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Specimens for Investigating Biaxial Fracture and Fatigue Processes

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ABSTRACT Whereas investigations on the fracture behaviour of cracked specimens exposed to normal stresses have already produced remarkable results, little is known about the effect that superimposed normal and shearing stresses have on stable and unstable crack growth. Numerous types of specimen have been proposed especially for acquiring fracture toughness values or examining mixed-mode fracture criteria and measuring the growth of fatigue cracks exposed to multi-axial loading. Some of these specimens are presented and critically analysed on the basis of a catalogue of requirements derived from practical considerations. It will be shown that the CTS specimen is well suited for mixed-mode experiments. Some test results obtained with this specimen are presented.

Introduction

Until a few decades ago, structures and construction components were designed exclusively on the basis of the classical strength calculations. Although these calculations were made with great care, damage and fracture occurred repeatedly, sometimes with disastrous consequences. Most of the damage is caused by small cracks which are present in the material as defects or develop as a result of service conditions. Further damage is then caused by such cracks growing larger as a result of operating conditions, e.g., fatigue loading which finally leads to a fracture. Knowledge of the conditions prevailing when a crack develops and grows under fatigue loading or factors that cause cracks to become unstable is of great importance for predicting damage, evaluating its progress, and devising measures and concepts for preventing similar damage in the future.

Whereas investigations of fracture behaviour of cracks under Mode I stresses have already produced good results, little is known about the effect of mixed-mode loading on the growth of fatigue cracks and the factor causing cracks to become unstable. In order to acquire definite knowledge concerning these practically-relevant problems, increasing research efforts are now being made. Thus numerous fracture hypotheses now exist and a list is reproduced under (1), together with several hypotheses for pre-calculating the growth of fatigue cracks, see, e.g., (2)–(4). However, not all of these hypotheses lead to

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uniform predictions, and additional knowledge on mixed-mode problems needs to be obtained by experiment. For this purpose, numerous types of specimen have been proposed in recent years (see, in particular, (5)(6)). Some of these will be presented and critically analysed in this paper. In addition, the results of fracture and fatigue tests made with the CTS specimen will be presented.

Notation

a	Crack length
b, c, e, f, k, l, w, D, B	Specimen dimensions
F, F_1, F_2, F_3	Forces
K_I	Stress intensity factor for Mode I
K_{II}	Stress intensity factor for Mode II
K_e	Effective stress intensity factor
K_{IC}	Fracture toughness for Mode I
K_{Io}, K_{IIo}	Mode I/Mode II overload stress intensity factor
K_{Imax}, K_{Imin}	Maximum/Minimum stress intensity factor applicable to mode I fatigue loading
M_T	Torsional moment
N	Load cycles
α	Load application angle
α_1	Material constant
γ, β	Crack inclination angles
σ	Tensile stress
ϕ_o	Crack deflection angle

Specimens

Since no standardized test procedure exists, different test conditions have been used for most of the experiments made until now and numerous types of specimen have been proposed in recent years for investigating fracture toughness, examining the many mixed-mode fracture criteria, measuring threshold values and determining the crack growth laws applicable to cyclic multi-axial stress situations. The most important of these include, with reference to Fig. 1:

S1: Tensile-stress specimen with oblique central crack;
S2: Tensile-stress specimen with oblique edge crack;
S3: Disc specimen with oblique inside crack;
S4: Cruciform specimen with oblique central crack;
S5: Shear specimen with oblique central crack;
S6: Pipe specimen with oblique through crack;
S7: Pipe specimen exposed to superimposed tensile and torsional stresses;
S8: Three-point bend specimen with offset crack (This specimen can also be used as a four-point shear specimen);
S9: Compact tension shear specimen (CTS).

The investigations made with some of these specimens more often than not pursued different objectives. Several specimens were used to study brittle plastics, metals, wood, and composite materials.

