## THE USE AND THE LIMITATIONS OF $C^*$ IN CREEP CRACK GROWTH TESTING

Hermann Riedel
Max-Planck-Institut für Eisenforschung, D-4000 Düsseldorf, FRG

Within the framework of fracture mechanics, creep crack growth is described by macroscopic load parameters like $K_I$, J and $C^*$. The idea is that identical values of the appropriate load parameter in differently shaped specimens or structures generate identical conditions of stress near the crack tip so that the crack growth rates must be the same provided that the material, the temperature and the chemical environment near the crack tip are the same. If the right load parameter that unifies the behavior of different specimens can be identified, one need not care about the micromechanisms of creep crack growth (e.g. grain boundary cavitation, corrosion). It suffices to measure the crack growth rate as a function of the load parameter in the laboratory, and to calculate the value of the load parameter for the crack in the structure under consideration. This gives the expected crack growth rate in future service.

The following discussion is grouped around the $C^*$-approach which has been applied successfully to many of the more ductile materials in the past few years, e.g. [1-7]. Its theoretical basis and the rules for its use are summarized. An attempt is made to identify some of the limitations to the $C^*$-approach caused by crack-tip blunting, elastic deformation, instantaneous plasticity, transient creep and cavitation. The goal is to establish a set of conditions for the applicability of $C^*$ and other load parameters, quite in the spirit of the ASTM-rules for the validity of linear elastic fracture mechanics or of J-controlled fracture testing.

### I NONLINEAR VISCOUS MATERIALS AND THE USE OF $C^*$

The continuum-mechanical discussion is based on the constitutive equations for equilibrium and compatibility

$$\nabla_i \sigma_{ij} = 0, \qquad \nabla_k \nabla_l \dot{\varepsilon}_{ij} + \nabla_i \nabla_j \dot{\varepsilon}_{kl} = \nabla_j \nabla_l \dot{\varepsilon}_{ik} + \nabla_i \nabla_k \dot{\varepsilon}_{jl} \tag{1}$$

together with various material laws. A material is called (generally nonlinear) viscous if the strain rate is a unique function of the current stress, $\dot{\varepsilon}_{ij} = f(\sigma_{ij})$. Equation (1) is understood with the summation convention for repeated indices, $\nabla_i$ is the gradient operator, and the superposed dot denotes the time derivative.

It has long been recognized that nonlinear viscous materials are analogous to the corresponding nonlinear elastic materials described by $\varepsilon_{ij} = f(\sigma_{ij})$. The constitutive equations for the two types of materials are identical if strain and strain rate are exchanged. Therefore, if the applied loads are the same, also the stress fields in elastic and viscous bodies are the same while strain and displacement in elastic materials correspond to their time rates in viscous bodies.

In the present paper, we consider two-dimensional problems (plane strain or plane stress). A planar crack with traction-free faces is subjected to remote symmetric loading (Mode I). If $\Gamma$ is a path around the crack tip starting at an arbitrary point on the lower crack face and ending at an arbitrary point on the upper face, the integral

$$C^* = \int_\Gamma (W^* dx_2 - \sigma_{ij} n_i \partial \dot{u}_j / \partial x_1 ds) \qquad \text{with} \qquad W^* = \int^{\dot{\varepsilon}_{ij}} \sigma_{ij} d\dot{\varepsilon}_{ij} \tag{2}$$

is the creep analogue of Rice's J-integral; $\dot{u}_j$ is the displacement rate field, $x_1$ and $x_2$ are Cartesian coordinates parallel and normal to the plane of the crack, s is arc length and $n_i$ is the outward normal unit vector on $\Gamma$. Since J is independent of the choice of the path, $C^*$ is also path-independent. In principle, the value of $C^*$ is measurable at the load pins of a pair of specimens which have slightly different crack lengths but are otherwise identical [1]. Using the analogy with J gives

$$C^* = -(\partial/\partial a) \int P d\dot{\Delta} \tag{3}$$

where P is load per unit specimen thickness, $\dot{\Delta}$ is the displacement rate at the load pins and a is crack length. Alternative methods of determining $C^*$ will be described below for power-law viscous materials.

