DISLOCATION SHIELDING OF CRACKS AND THE FRACTURE CRITERION

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Fracture criteria is a generally frustrating subject to the fracture mechanics theorist because due to an early theorem of Rice [1], in a medium where the stress at the crack tip saturates to a finite value, the energy absorbed by the underlying crack, as it moves, from the external driving mechanisms is zero. That is, the underlying crack is effectively disconnected from the process, and the fracture criterion becomes difficult to address on the continuum level. On the other hand, physical intuition suggests that local conditions at the crack tip must have a great deal to do with crack advance.

Physical insight into this problem is provided when one adopts the point of view of the defect theorist, and addresses the fracture problem in terms of sharp cracks and individual dislocations as singularities interacting with one another through the means of the elastic medium. On this view, the plastic zone is not a smeared out distribution of continuum plastic strain (at least not at first), but a distribution of dislocations created by discrete sources distributed in a random fashion. It is important to note that the source distribution is very sparce on an atomic scale in consonance with the findings of electron microscope studies of plasticity in materials.

We suppose then for purposes of developing a simple model that a mode III crack has been introduced into the medium, and a set of dislocations is introduced by a source in the immediate vicinity of the crack. Expressions have been derived giving the force on the crack and on each of the dislocations [2], respectively,

$$\bar{f}_c = \frac{k^2}{2\mu}$$

$$k = K - \sum_j \frac{\mu b_j}{2\sqrt{2\pi}} \left\{ \frac{1}{\sqrt{\zeta_j}} + \frac{1}{\sqrt{\bar{\zeta}_j}} \right\}$$

$$\bar{f}_d = \frac{Kb}{\sqrt{2\pi\zeta}} - \frac{\mu b^2}{4\pi} \left\{ \frac{1}{2\zeta} + \frac{1}{\zeta - \bar{\zeta}} - \sqrt{\frac{\bar{\zeta}}{\zeta}} \frac{1}{\zeta - \bar{\zeta}} \right\}$$

$$+ \sum_j' \left\{ \frac{1}{\zeta - \zeta_j} - \frac{1}{\zeta - \bar{\zeta}_j} + \sqrt{\frac{\zeta_j}{\zeta}} \frac{1}{\zeta - \zeta_j} + \sqrt{\frac{\bar{\zeta}_j}{\zeta}} \frac{1}{\zeta - \bar{\zeta}_j} \right\} \frac{bb_j \mu}{4\pi} \qquad (1)$$

The force is given as a vector f in the complex plane on crack and dislocation, respectively, K is the applied stress intensity factor, b is the Burgers vector of a reference dislocation, and b_j is the Burgers vector of all other dislocations. Note that b has a sign and may be plus or minus for the screw dislocations envisioned, which are at position ζ in the complex plane with the crack at the origin. Both crack and dislocations are parallel to the x_3 axis. The sum over j is a sum over the dislocation distribution. Σ' denotes a sum over all dislocations except that for which the force is being calculated. k is a local stress intensity factor for the crack, which is a shielded value relative to the external stress intensity if the Burgers vectors have a positive sign. Negative Burgers vectors denote antishielding dislocations.

Generally, sources of dislocations emit as many positive as negative Burgers vectors (conservation of total Burgers vector), and if only one sign predominates in a distribution, the remainder have disappeared at some nearby surface--such as the crack surfaces. The antishielding dislocations are in fact attracted to the crack, and one would expect a distribution of dislocations to thus be composed of an excess of shielding dislocations over antishielding ones.

In equilibrium, each defect will seek a position of total net force.

That is, the elastic forces in (1) will be balanced by other lattice forces, such as surface tension effects at the crack tip (Griffith's relation) or Peierls forces, etc., at the dislocations. Thus

$$f_c = 2\gamma$$

$$\bar{f}_d = \sigma_f b \qquad (2)$$

γ is the intrinsic surface tension of the open surface, while σ_f is an assumed friction stress acting on a dislocation. The general problem posed cannot be solved, of course, when the number of dislocations is large, but a variety of simple cases can be addressed, such as one dimensional dislocation pileups, with various assumptions about σ_f and pileup geometry.