Criteria for evaluating mixed-mode specimens

In order to be able to evaluate the specimens systematically, it appears appropriate to proceed according to a catalogue. According to this, an optimal mixed-mode specimen, designed for fracture and fatigue tests, should meet the following conditions:

C1: Full range of mixed Mode I/Mode II combinations;
C2: Compactness of specimen;
C3: Ease of manufacture;

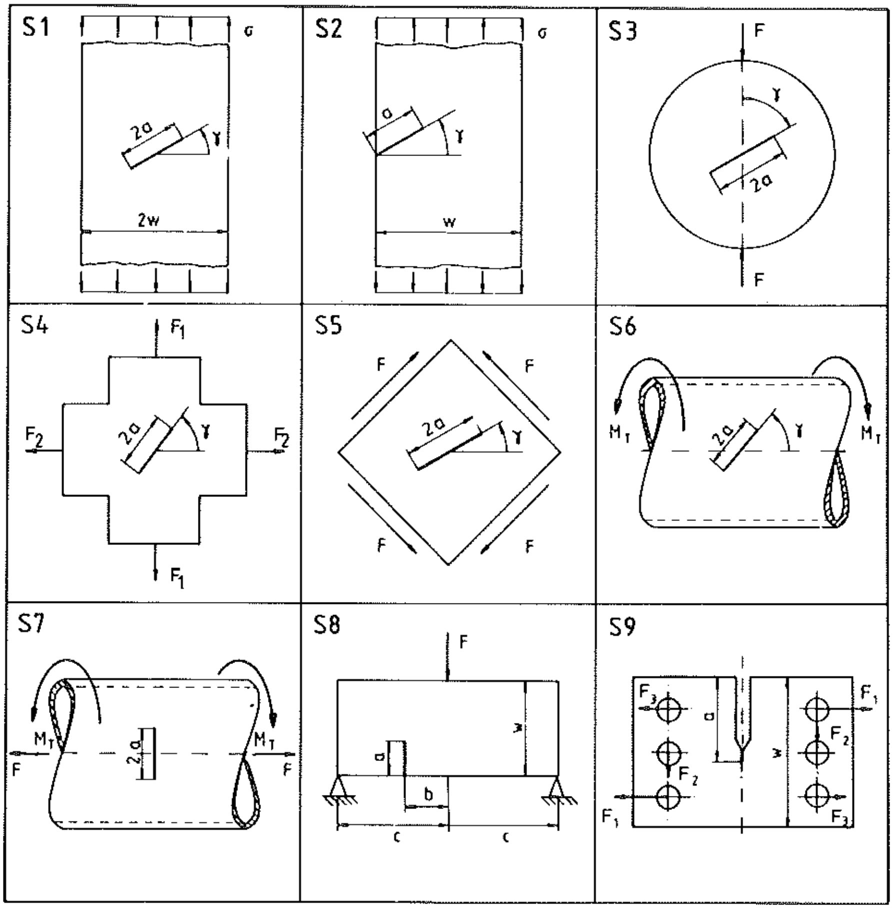


Fig 1 Specimens for mixed mode experiments

C4: Ability to form fatigue precracks under Mode I loading;
C5: Clamping and loading conditions must be easy to achieve;
C6: Simple test procedure and evaluation;
C7: Realizability of a state of plane strain;
C8: Small minimum dimensions to give small fracture loads.

Requirement C1 demands that the mixed-mode stresses at the crack tip can be varied between pure Mode I stresses and pure Mode II stresses. Whereas pure Mode I stresses are relatively easy to generate, the transition to Mode II stresses causes some problems.

The geometric compactness of the specimen (Requirement C2) facilitates the making of small specimens and effects a saving in material and costs. Because of the somewhat complicated load application, most mixed-mode specimens are not as compact as the CT specimen for Mode I.

Requirement C3, which calls for simple production of the specimen, reduces both the time and the costs of production. Care should be taken that the specimen contour and the starting notch for the fatigue crack are easy to produce.

