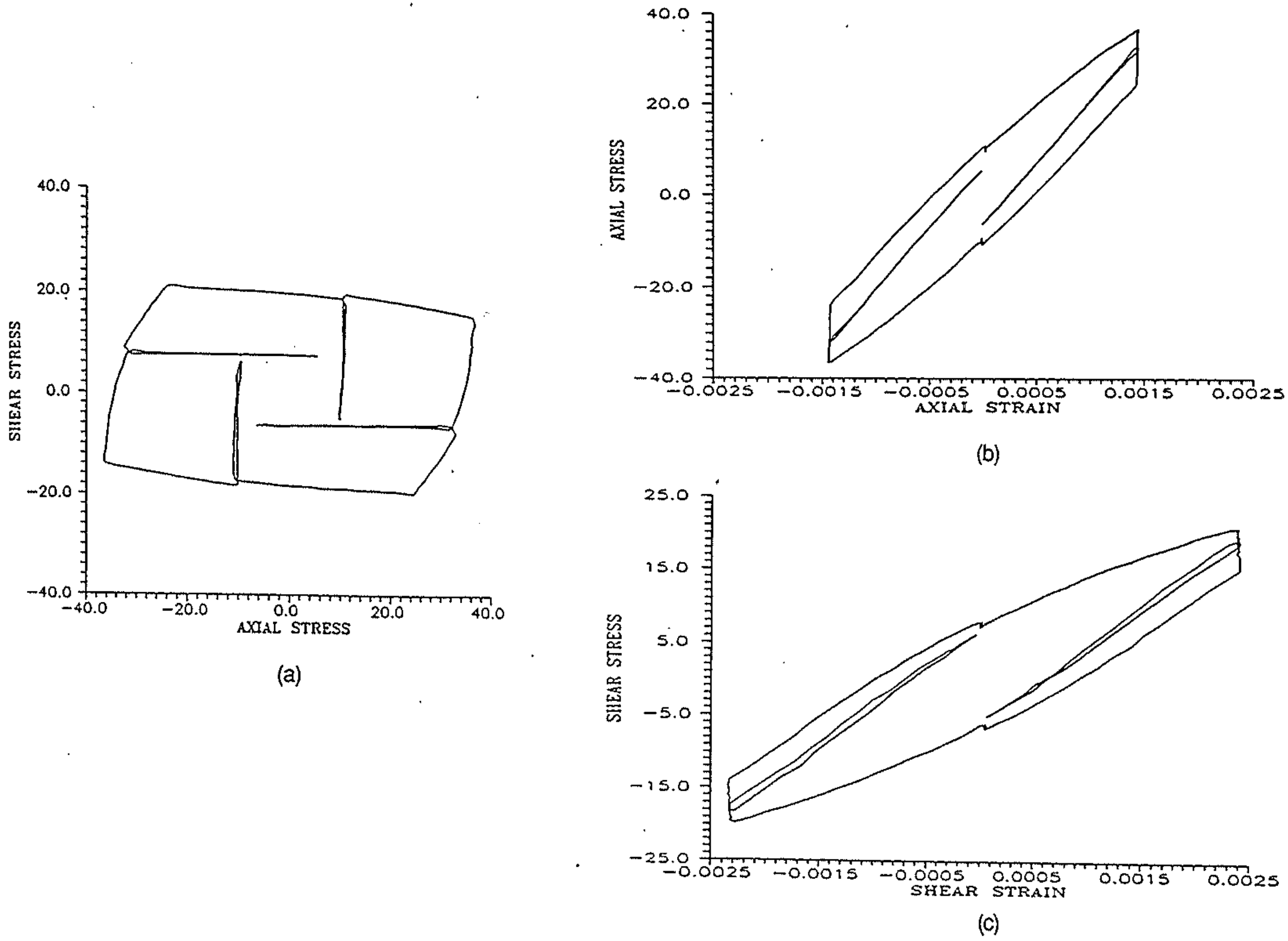

Figure 2 (a) Axial-Shear Stress Response (b) Axial Stress-Strain Response (c) Shear Stress-Strain Response

