THE CORRELATION OF TRIAXIAL STATES OF STRESS AND THE FAILURE STRAIN

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The development and the subsequent growth of voids discussed recently (1) is strongly dependent on the triaxial state of stress. The decrease of the failure strain with increasing triaxiality, reported elsewhere (2) confirms this. Cylindrical tensile specimens with the pre-machined circumferential notches are particularly suitable for the investigation of the effect of stress states (3,4,5). In the present work an approach similar to that derived by Rice (6) has been applied. It is suggested here, that the effect of stress state may be expressed in the form

$$C_V = \varepsilon_f \, exp \left(\frac{3}{2} \frac{\sigma_m}{\sigma} \right) \qquad (1)$$

and C_V is a material constant.

For the geometry of the tensile specimens used in our tests two assumptions were made:

1. The strain ε_p at the failure initiation (ie voids coalescence on a certain low scale) corresponds to the critical growth rate of voids, or, to the relative volume of voids;

2. The fracture strain ε_f is proportional to the initial failure strain ε_p; it is possible to measure fracture strain experimentally, whereas the initial failure strain can not be actually determined.

The geometry of plain cylindrical and notched tensile specimens is shown in Figure 1. and Table 1. These specimens were used to study the influence of triaxial states on failure strain ε_f. The axis of the specimens was parallel to the LT direction. The results from both unnotched and notched tests are presented in Figures 2, 3 and 4 and Table 2. In Figure 2. the average instant stress at the minimum cross section σ_A is plotted against $2 \, ln \, (a_o/a)$, where a_o is the original, and a is the current radius at the minimum section. The average stress σ_A at the fracture of the notched specimens was higher than that recorded for the unnotched specimens. However, the terminal strain was much smaller than that reached in the unnotched specimens and never more than 50% of ε unnotched. In Table 1, a_f and R_f denote the value of the radius of minimum cross section and the curvature of the notch root (neck) at the complete failure of the specimen respectively.

In Table 2, Bridgman's analysis (7) was used as an approximate solution for the estimation of the stress state parameter $\sigma_m/\bar{\sigma}$ and the

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effective plastic strain ε_p in the central region of the specimen where failure initiated. Referring now to Equation (1), the original and the current values of C were expressed as

$$C_{V_o} = \varepsilon_f \exp\left(\frac{3}{2}\frac{\sigma_m}{\bar{\sigma}}\Big|_o\right) \tag{2}$$

$$C_V = \varepsilon_f \exp\left(\frac{3}{2}\frac{\sigma_m}{\bar{\sigma}}\Big|_f\right)$$

and $\bar{C}_{V_o}$ and $\bar{C}_V$ are the mean values of C_{V_o} and C_V respectively. Using Table 2 and Figure 4, it can be concluded that the value $\varepsilon_f \exp\left(\frac{3}{2}\frac{\sigma_m}{\bar{\sigma}}\right)$ was calculated as a material constant. This material constant applies for tensile specimens with notches of different radii.

The stress state parameter $(\sigma_m/\bar{\sigma})$, estimated from the final value of the ratio (a_f/R_f), is better than that derived from the original values of (a_o/R_o). It is worth pointing out the dashed line in Figure 3, representing a typical axisymmetric deformation history; the difference in the C_V values for the notched and unnotched specimens will require further study.

CONCLUSION

It was shown that the expression Equation (1) provided the value of a constant C_V for the low alloy steel BS4360-50D as follows:

$$C_V = \varepsilon_f \exp\left(\frac{3}{2}\frac{\sigma_m}{\bar{\sigma}}\Big|_f\right) = 2.63$$

with the error less than ± 9%.

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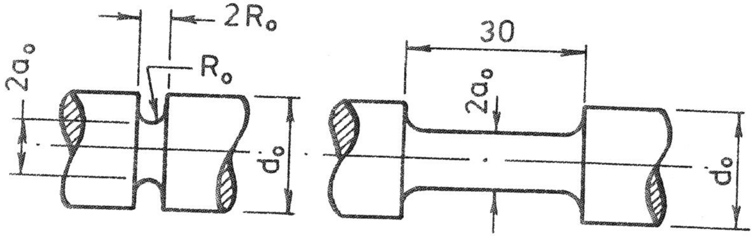