A fatigue crack is required in order to be able to obtain valid fracture limit curves from mixed-mode specimens exposed to static stresses, or threshold values as well as crack growth laws from such specimens exposed to vibrating stresses. For this reason, the specimens and the load application conditions must be such that, beginning with a starting notch, a fatigue crack can be initiated in a relatively easy manner (Requirement C4). The use of specimens with saw cuts or wide slots produces inaccurate results which are inadmissible at least for determining threshold values.

It is perhaps advantageous to apply load to a defined standard specimen (Requirement C5) by means of a loading device operating in conjunction with a standard compression–tension testing machine. A favourable solution is to effect the load transfer between the device and the specimen in a unique but reproducible and, if possible, statically determined manner. In addition, the connection between the device and the specimen should be relatively easy to make and to break. Tensile- and torsional-stress tests or tests involving specimens exposed to two-axis stresses cannot be carried out in every laboratory.

Requirement C6, which calls for a simple test procedure, goes beyond Requirements C4 and C5. Thus, when fracture tests are made with a compression–tension testing machine, tensile stresses are to be preferred to compressive stresses because there is a danger that these compressive stresses may unintentionally cause damage to the fracture surface which in turn, would interfere with the determination of the crack deflection angle and the analysis of the fracture surfaces. Furthermore, during fracture tests only the fracture process originating from the fatigue crack should be plotted on the load–displacement record and the force leading to critical crack growth must be clearly indicated.

It should be possible to realize a state of plane strain (Requirement C7) so that the results obtained from fracture tests constitute material limit values. It should be pointed out, however, that the question of what the minimum thickness of specimen for mixed-mode tests should be has not yet been answered in principle.

According to Requirement C8 the minimum dimensions, e.g., length of ligament, of the specimen to give small scale yielding and the fracture loads applied during the tests should be as small as possible. This is the case, for example, when the effective or comparative stress intensity factor K_e of the specimen is very high. This can be determined from stress intensity factors K_I and K_{II} of the specimen (7), e.g.

$$K_e = \frac{K_I}{2} + \frac{1}{2}\sqrt{\{K_I^2 + 4(\alpha_1 K_{II})^2\}} \tag{1}$$

where $\alpha_1 = 1.155$ (corresponding to the assertion of the tangential stress criterion). If a comparatively small fracture load is required, this makes it possible to use smaller testing machines for the fracture test and reduces the problems which may occur in connection with the application of the load to the specimen. A high K_e factor furthermore causes plastic zones to occur only in the region of the crack tip before the specimen breaks.

Critical evaluation of the specimens

Specimens S1 to S9 (Fig. 1) will now be critically analysed on the basis of the criteria for evaluation mixed-mode specimens mentioned above.

Specimen S1

This specimen consists of a plate with an oblique central crack which is exposed to tensile stresses. Mixed-mode stresses occur at the two crack tips (8)(9), and it is possible to vary the K_{II}/K_I ratio of the stress intensity factors by changing the crack inclination angle. If $\gamma = 0$ degrees, then $K_{II} = 0$, i.e., pure Mode I stresses occur at the crack, whereas for $\gamma = 90$ degrees the stress intensity factors $K_I = K_{II} = 0$. No transition from pure Mode I to pure Mode II stresses is possible on this specimen. Furthermore, the generation, of fatigue cracks is problematic because the crack would instantly be deflected when the envisaged tensile stresses are applied. The procedure according to which horizontal fatigue cracks are generated in a larger plate from which the specimens are then made does not appear to be practical for a series of investigations. Problems may also occur during the test because there are two crack tips involved. Such problems may occur, for example, when, while carrying out fracture tests, the unstable crack propagation does not start at both crack tips simultaneously. Moreover, the relatively small stress intensity factors lead to very large specimen dimensions and fracture loads.