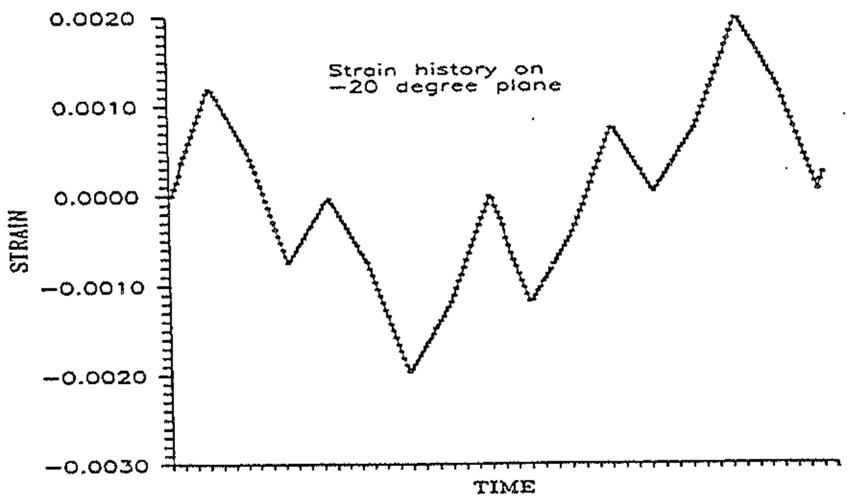

Figure 3 Normal Strain History on Plane Rotated -20 Degrees from Horizontal

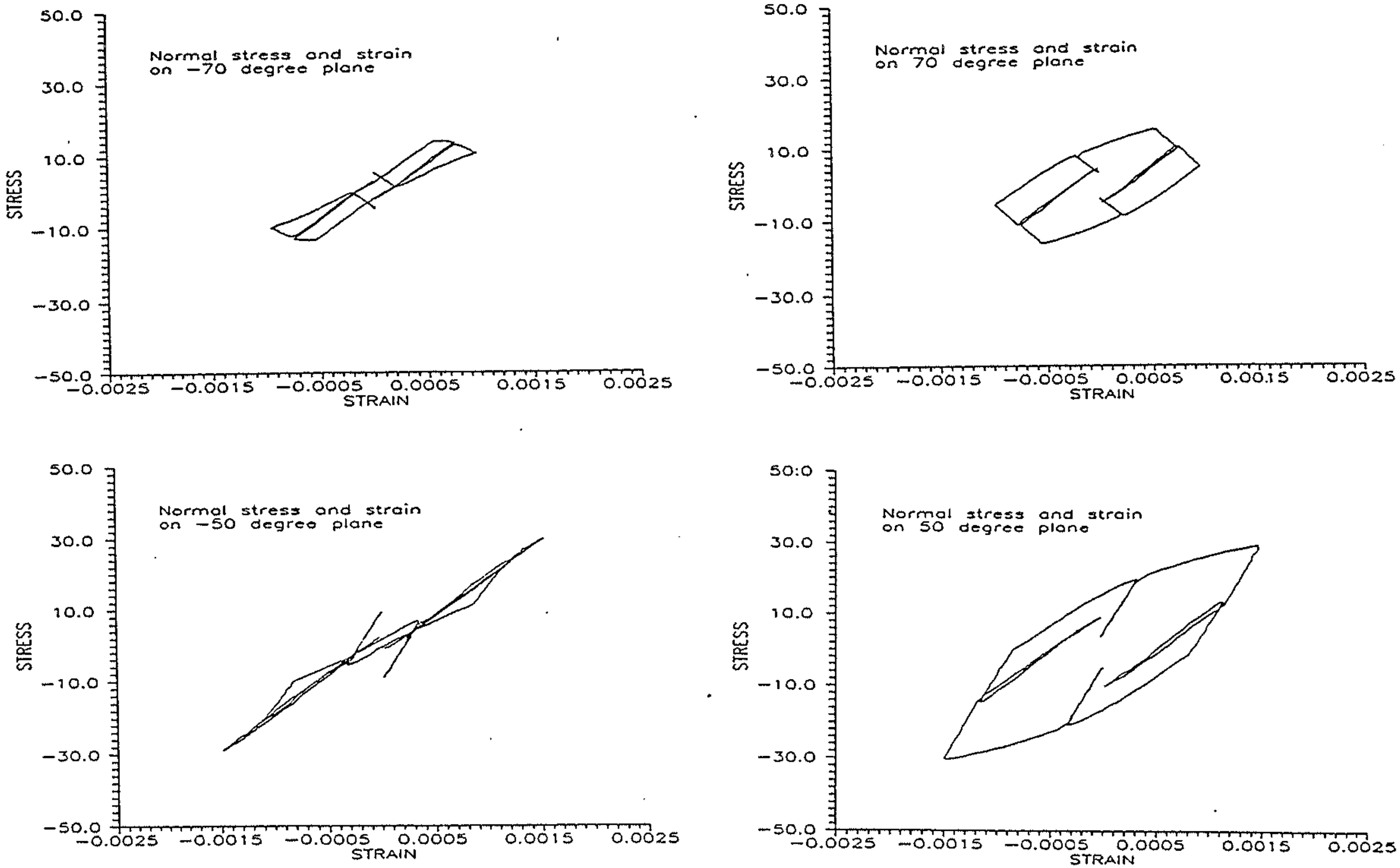

Figure 4 (a) Normal Stress-Strain Response on Planes Rotated -70°, 70°, -50°, 50° from Horizontal

