A FATIGUE LIFE PREDICTION METHODOLOGY FOR INCLINED CRACK IN A CYLINDRICAL SHELL UNDER TORSION

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INTRODUCTION

There are a number of engineering components subjected to torsional loading such as the turbogenerator axle in powerstations. It is important to carry out the safety evaluation and the life prediction for these components with flaws. This problem has been concerned by some experts. Erdogan and Ratwani[1] have studied torsion fatigue for a cylindrical shell with a small circular hole.

In this paper, torsion fatigue experiments of the cylindrical shell specimems with inclined cracks are performed. The effect of different initial angles and different shell curvatures on fatigue crack growth rate are considered. The mixed mode fatigue crack growth law has been studied.

Refering to the crack tip stress field solutions by Lakshmiaray and Murthy[2], a dimensionless curvature parameter, β, is introduced. The stress field of the crack tip and thus the mixed mode fracture parameters can be expressed as a funtion of β. The mixed mode fracture criteria correlated with an arbitrarily oriented crack in a cylindrical shell under torsion are devoloped and the mixed mode parameters are extended to describe the fatigue crack growth law.

Finally, a methodology for fatigue life prediction is suggested. The prediction and experimental results agree well and the prediction is on the safe side.

SPECIMENS AND EXPERIMENT

Th material tested is 20MnSi steel which is used for axle of water turbogenerator. The geometry of the cylindrical shell(CS) specimen is shown in Fig.1 and the dimensions R, d, t, a_o and L are listed in Table 1. Speciments in Group A have different initial crack angles α_o: 0^o, 20^o,

45°, 70° and 90°. Specimen groups A, B, C and D have different curvatures.

Table 1. Specimen geometry and curvature parameter

Specimen Groups	d (mm)	t (mm)	R (mm)	a_o (mm)	L (mm)	Initial crack angles ($\alpha_o{}^{\circ}$)	Curvature parameter β
A	40	4.0	22.0	5	100	0,20,45,70,90	0.343
B	35	3.5	19.3	4	90	0	0.313
C	30	3.0	16.5	4	80	0, 45	0.365
D	40	3.0	21.5	5	100	0	0.397

The applied torsional moment ΔT has a sinusoidal wave and is controlled by the condition that the equal shear stress in net section of each specimen took place. Thus the experimental results could only reflect the influence of the initial crack angle and the shell curvature. An optcal microscope with 30X was used to measure the crack length.

Experimental results are shown in Fig.2 to 5. Fig.2 and Fig.3 give the crack growth pathes and the a-N curves of the speciments with different initial crack angles. We can see that only in the early stage, the crack growth pathes are effected by their initial crack angles. Afterward, they all approached to 45° direction (Fig.2). The 45° oriented crack grew much faster then others and its fatigue life was the shortest(Fig.3). Fig.4 and Fig.5 give the crack growth pathes and a-N curves of the specimens with different curvatures. Evidently, all growth pathes are nearly the same and approximatly in the 45° direction (Fig.4), and the larger the curvature, the less the life (Fig.5).

MODEL AND ANALYSIS

Referring to crack tip stress field solution by Lakshminarayana et al.[2], a dimensionless curvature parameter β is introduced and defined as

$$\beta^2 = \frac{a^2 [12(1-\nu^2)]^{1/2}}{8Rt} \tag{1}$$

where R,t -- radius and thickness of CS specimen, respectively

a -- half crack length and ν -- Poisson's ratio.

The stress field near the crack tip must be expressed as a function of

β and α_o. For an inclined crack in a cylindrical shell, the stresses consist of membrane stresses $\sigma_{ij}{}^m$ and bending stresses $\sigma_{ij}{}^b$. Correspondingly, the stress intensity factors $K_i{}^m$ and $K_i{}^b$ are also expressed in termx of β and α_o.