Specimen S2

On this specimen too, the K_{II}/K_I ratio can be varied by altering the crack inclination angle γ (**10**). If $\gamma = 0$ degrees, then Mode I loading occurs at the crack, whereas for $\gamma = 90$ degrees, no crack exists any more. This means that no pure Mode II loading can be obtained on this specimen either. What has been stated previously about the generation of fatigue cracks on specimen S1 also applies to this specimen.

Specimen S3

This very compact specimen (**11**) permits a transition from pure Mode I stresses to pure Mode II stresses in the range 0 degrees $\leqslant \gamma \leqslant$ 25 degrees ($\gamma = 0$ degrees: Mode I, $\gamma \approx 25$ degrees; Mode II). In the range $\gamma > 25$ degrees, compressive stresses are present at the crack. However, in the angle range 0 degrees $\leqslant \gamma \leqslant$ 25 degrees, a small angular deflection means a considerable alteration of stress intensity, which may lead to problems during the test and also to a loss in accuracy. Owing to the occurrence of compressive stresses in the specimen, there is an additional danger that the fracture surface may be damaged when the specimen breaks. Again the presence of two crack tips is unfavourable.

Specimen S4

This specimen permits (**12**) variation of the K_{II}/K_I ratio by changing angle γ and of the ratio of forces F_2/F_1 acting on the specimen. If F_1 and F_2, are tensile forces, no Mode II stresses can be obtained at the crack. Only if $F_2 = -F_1$, can pure Mode II stresses be generated at $\gamma = 45$ degrees. But in this case there is a danger that thin plates may buckle. The load application (which can only be accomplished with the aid of a special testing machine) and the performance of the test are complicated, however.

Specimen S5

This specimen consists, for instance, of a square plate to which shearing forces are applied (**13**) via studs located at its edges. Mode I stresses occur at a crack angle of $\gamma = 0$ degrees, while pure Mode II stresses occur at $\gamma = 45$ degrees, so that the K_{II}/K_I ratio can be varied at will. What is problematic with this specimen are the load application conditions and the fatigue crack generation. In addition, the central crack problem exists here, too. Furthermore, owing to the weak stress intensity field, large specimen dimensions and testing forces are required.

Specimen S6

This specimen consists of a thinwalled pipe to which torsional stress is applied (**14**). By varying crack angle γ, pure Mode II stresses are obtained at $\gamma = 0$

degrees, while pure Mode I stresses are obtained at $\gamma = 45$ degrees. This means, the K_{II}/K_I ratio can be varied in the range $0 \leq K_{II}/K_I \leq \infty$. With this specimen it is practically impossible to initiate a fatigue crack in the direction of the starter notch. In order to obtain an almost plane crack problem, the ratio between wall thickness and pipe radius must be very small. On the other hand, tests made to determine fracture toughness values require a plane strain condition, i.e., a relatively large wall thickness. If both conditions are to be met, extremely large specimen dimensions are required.

Specimen S7

This specimen consists of a thin-walled pipe which is simultaneously exposed to tensile and torsional stresses (**15**). When exposed to tensile stresses, Mode I occurs, and when exposed to torsional stresses, Mode II occurs at the crack. By superimposing tensile and torsional strains mixed-mode stresses occur and so the K_{II}/K_I ratio can be varied at will. However, the generation of fatigue cracks is problematic with this specimen also, and tests can only be carried out with the aid of special machines. Furthermore, it is difficult to obtain a state of plane strain.

Specimen S8

This specimen (**16**) has been derived from the three-point bend specimen which is generally used for determining K_{IC} values. By varying the b/c ratio or, for $b/c > 0$, by altering a/w, the stress intensity factors K_I and K_{II} can be varied to a certain extent. Whereas at $b/c = 0$ pure Mode I stresses occur, it is not possible to obtain pure Mode II stresses with this load application method. The K_{II}/K_I ratio can only be increased to approximately a value of 1. However, this specimen can also be used as a four-point shear specimen with offset crack (**6**), in which case only predominantly Mode II stresses can be produced, while the generation of pure Mode I stresses is difficult (**17**). With this specimen the full range of stress states from Mode I to Mode II can be obtained with the use of three-point bend and four-point shear specimens in conjunction. Furthermore, this specimen is very compact, enables a pre-crack to be formed under Mode I loading, provides a simple loading condition, and permits a state of plane strain. However, in the event of fatigue stresses, the $K_{\min}/K_{\max}$ relationship of stress intensity factors cannot become negative.

