A STUDY ON FRACTURE MODEL OF CROSS-PLY COMPOSITE LAMINATES

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I. INTRODUCTION

In reference [1] the fracture toughness values of various composite laminates obtained by compact tensile specimen were given and the spread process of the damage area in front of cracks was expounded. In reference [2] the influence of the stacking sequence on the cracks spread direction was discussed. In this paper the fracture model of cross-ply laminates is studied and a method for evaluating fracture toughness K_c is proposed, provided that the crack propagation is of self-similar type.

II. MECHANICAL MODEL

As is well-known, there are two kinds of the fracture models for composite materials. Some researchers assert that there is an equivalent crack length or an average stress area around the main crack, hence the conventional macroscopic fracture method for orthotropic material may be applied to the composite materials[3]. Others maintain that owing to the weak interface-strength between fibers and resin the interface will be debonding before catastrophic propagation of main crack. The interface cracks will absorb energy and lessen the stress conentration on the fibers around the tip of main crack. That is the reason why the toughness of composite materials is much higher than matrix's[4]. Reference [5] suggested a material model: a main crack perpendicular to the 0° fiber is introduced, in front of which there is a branch crack simulating "interface debonding". Clearly, the second viewpoint has provided an explicit physical concept, however the first one is simpler for calculation as it enables to employ the conventional fracture formula. In this paper we try to combine the merits of both.

According to our experimental observation, in case of self-similar crack propagation in cross-ply laminates (with about one third of fibers laid in 90° direction) there are some "small terraces" before steady propagation of main crack in the load-crack opening displacement curve (P-Δ). And on the crack propagation pictures of cross-ply laminates a triangular damage area is always observed in front of the main crack. It means that in front of the main crack there exist several longitudinal branch cracks with different length forming a triangular area, which yields accordingly some terraces in the P-Δ curve. After the appearance of the first branch crack, both the transverse normal stress σ_x and shear stress τ_{xy} on the branch crack surface are nullified, so the peak stresses move forward, and thus producing the second and more branch cracks. Since the length of cracks are reduced one by one, as a result a somewhat triangular damage area is formed.

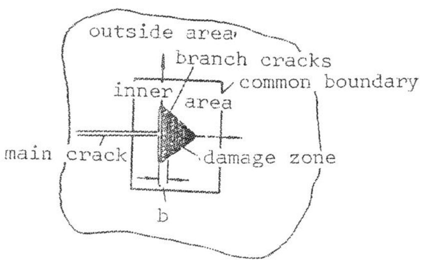

Fig. 1 A fracture mechanical model.

By analogy to the fracture model of the metals where a plastic zone is always formed in front of the crack tip, we tried to insert a triangular damage area instead for the case of composite laminates. (Fig. 1). The stress field and displacements outside the damage area are calculated according to the anisotropic fracture formula. Since the stress analysis inside the damage area is quite different from that of plastic zone in metal, present paper investigates the mechanism of the multi-branch cracks and the stress relaxation in the fiber immediatly ahead of the crack tip, and proposes a method for evaluating the related K_c for composite material.

III. COMPUTING METHOD

It is extremely difficult to solve a multi-boundary problem of a body with multi-cracks by analytical method. P.E. Chen [6] has studied such problem by the relaxation-finite difference method, and he concluded that if the multi-cracks were disposed properly the load-bearing capacity would be higher than that of one crack. In the present paper, similar to that

used in reference [7] the analytical-finite element method is used. The cracked body is divided into two regions-inner area and outside area. We first obtain the analytical solution in the outside area in which the composite laminates is treated as the orthotropic material. The displacements along the boundary common to both regions are thus obtained from this analytical solution. Then the finite element method is used for solution in inner area. The method, however, is different from that of reference [7]

The displacement for crack problems of orthotropic material can be expressed as

$$u(r,\theta) = K_I\left(\frac{2r}{\pi}\right)^{\frac{1}{2}}\mathrm{Re}\left\{\frac{1}{\mu_1-\mu_2}\left[\mu_1 P_2(\cos\theta + \mu_2\sin\theta)^{\frac{1}{2}} - \mu_2 P_1(\cos\theta+\mu_1\sin\theta)^{\frac{1}{2}}\right]\right\}$$

$$v(r,\theta) = K_I\left(\frac{2r}{\pi}\right)^{\frac{1}{2}}\mathrm{Re}\left\{\frac{1}{\mu_1-\mu_2}\left[\mu_1 q_2(\cos\theta + \mu_2\sin\theta)^{\frac{1}{2}} - \mu_2 q_1(\cos\theta+\mu_1\sin\theta)^{\frac{1}{2}}\right]\right\}$$

where P_1, P_2, q_1, q_2 are the functions of elastic constants.

$$P_1 = a_{11}\mu_1^2 + a_{12} - a_{16}\mu_1$$

$$P_2 = a_{11}\mu_2^2 + a_{12} - a_{16}\mu_2$$

$$q_1 = \frac{1}{\mu_1}(a_{12}\mu_1^2 + a_{22} - a_{26}\mu_1)$$

$$q_2 = \frac{1}{\mu_2}(a_{12}\mu_2^2 + a_{22} - a_{26}\mu_2)$$

μ_1, μ_2, and $\bar{\mu}_1$, $\bar{\mu}_2$ are the roots of the following characteristic equation.

$$a_{11}\mu^4 - 2a_{16}\mu^3 + (2a_{12} + a_{66})\mu^2 - 2a_{26}\mu + a_{22} = 0$$

a_{ij} are the elements of flexibility matrix $[a]$, $\{\varepsilon\} = [a]\{\sigma\}$. The displacements obtained on common boundary are employed for solution of inner area as prescribed displacement boundary condition. Some supporting rods with certain stiffness are introduced. Then we can prescribe with aid of the above mentioned displacements the effective node forces on the boundary nodes. This makes it possible to assemble the stiffness matrix of boundary element on to total stiffness matrix of inner area and solve the equations. The rectangle element is used here to solve the equations for inner area.