According to above stress field analysis, the mixed mode fracture criteria, such as $\sigma_{\theta max}$ criterion, S_{min} criterion, r_{pmin} criterion ect., are extened to the cylindrical shell with inclined through crack. The mixed mode fracture parameters, Circumferential tensile stress σ_θ, Strain energy density factor S and Plastic range size factor r_p, can be obtained as [3]

$$\left.\begin{aligned}
\sigma_\theta &= \frac{1}{\sqrt{2\pi r}}\left[K_i^m g_i^m + K_i^b g_i^b \right] \\
S &= \frac{1}{4\pi E}\{[g_{xi}^m g_{xj}^m + g_{yi}^m g_{yj}^m - 2\nu g_{xi}^m g_{yj}^m + 2(1+\nu) g_{xyi}^m g_{xyj}^m] K_i^m K_j^m \\
&\quad + [g_{xi}^b g_{xj}^b + g_{yi}^b g_{yj}^b - 2\nu g_{xi}^b g_{yj}^b + 2(1+\nu) g_{xyi}^b g_{xyj}^b] K_i^b K_j^b \\
&\quad + 2[g_{xi}^m g_{xj}^b + g_{yi}^m g_{yj}^b - \nu (g_{xi}^m g_{yj}^b + g_{yi}^m g_{xj}^b) + 2(1+\nu) g_{xyi}^m g_{xyj}^b] K_i^m K_j^b \} \\
r_p &= \frac{1}{2\pi\sigma_s^2}\{[g_{xi}^m g_{xj}^m + g_{yi}^m g_{yj}^m - g_{xi}^m g_{yj}^m + 3 g_{xyi}^m g_{xyj}^m] K_i^m K_j^m \\
&\quad + [g_{xi}^b g_{xj}^b + g_{yi}^b g_{yj}^b - g_{xi}^b g_{yj}^b + 3 g_{xyi}^b g_{xyj}^b] K_i^b K_j^b \\
&\quad + [2 g_{xi}^m g_{xj}^b + 2 g_{yi}^m g_{yj}^b - (g_{xi}^m g_{yj}^b + g_{yi}^m g_{xj}^b) + 6 g_{xyi}^m g_{xyj}^b] K_i^m K_j^b \}
\end{aligned}\right\} \quad (2)$$

where i,j = I,II, K's are stress intensity factors (SIF) which can be obtained from Fig.6, and g's are dimensionless distribution function of angle.

For mixed mode fatigue problem , it is followed by kinking as soon as the crack initiates. No formulas have been used to determine the SIF of kinking crack tip in a shell up to now. It is necessary to establish the simplified equivalent crack model from which one can use the imaginary inclined crack to take the place of the actual kinking crack. The three equivalent crack models taken here are:

Model A (Fig.7)— straigth line equivalent crack model ,

Model B (Fig.7)— tangent projection equivalent crack model ,

Model C (Fig.8)— 45° equivalent crack model .

Assuming crack extension step by step, the crack growing path prediction could be obtained (Fig.9). It is found that the result predicted by S_{min} criterion is much closer to the expetimental data and 45^{o} equivalent crack model is reasonable and simplier. It will be used in following analysis.

The mixed mode fatigue crack growth rate can be expressed as follows :

$$\left.\begin{aligned} \frac{da}{dN} &= C_\sigma \ (\Delta\Sigma_{\theta max})^{n_\sigma} \\ \frac{da}{dN} &= C_s \ (\Delta S_{min})^{n_s} \\ \frac{da}{dN} &= C_r \ (\Delta r_{pmin})^{n_r} \\ \frac{da}{dN} &= C_k \ (\Delta K_{eff})^{n_k} \end{aligned}\right\} \quad \frac{da}{dN} = C_i \ (\Delta I)^{n_i} \tag{3}$$

where $\Delta\Sigma_{\theta max} = \Delta\sigma_{\theta max}\sqrt{r}$, ΔS_{min}, Δr_{pmin} and $\Delta K_{eff} = (K_I^4 + K_{II}^4)^{\frac{1}{4}}$ [4] are mixed mode range parameters. Because of the bending stress distributes linearly along the thickness of the cross section , therefore

$$\sigma_{ij} = \sigma_{ij}^m + \frac{1}{2}\sigma_{ij}^b$$

is regarded as equivalent stresses. Correspondingly, the equivalent SIF's are

$$\left.\begin{aligned} K_I &= K_I^m + \frac{1}{2} K_I^b \\ K_{II} &= K_{II}^m + \frac{1}{2} K_{II}^b \end{aligned}\right\} \tag{4}$$

Treating the exparimental data with these four parameters and plotting the points in da/dN versus ΔI, the slops n_i, the intercepts C_i, the correlation coefficients r and remainder standard deviations σ of these regression lines are summarized in Table 2.