Specimen S9

This rectangular specimen has an edge crack and six holes to which forces are applied in such a way that mixed-mode stresses can optionally be generated at the crack (**18**). Load is applied to this specimen by means of a suitable loading device (Fig. 2). To this, in turn, a tensile force is applied in, for example, a standard compression–tension testing machine. The two parts of the loading

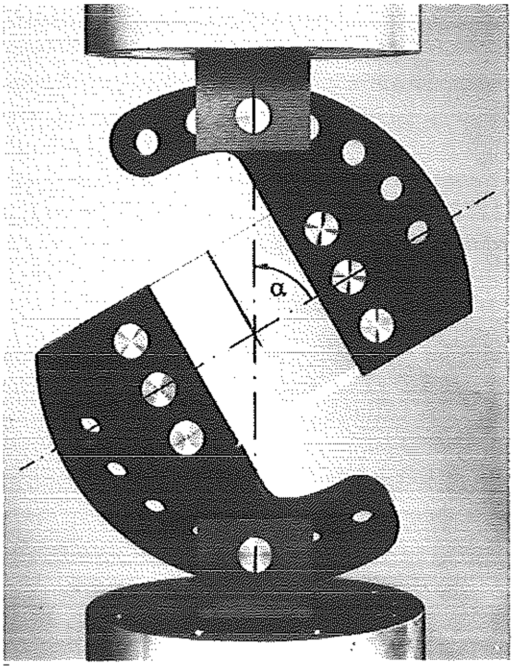
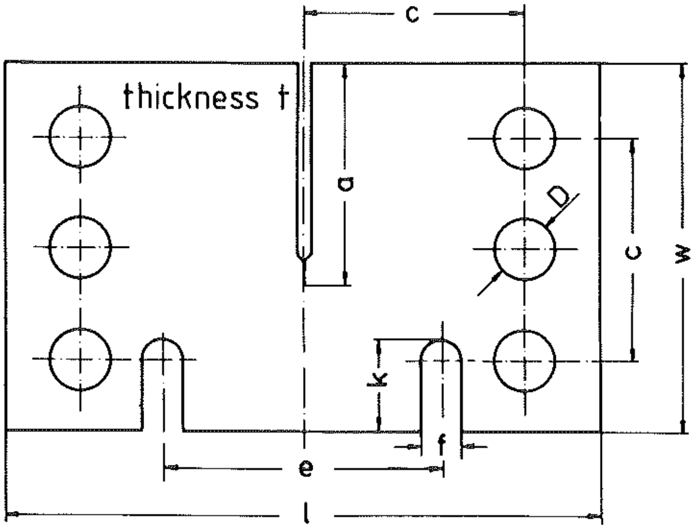


Fig 2 Loading device and a CTS specimen in a tensile testing machine



a ≈ 5.4t c = 5.4t w = 9t l = 14.5t D = 1.5t
e = 6t k = 2t f = t

Fig 3 CTSM specimen

device are connected to the specimen by studs, the chosen arrangement of the elongated holes in the loading device resulting in a statically determined load transfer. Depending on the size of load application angle α, various load conditions can be generated at the crack tip. At $\alpha = 0$ degrees, pure Mode I stresses occur at the crack. At $\alpha = 90$ degrees pure Mode II stresses occur, while in the range 0 degrees $\leqslant \alpha \leqslant 90$ degrees, the K_{II}/K_I ratio can be varied at will. Furthermore, the CTS specimen is very compact, enables initiation of a fatigue crack (under Mode I conditions), provides a simple loading condition and an uncomplicated test procedure, permits a state of plane strain, and, owing to a relatively high stress intensity field, requires but small specimen dimensions and a small test load. However, the CTS specimen requires an accurate manufacture with higher costs.