In order to determine the element size on interface, and to evaluate the elastic modulus of both interface element and damage area after interface debonding, we first calculated a simple model in which there is a main crack with a branch crack only. And we choose the width of interface elements between two fibers be one tenth of the fiber width. After interface debonding the modulus E_x, E_y, G_{xy} in the interface elements are all given a reduction by the order of three magnitudes. The longitudinal modulus E_y in the damage area between two branch cracks are kept unchanged, while E_x, G_{xy} are reduced by three magnitudes as well. The finite element model used for calculating in inner ares has 160 rectangle elements (187 nodes). The elements introduced to simulate the boundary between fibers are allowed up to 5 branch cracks.

It should be pointed out that when we evaluate the displacements on boundary of inner-outside areas, the displacement increment caused by the increase of flexibility in specimen has been ignored. It means that the linear fracture hypothesis is assumed, so that the method is suitable for the cases where the damage area is relatively small. If the dimension of damage area is significantly large the nonlinear effect should be considered.

IV. RESULTS AND DISCUSSION

The fracture model and computing method mentioned above were used for calculating the displacement field of branch cracks and the fiber stresses in front of the cracks. The data used for computation is as follows: $E_1 = E_2 = 1.52 \times 10^3$ kg/mm², $\nu_{12} = \nu_{21} = 0.13$, $G_{12} = 0.3 \times 10^3$ kg/mm², the fiber width $b = 0.625$ mm.

The displacement field obtained for the case in which there are five branch cracks in front of the main crack is shown in Fig. 2. The displacement of a branch crack thus obtained is similar to that of the reference [8]

Fig. 3 and Fig. 4 show the variation of stresses σ_y in the first fiber in front of crack along the x and y direction, respectively. If there is only a main crack then $\sigma_y \to \infty$ as $x \to 0$ and $y \to 0$, i.e. the stress is singular at that point. However once the branch crack appears the originally singular stress turns to be a finite value and keeps relatively uniform value along the y direction. From Fig. 3 it is evident that the uniform stress area increases as the branch crack is getting longer, meanwhile the magnitude of this uniform stress decreases. From Fig. 4 we see that as the length and number of the branch cracks increases and the damage area of material between the branch cracks extends, the stress σ_y is getting smaller and finally approaching to a steady value. We refer it as critical relaxation stress.

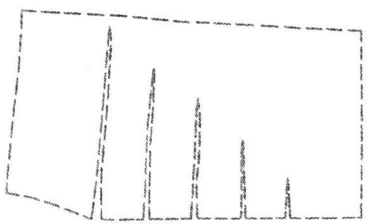

Fig. 2 The displacement field in front of the main crack with five branch cracks

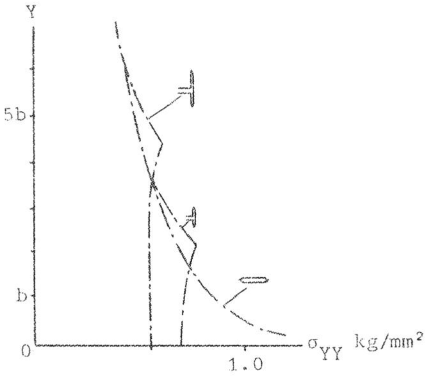

Fig. 3 The curve of σ_y-Y

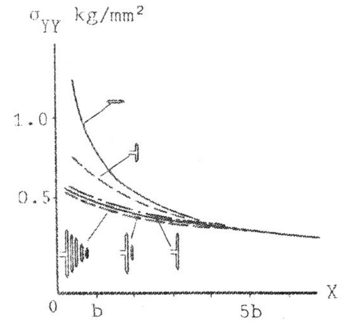

Fig. 4 The curve of σ_y-X

If the plane elastic modul E_{xx}, E_{yy}, μ_{xy} and G_{xy}, the longitudinal ultimate tensile stress σ_{yB} and the width b of a single fiber are given, the critical relaxation stress under the action of $K_1 = 1.0$ kg/mm$^{3/2}$ can be calculated. For the material constants computed given in this paper, the critical relexation stress is 0.48 kg/mm². So when the critical relaxation stress reaches the tensile strength σ_{yB} (for material given in this paper σ_{yB} = 25–30 kg/mm²), we obtain the critical value of K_1, i.e. the fracture toughness K_c = 50–60 kg/mm$^{3/2}$. This value is in agreement with the experimental results. This is the method to evaluate K_c for the cross-ply laminate by relaxation value of stress σ_y obtained by the "debonding damage area" fracture model.

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