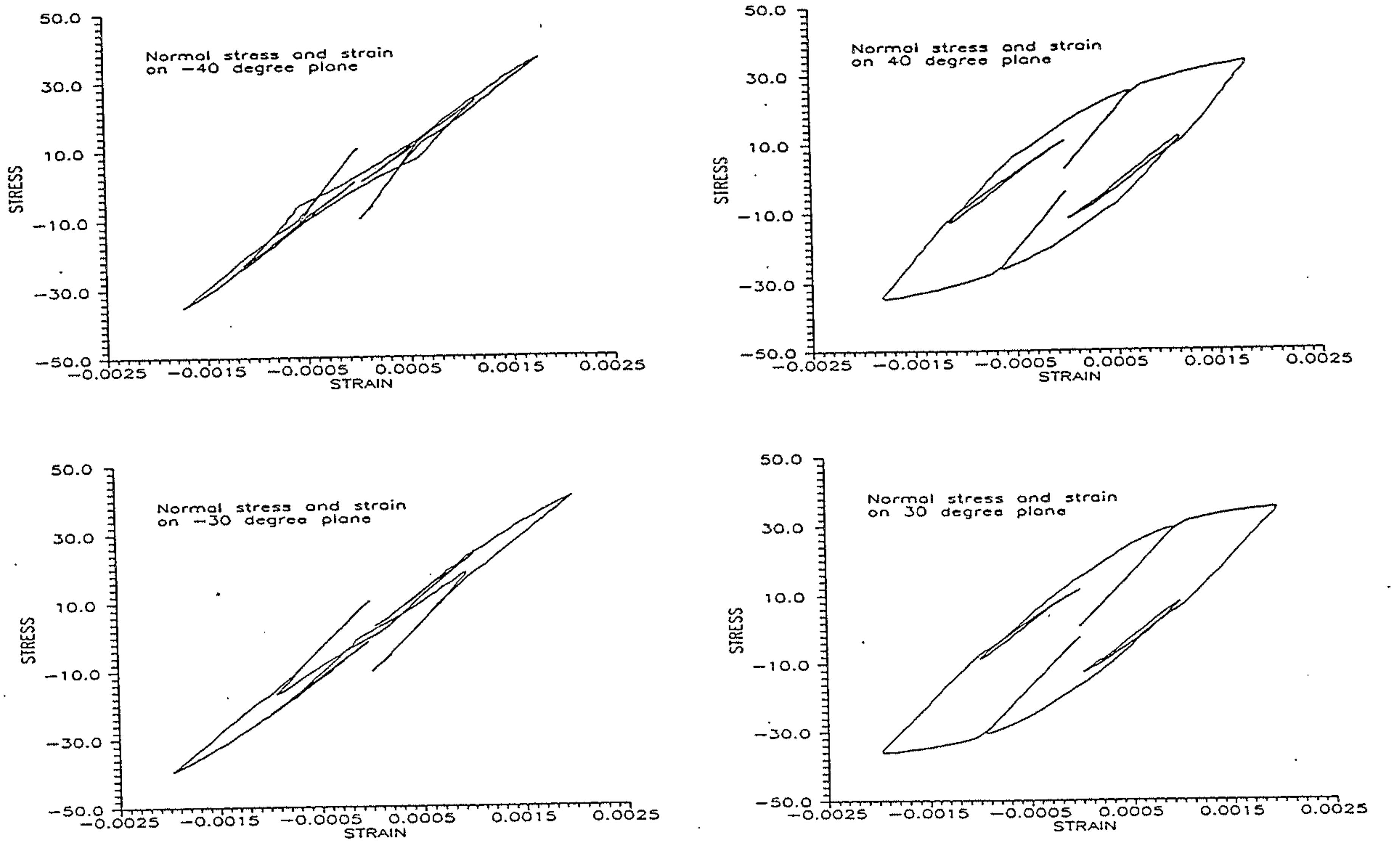

Figure 4 (b) Normal Stress-Strain Response on Planes Rotated -40°, 40°, -30°, 30° from Horizontal