Apart from this CTS type specimen, a CTSM (modified CTS) specimen can be used especially for testing tougher materials ((**1**), Fig. 3). This, among other factors, permits even higher stress intensity values. This is indicated in Fig. 4, in which the effective stress intensity factors of several specimens are compared.

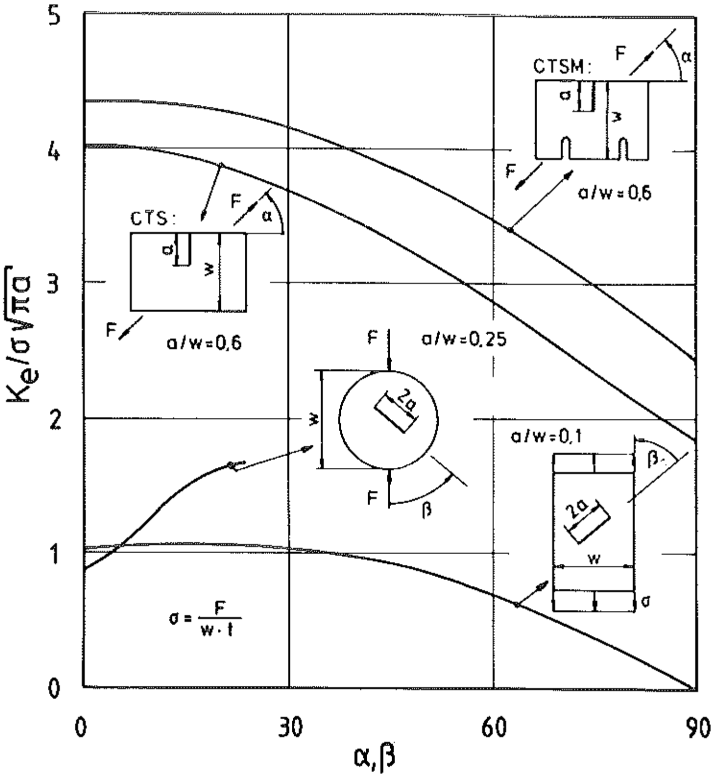


Fig 4 Comparison of the effective stress intensity factors of a number of mixed mode specimens

Experiments

As has been shown, the CTS specimen or the CTSM specimen, together with the corresponding loading device are particularly well suited for carrying out fracture or fatigue tests under mixed-mode stresses.

Fracture tests were conducted according to a testing procedure described in (1), which incorporates a considerable number of modifications compared with the ASTM E 399 standard.

Previously fracture limit curves have been defined for various materials (plastics and aluminium alloys) by experiments made at the University of Paderborn. The values measured for the PMMA materials (Plexiglas) during these experiments are indicated in Figs 5 and 6. The results derived from the fracture limit curves at $\alpha_1 = 1.08$ can be expressed by the generalized fracture criterion or effective stress intensity criterion (ESIF criterion) (7), thus

$$K_e = \frac{K_I}{2} + \frac{1}{2}\sqrt{\{K_I^2 + 4(\alpha_1 K_{II})^2\}} = K_{Ic} \tag{2}$$