Table 2. Fitting parameters

Specimens	Parameters	C_i	n_i	r	σ
	ΔK_{eff}	2.723×10^{-9}	2.416	0.9621	0.0942
A	$\Delta\Sigma_{\theta max}$	4.592×10^{-8}	2.221	0.9609	0.0956
($\alpha = 0^{o}$)	ΔS_{min}	3.289×10^{-3}	1.234	0.9613	0.0951
	Δr_{pmin}	8.320×10^{-5}	1.197	0.9609	0.0959

B	ΔK_{eff}	2.877×10^{-9}	2.398	0.9839	0.0650
($\alpha = 0^o$)	$\Delta\Sigma_{\theta max}$	5.058×10^{-8}	2.183	0.9835	0.0658
	ΔS_{min}	3.020×10^{-3}	1.213	0.9836	0.0655
	Δr_{pmin}	5.310×10^{-5}	1.101	0.9833	0.0661
C	ΔK_{eff}	2.433×10^{-9}	2.621	0.9989	0.0149
($\alpha = 0^o$)	$\Delta\Sigma_{\theta max}$	1.799×10^{-8}	2.403	0.9989	0.0151
	ΔS_{min}	3.304×10^{-3}	1.377	0.9989	0.0149
	Δr_{pmin}	6.080×10^{-5}	1.296	0.9989	0.0149
D	ΔK_{eff}	2.520×10^{-9}	2.625	0.9785	0.0968
($\alpha = 0^o$)	$\Delta\Sigma_{\theta max}$	7.278×10^{-8}	2.066	0.9782	0.0975
	ΔS_{min}	2.234×10^{-3}	1.128	0.9784	0.0970
	Δr_{pmin}	7.740×10^{-5}	1.087	0.9783	0.0973

From the values of r and σ in Table 2, it is evident that all of the four mixed mode parameters could be used to discribe the torsion fatigue crack growth law of an inclined crack in a cylindrical shell.

LIFE PREDICTION

In order to meet the engineering needs, a simplified formula is presented from the above study. The effect of shell curvature on the da/dN has been taken account in the mixed mode parameters, thus the material constants C_i and n_i should be the same as that of the plate. Actualy, it is found that when the same parameter is used, the average values of C_e and n_e obtained from Table 1 are very close to the values of C and n obtained from the fatigue experimental results of a cracked plate. Example, using the parameter ΔK_{eff}, the average values $C_e = 2.64\times10^{-9}$, $n_e = 2.51$ for shell and $C = 2.73\times10^{-9}$, $n = 2.52$ for plate. Therefore, C_e and n_e can be replaced with C and n . The torsion fatigue crack growth rate of inclined crack in a cylindrical shell may be expressed as

$$\frac{da}{dN} = C\ (\ \Delta K_{eff}\ )^{n} = 2.730\times10^{-9}\ (\ \Delta K_{eff}\ )^{2.52} \quad \text{mm/cycle} \tag{5}$$

Integrating Eq.(5), the fatigue life is obtained as

$$N = \int_{a_o'}^{a_i'} \frac{da}{C(\Delta K_{eff})^{n}} \tag{6}$$

where a_o' and a_i' are initial and current equivalent crack length, respectively. They are determined by 45° equivalent crack model(Fig.8).

According to the Romberg's numerical integral method, the predicted fatigue life is given by Eq.(6). Both the predicted life and the experimental results are listed in Table 3. It is evident that they agree very well and the prediction is on the safe side.

Table 3. Life prediction and experimental results

Specimens	a_o' — a_i' (mm)	prediction N_f (cyc.)	Experiment N_f' (cyc.)	Errors (%)
A — 0°	4.5 — 10	68000	72000	5.6
B — 0°	3.8 — 10	83000	84000	1.2
C — 0°	3.8 — 10	69000	85000	18.5
D — 0°	4.5 — 10	56000	57000	1.6
A — 20°	5.7 — 10	45000	57000	22.4
A — 45°	6.0 — 10	39000	62000	37.1
A — 70°	5.7 — 10	45000	61000	26.2
A — 90°	4.5 — 10	68000	77000	11.7

CONCLUSIONS

The following conclusions are drawn from the present study:

1) The fatigue life prediction of an inclined crack in cylindrical shell under torsional loading could be made by using the 45° equivalant crack modle.