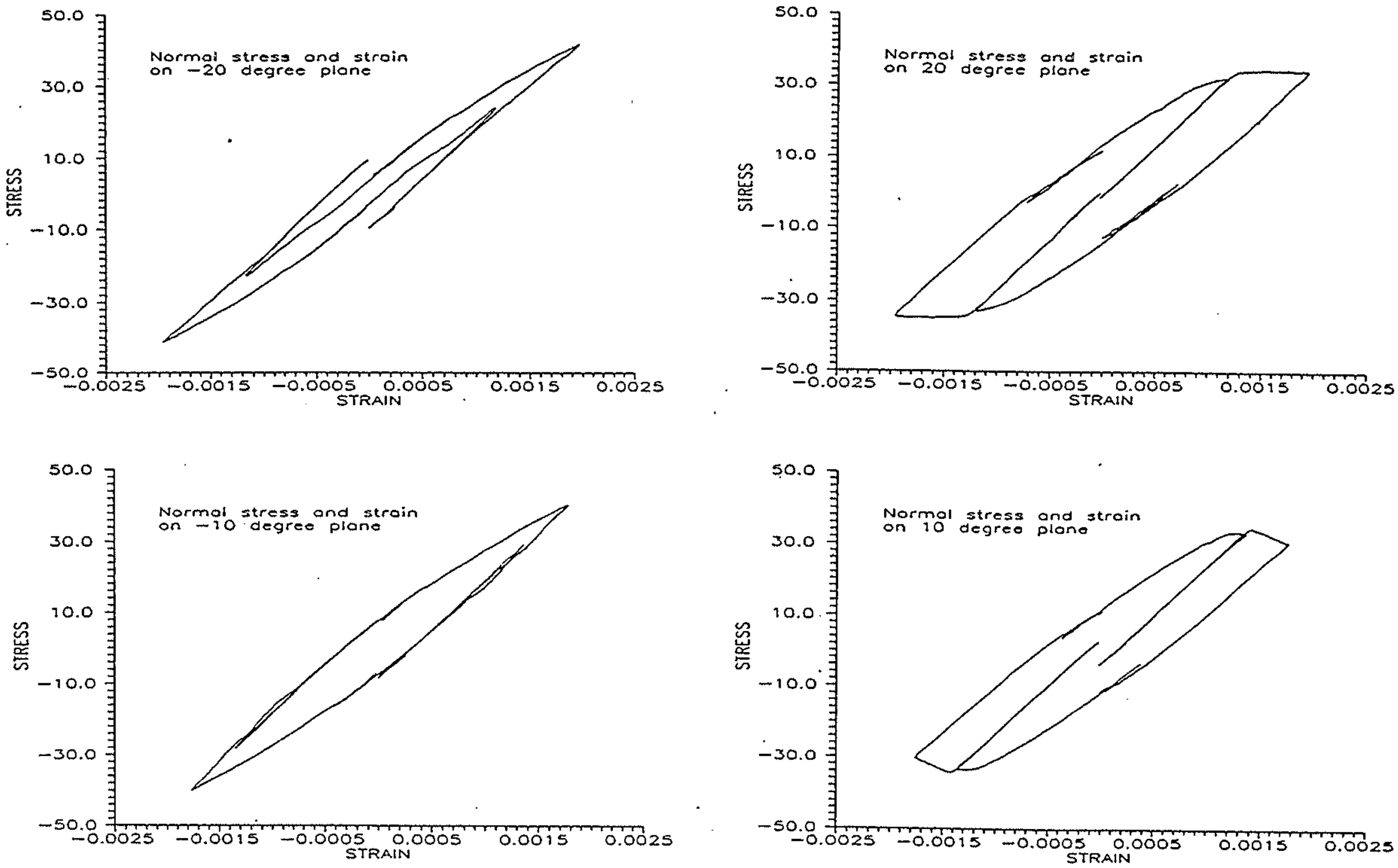

Figure 4 (c) Normal Stress-Strain Response on Planes Rotated -20°, 20°, -10°, 10° from Horizontal