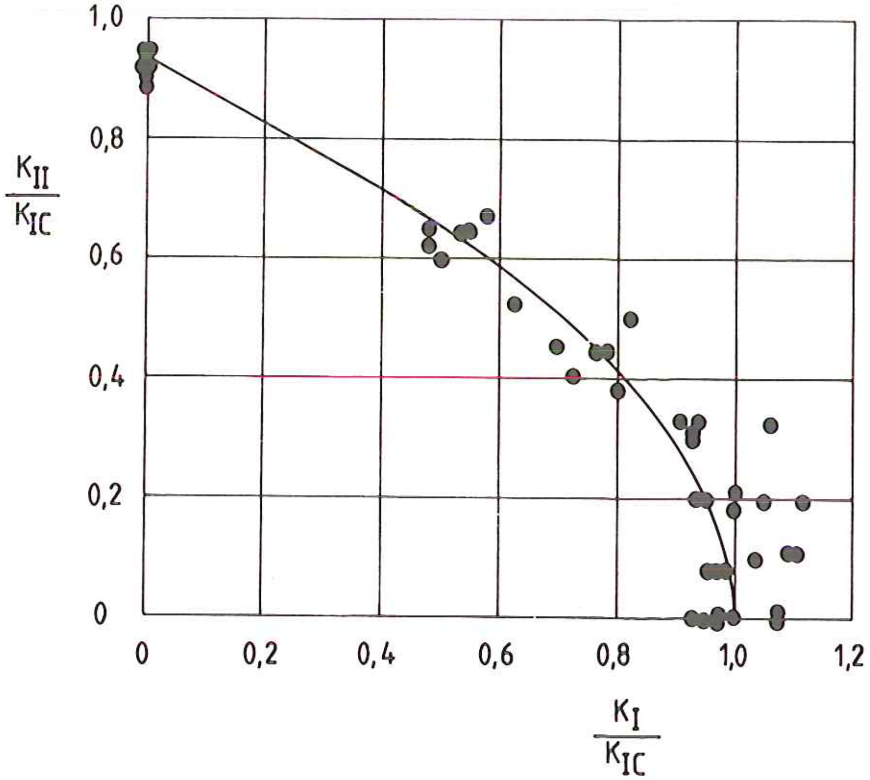


Fig 5 Fracture values on PMMA compared with the ESIF fracture limit criterion

and the crack deflection angle can be expressed by the equation (1)

$$\phi_o = -2.714 \frac{|K_{II}|}{|K_I| + |K_{II}|} - 1.456 \left(\frac{|K_{II}|}{|K_I| + |K_{II}|} \right)^2 \qquad (3)$$

where for $K_{II} > 0$ the angle $\phi_0 < 0$, and for $K_{II} < 0$ the angle $\phi_0 > 0$. Equation (3) is an empirical expression, which agrees well with experimental results for many materials (1).

Fatigue tests on the aluminium alloy AlCuMg 1 (2017A) have shown that Mode I fatigue crack growth is little influenced by a single Mode II overload, whereas a considerable delay in crack growth can be observed when a Mode I overload of equal size ($K_{Io} = K_{IIo}$) occurs (Fig. 7). This fact is of special importance for evaluating practically-relevant fatigue and accidental stresses.

Conclusion

A critical comparison of a number of test procedures for mixed-mode experiments has been made. The investigations show that the three-point bend specimen in conjunction with the four-point shear specimen and the CTS and CTSM specimens are highly suitable for mixed-mode experiments. CTS and