It is interesting to note that in viscous materials the stress field is independent of whether the crack is stationary or whether it grows. Only the current specimen geometry determines the stress field.



## Power-Law Viscous Materials

The great practical advantage of a path-independent integral like $C^*$ is illustrated considering a power-law viscous material. Such a material is characterized by

$$\dot{\varepsilon}_{ij} = (3/2)\, B\, \sigma_e^{n-1} \sigma'_{ij} \qquad \text{with} \qquad \sigma_e = (3\sigma'_{ij}\sigma'_{ij}/2)^{1/2}, \tag{4}$$

where the prime denotes the deviator and $\sigma_e$ is the equivalent tensile stress. The material parameters B and n can be determined in uniaxial creep tests. In this case, eq. (4) takes the form $\dot{\varepsilon} = B\sigma^n$, which is known as Norton's creep law. The multiaxial generalization (eq. 4) is based on von Mises' flow rule for incompressible materials.

In a power-law viscous material it can easily be verified by insertion of eq. (5) below into the constitutive equations that, for proportional loading P(t), the stress field is also proportional, i.e.,

$$\sigma_{ij}(r_i, t; P(t), a) = \sigma_{net}(t)\, F_{ij}(r_i/a). \tag{5}$$

Here, for mere convenience the load has been replaced by the net section stress $\sigma_{net}$ which is defined as load divided by the area of the uncracked ligament of the specimen. The dimensionless function $F_{ij}$ is a function of the spatial variables $r_i$ only but does not depend on the magnitude of the load. For self-similar specimen shapes the stress fields are also self-similar so that $F_{ij}$ depends on the dimensionless ratio $r_i/a$ only. (This latter property is not confined to power-law viscous materials).

While the calculation of the stress distribution $F_{ij}$ in the whole body generally requires numerical techniques, the asymptotic stress field near the crack tip is analytically known from the analysis of power-law elastic materials. Hutchinson [9], and Rice and Rosengren [10] (often jointly referred to as HRR) show that the stress field approaches

$$\sigma_{ij}(r,\theta) = \left(\frac{C^*}{I_n B r}\right)^{1/(n+1)} \tilde{\sigma}_{ij}(\theta) \qquad \text{for } r/a \rightarrow o, \tag{6}$$

independent of the outer specimen geometry. Here, r and $\theta$ are polar coordinates centered at the crack tip with $\theta$=0 directly ahead of the crack. The dimensionless constant $I_n$ is chosen such that the maximum of the dimensionless angular function $\tilde{\sigma}_e = (3\tilde{\sigma}'_{ij}\tilde{\sigma}'_{ij}/2)^{1/2}$ is normalized to unity. Values of $I_n$ and $\tilde{\sigma}_{ij}(\theta)$ are given by HRR and subsequent workers.

The above equation (6) explicitly gives the near-tip stress field in terms of $C^*$ which is measurable at the load pins of the specimen. This illustrates that the conditions at the crack tip, and therefore crack growth behavior in viscous materials, are determined by $C^*$ rather than by any other load parameter. This statement is the basis for the application of $C^*$ to creep crack growth.

## Determination of $C^*$ Based on Stress Analysis

The experimental determination of $C^*$ based on eq. (3) is often inconvenient. Therefore a few alternative equations will be given which are based on explicit (mostly numerical) stress analyses of power-law materials. From the general form of the stress field (eq. 5), together with the line-integral definition of $C^*$ (eq. 2) and the material law (eq. 4), it follows that in a power-law viscous material $C^*$ must have the form

$$C^* = a\, B\, \sigma_{net}^{n+1}\, g_1(a/W,n) \tag{7}$$

where the dimensionless quantity $g_1(a/W,n)$ is an abbreviation for the line integral over the $F_{ij}$'s. Kumar et al [11] have evaluated these integrals based on finite element solutions for various specimen geometries and a/W-ratios where a is crack length and W is specimen width. Their notation differs from eq. (7). For standard ASTM compact specimens, for example, they express $g_1$ as

$$g_1 = h_1(\, W/a-1\, )/(\, 1.455\ n\, )^{n+1} \tag{8}$$

The function $h_1$ is tabulated for plane stress and plane strain while $n$ is given analytically. If in a creep crack growth test the load line deflection rate $\dot{\Delta}$ is measured, it is advantageous to use the fact that $\dot{\Delta} \propto aB\sigma_{net}^n$ with a geometry-dependent factor of proportionality which is also given by Kumar et al [11]. Then eq. (7) can be written in one of the forms