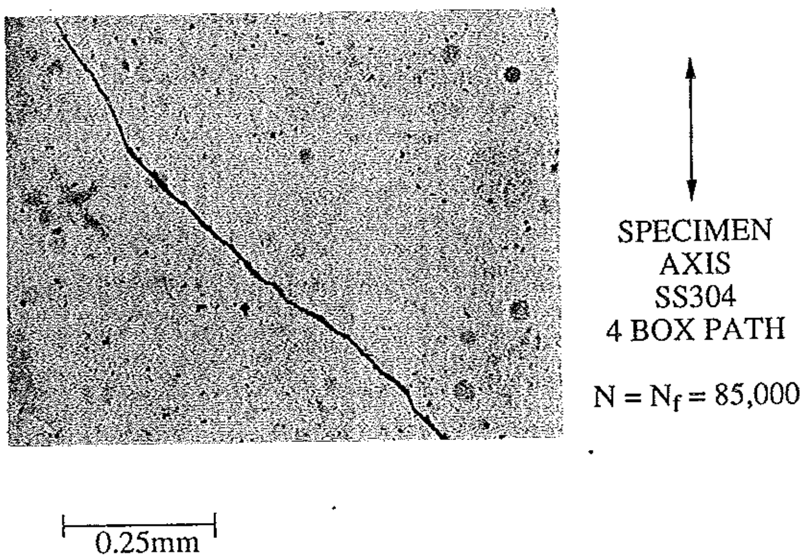

Figure 5 Cracking Observed in Thin Walled Tube
Subjected to Four Box Loading Path