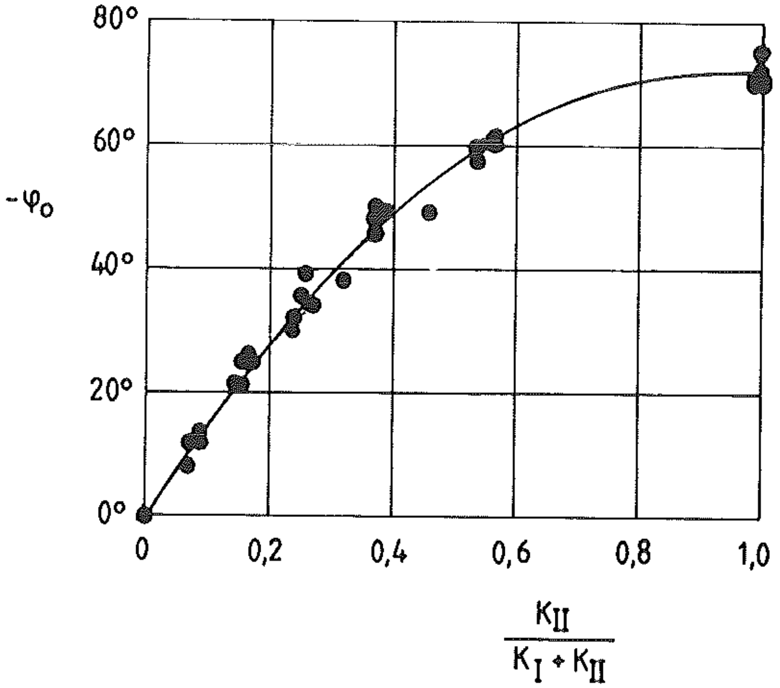


Fig 6 Crack deflection angle measured on PMMA compared with the approximation formula used for calculating the crack deflection angle of mixed-mode fractures

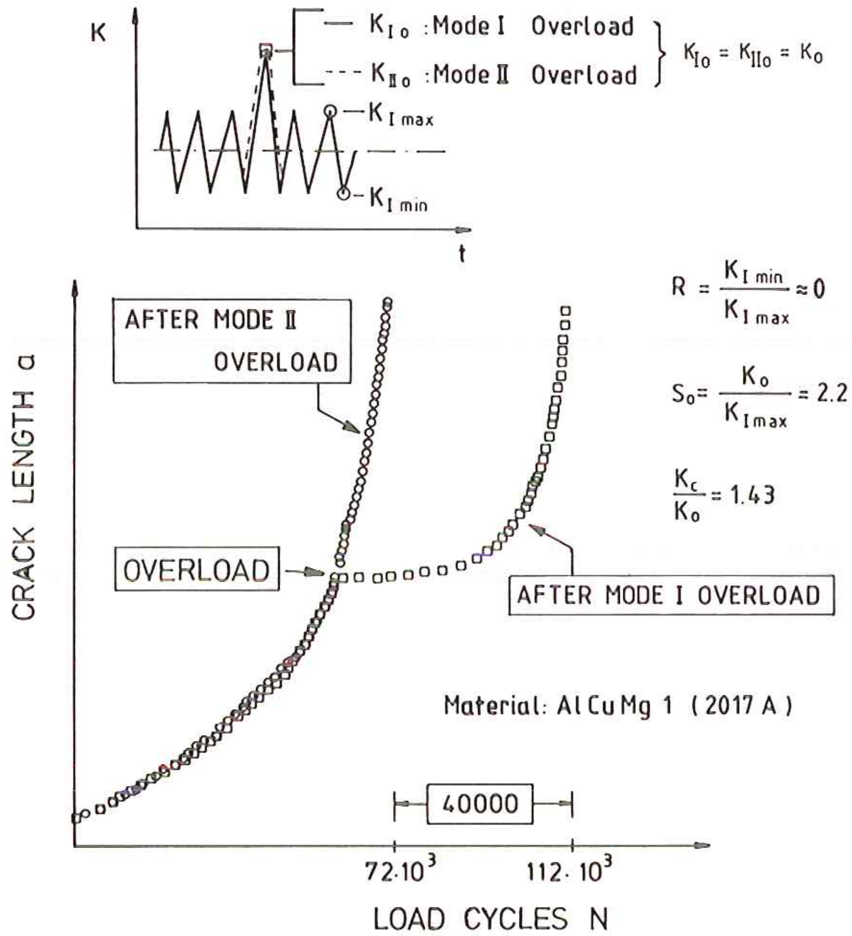


Fig 7 Effect of Mode I and Mode II overloads on Mode I fatigue crack growth

CTSM specimens apply not only to determining fracture toughness values and crack deflection angles, but also to tests for investigating crack growth under fatigue loading. The test results described in this paper illustrate that mixed-mode experiments are of practical importance.

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