2) All of the four mixed mode range parameters can be used to desctibe the fatigue crack growth law. The range parameters should be calculated using the approach suggested by this paper in order to take account of the effect of the shell curvature.

3) It is proved that the approximate approach of life prediction given by present study is convenient and reasonable.

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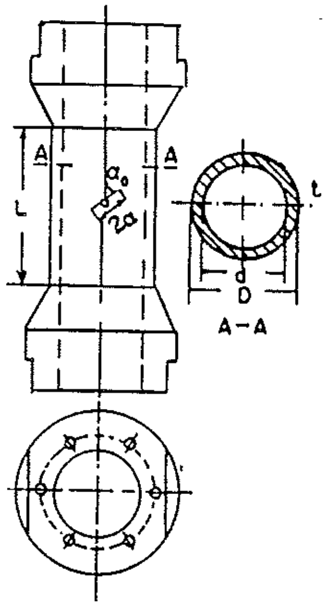

Fig.1 CS specimen

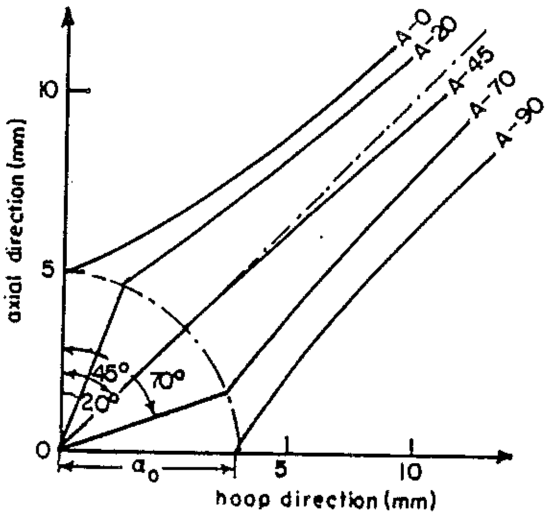