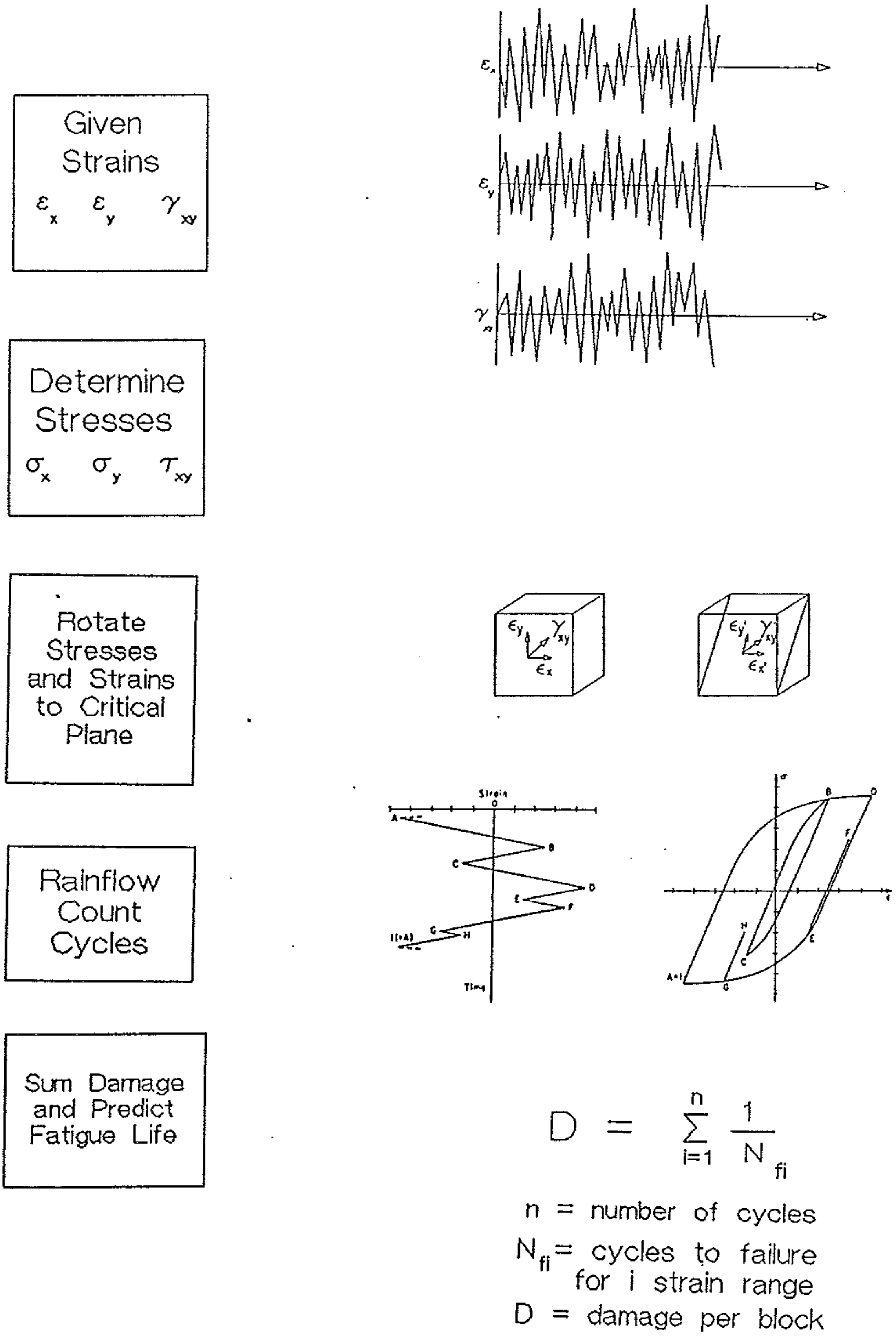

Figure 6 Schematic Outline of Variable Amplitude Multiaxial Fatigue Life Prediction Computer Program

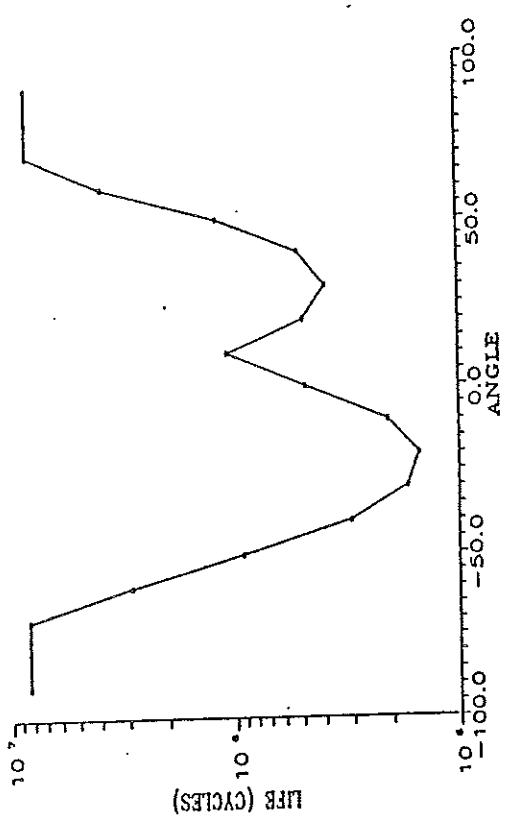

Figure 7 Life Prediction Results from Computer Program