Figure 1. Specimen Geometry

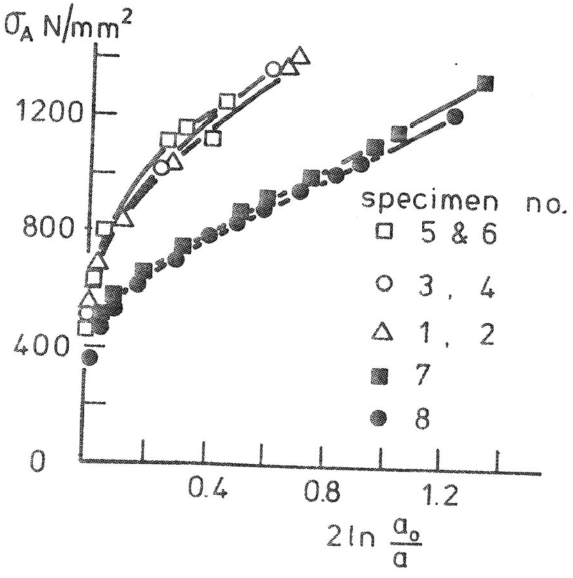
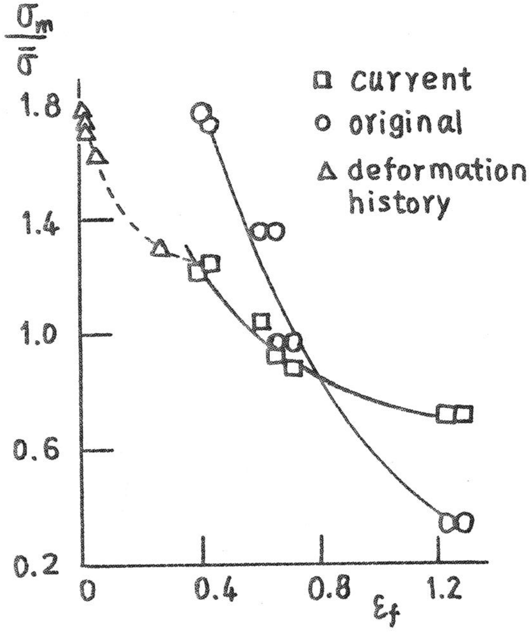
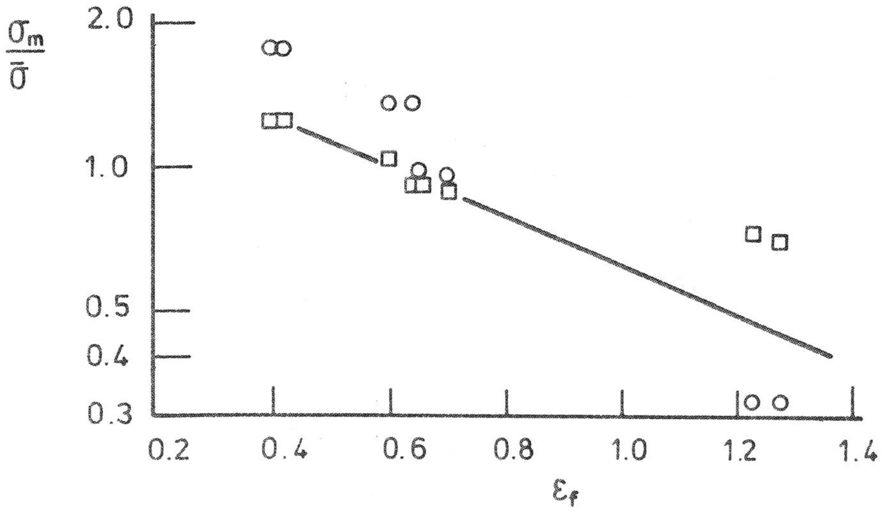


Figure 2. σ_A vs ln a_o/a





Figs. 3 and 4 $\sigma_m/\bar{\sigma}$ vs ε_f

TABLE 1.

Type	No	d_0 mm	A_0 mm	R_0 mm	A_0/R_0	A_f mm	R_f mm	A_f/R_f
Notched	1	11.28	3.50	2.0	1.75	2.47	1.69	1.46
"	2	11.28	3.55	2.0	1.78	2.57	1.63	1.58
"	3	11.28	3.53	1.0	3.53	2.56	1.61	1.59
"	4	11.28	3.54	1.0	3.54	2.63	1.27	2.07
"	5	11.28	3.63	0.60	6.05	2.94	0.99	2.97
"	6	11.28	3.75	0.59	6.36	3.08	1.08	2.85
Unnotched	7	11.28	3.59	∞	0	1.89	2.16	0.88
"	8	11.28	5.64	∞	0	3.04	3.3	0.92

TABLE 2.

No	ε_f	$\frac{\sigma_m}{\sigma}\Big\rvert_o$	c_{v_o}	$\frac{\sigma_m}{\sigma}\Big\rvert_f$	c_v
1	0.70	0.96	2.95	0.88	2.62
2	0.65	0.97	2.78	0.92	2.58
3	0.64	1.35	4.85	0.92	2.54
4	0.60	1.35	4.55	1.04	2.86
5	0.42	1.73	5.63	1.24	2.70
6	0.40	1.76	5.61	1.22	2.49
		$\overline{c}_{v_o} = 4.40 \pm 36\%$		$\overline{c}_v = 2.63 \pm 9\%$	
7	1.28	0.33	2.10	0.70	3.66
8	1.23	0.33	2.02	0.71	3.57