Fig.2 Growth pathes of CS specimens with different initial crack angles

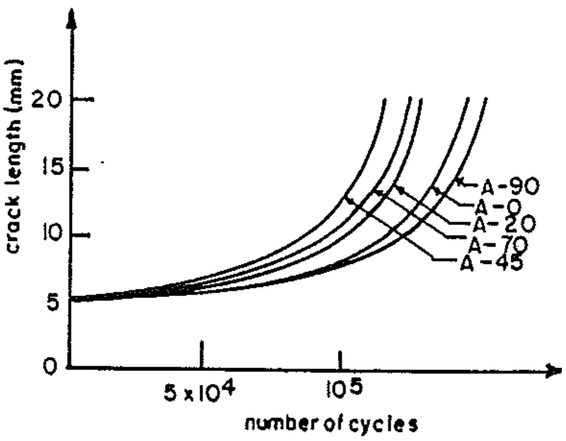

Fig.3 a-N curves of CS specimens with different initial crack angles

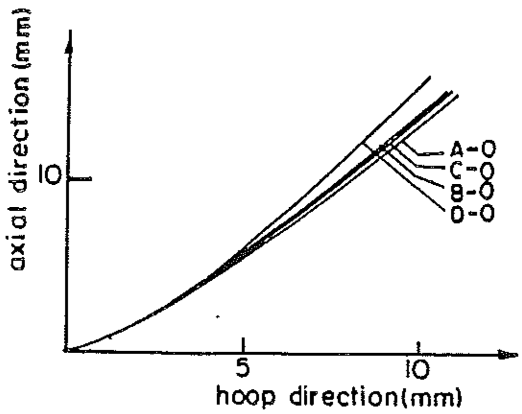

Fig.4 Growth pathes of CS specimens with different curvatures

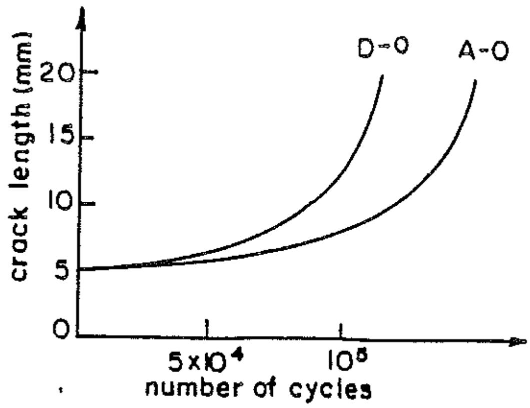

Fig.5 a-N curves of CS specimens with different curvatures

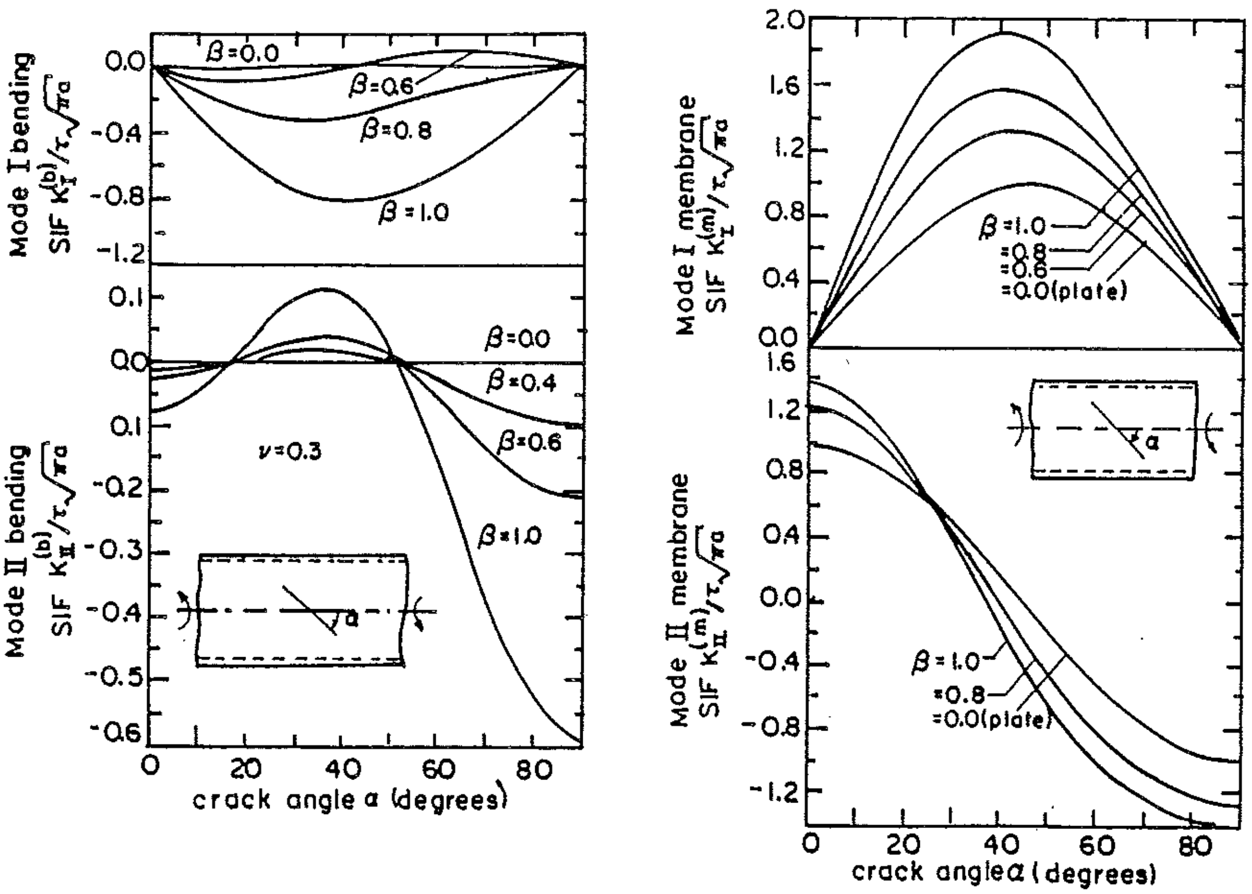

Fig.6 SIF of arbitrarily oriented crack in a cylindrical shell subjected to torsional loading