The simplest such problem is given by the one dimensional pileup of a set of dislocations on the crack plane, with the nearest dislocation at x_1=c and the furthest one at x_1=d. When d>>c, this problem has a very simple solution [3],

$$\frac{k}{K} = \frac{3}{2\pi} \sqrt{\frac{c}{d}} \left(\ell n \frac{4d}{c} + \frac{4}{3} \right)$$

$$K = 2\sqrt{2/\pi}\ \sigma_f \sqrt{d}$$

$$B = \int_c^d \beta(x)\, dx = \frac{2K}{\mu} \sqrt{\frac{d}{2\mu}} \qquad (3)$$

B is the total integrated Burgers vector content of the dislocation distribution, and σ_f has been assumed to be a constant in order to obtain the solution. The second and third equations are the same as the BCS result [4] for a crack and dislocation distribution. However, if c≠o, the crack tip has a local k associated with it, and the problem of the fracture criterion is solved when k is set to the Griffith value, (2).

This solution of the problem is fundamentally satisfying, in the sense that the properties of the crack tip and the local balance of forces on the atoms at the tip determine (in conjunction with similar properties of the dislocation) the total configuration. This point becomes explicit when K in the first equation is written in terms of γ, which shows that the equilibrium, or critical K is in general a function of γ (a local property of the sharp crack) and the plastic variables d and c. c is important in this picture, because choosing c>o saves us from the Rice catastrophe. In fact c will be greater than zero provided σ_f is finite, and the total number of dislocations is finite.

An illuminating second application is to assume the crack moves with its dislocation shielding cloud with a uniform velocity, v. Two cases emerge; in the first, a dislocation cloud of fixed number physically moves with the crack, and in the second, the crack generates a cloud as it translates from sources in the medium. We treat these cases in sequence.

In case 1, the dislocations are assumed to obey a stress velocity law $v(\sigma)$, which can be determined independently. In addition, the crack itself obeys such a law, but in a first approximation, we shall take this law to be a step function. That is, once the Griffith criterion is reached, $k^2=4\mu\gamma$, the crack moves at arbitrary velocity. With the assumption of a one dimensional distribution, Eqn. (3) can be applied. We rewrite these questions in the approximate form

$$k = p\sigma_f \sqrt{c}$$

$$K^2 = 2\mu\sigma_f B$$

$$K^2 = \frac{8}{\pi} \sigma_f^2 d \qquad (4)$$

p is a numerical constant of order 1. When σ_f is given in terms of the velocity law of the dislocations, and B is predetermined, the problem is solved. d and c take the values as given. In particular, from the first equation, c fixes itself so that $\sigma_f(v)$ and the critical value of k given by the Griffith value are determined. That is, in order to achieve a state

of uniform motion, the dislocations distribute themselves in such a way to shield the inner crack so that its k(v) law is satisfied. This is achieved merely by moving the first dislocation in the distribution closer or farther from the crack tip. By elimination, then from Eqn. (4) a K(v) law can be obtained.

However, in reality, c cannot be chosen arbitrarily. In particular, the distribution in a real crystal will have dislocations on a variety of slip planes--in general none of them coinciding exactly with the crack plane. Thus, the closest any dislocation can come to the crack tip is the distance to the nearest slip plane in the distribution, and a maximum shielding value exists which the dislocations can provide. When the external K is raised above this value, the excess K leaks through the shielding cloud, to the core crack itself, and a discontinuous breakaway ensues in which the core crack breaks away from its shielding dislocations in a dramatic manner.

The overall conclusion is thus that a K(v) law will be a reasonably continuous function during the shielding regime, dominated by the $\sigma_f(v)$ function of the shielding dislocations. At a critical value, however, the crack breaks away from its shielding charge, and completely brittle catastrophic fracture ensues.

In the second case, space does not permit a full solution here, but a similar breakaway phenomenon transpires for the same reason: the cutoff in c occasioned by the random but sparce distribution of dislocation sources does not allow arbitrarily large amounts of shielding.

REFERENCES

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