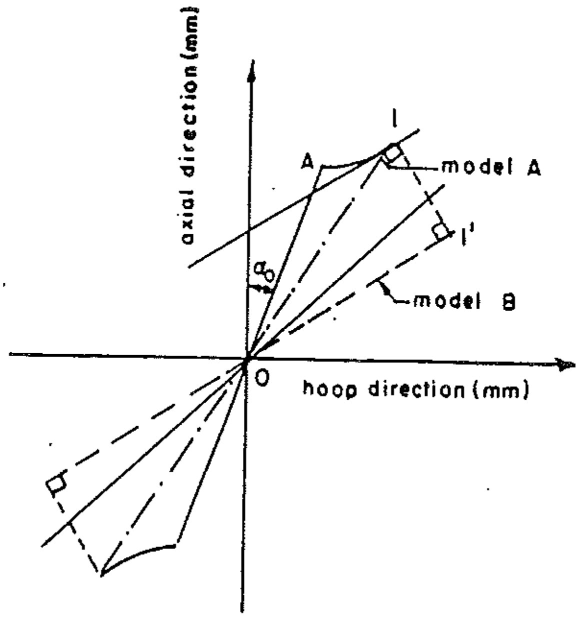

Fig.7 Model A and Modle B

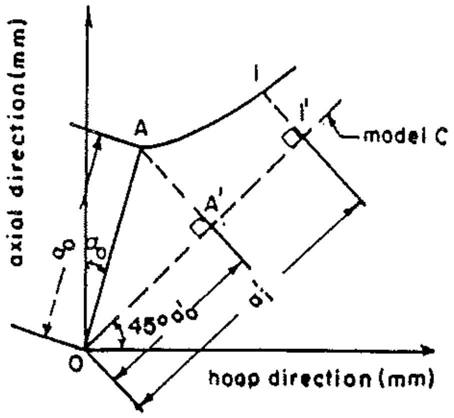

Fig.8 Model C

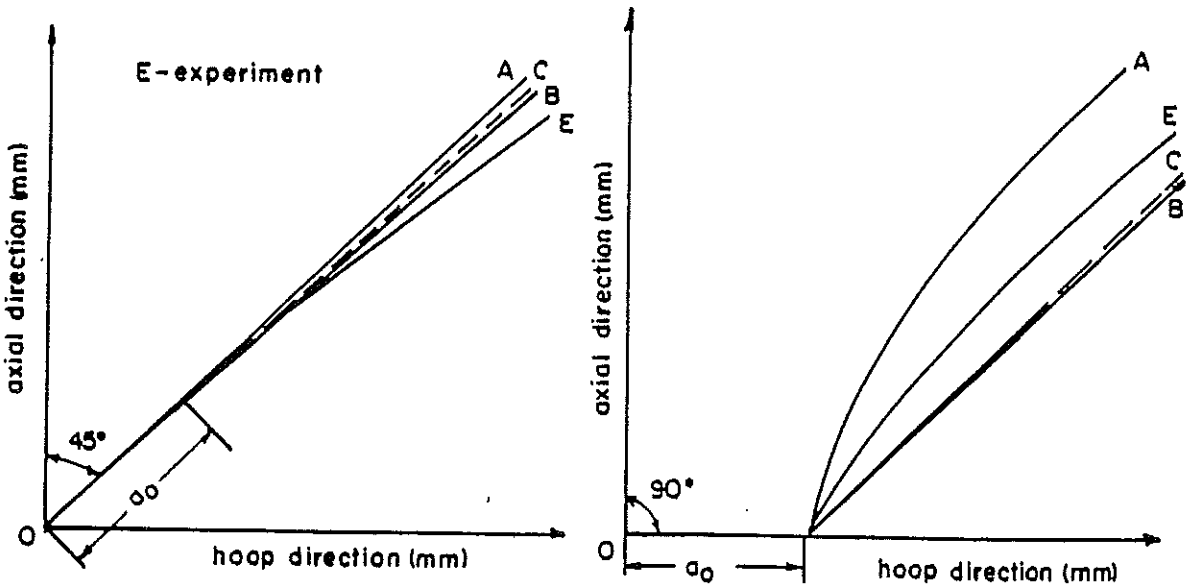

Fig.9 Crack growth path prediction by different models
(S_{min} criterion)