$$C^* = g_2(a/W,n)\, \sigma_{net}\dot{\Delta} = g_3(a/W,n)\, \dot{\Delta}^{(n+1)/n}/(aB)^{1/n} \tag{9}$$

where $g_2$ and $g_3$ are related to the notation of Kumar et al through



$$g_2 = (W/a-1)\, h_1/(\, 1.455\ n\ h_3), \qquad g_3 = (W/a-1)\, h_1/h_3^{1+1/n} \tag{10}$$

Musicco [8] gives plots of the dimensionless functions $g_2$ for several specimen geometries. When applying eq. (9) for the determination of $C^*$ one should make sure that the deflection rate $\dot{\Delta}$ is caused mainly by creep of the material rather than by crack growth. Crack growth also contributes to $\dot{\Delta}$ because of the increasing elastic and instantaneous plastic compliance of the specimen during crack growth. In many cases of $C^*$-controlled crack growth, creep is indeed the major contribution to $\dot{\Delta}$. A counter-example will be discussed in Section IV.

Besides the numerical solutions for various specimen geometries, also approximate analytic solutions are available for the plane-strain crack and the penny-shaped crack in an infinite body (He and Hutchinson [12]). For a plane-strain crack which is subjected to the remote stresses $\sigma_{22}^\infty$ and $\sigma_{11}^\infty$ normal and parallel, respectively, to the crack plane, the $C^*$-integral is approximately given by

$$C^* = a\, \sigma_e^\infty\, \dot{\varepsilon}_e^\infty\, (3\pi/4)\, \sqrt{n}\, (\sigma_{22}^\infty/\sigma_e^\infty)^2. \tag{11}$$

Here, $\dot{\varepsilon}_e^\infty = B(\sigma_e^\infty)^n$, and $\sigma_e^\infty = \sqrt{3}\, |\sigma_{22}^\infty - \sigma_{11}^\infty|\, /2$ for plane strain. Compared with numerical solutions, which are also developed in Ref. [12], the accuracy of eq. (11) decreases at higher values of $\sigma_{22}^\infty/\sigma_e^\infty$ and this occurs faster for greater n. The formula is exact for all $\sigma_{22}^\infty/\sigma_e^\infty$ if n=1. For plane strain tension ($\sigma_{11}^\infty$ = o), eq. (11) is accurate to within a few per cent if n = 5. Similarly, the $C^*$-integral for the penny-shaped crack under axisymmetric loading is given by

$$C^* = a\, \sigma_e^\infty\, \dot{\varepsilon}_e^\infty\, (6/\pi)\, (1+3/n)^{-1/2}\, (\sigma_{22}^\infty/\sigma_e^\infty)^2 \tag{12}$$

where $\sigma_{22}^\infty$ acts normal to the crack plane, $\sigma_{11}^\infty$ acts radially and $\sigma_e^\infty = |\sigma_{22}^\infty - \sigma_{11}^\infty|$. This formula is again exact for n = 1 and remains approximately valid for all n if $\sigma_{22}^\infty/\sigma_e^\infty \leq 2$. Note that uniaxial tension corresponds to $\sigma_{22}^\infty/\sigma_e^\infty = 1$ and is therefore described accurately by eq. (12).

## Crack Growth Behavior in Viscous Materials

So far, only the stress and strain distribution in cracked bodies has been considered. The response of the crack (in terms of crack growth rate) to the applied driving force $C^*$ will, in practice, be determined experimentally. It can also be treated theoretically provided that the micromechanisms of crack extension are understood well enough. Very often creep crack growth occurs by the coalescence of grain boundary cavities with the main crack. Although the details of cavity nucleation and growth are not yet perfectly clear it appears that in many instances failure by cavity coalescence can be described approximately by a critical strain criterion. This is supported by the empirical Monkman-Grant rule which is in essence a critical strain criterion. The most convincing explanation for the applicability of a critical strain criterion appears to be Dyson's model of constrained diffusive cavity growth [13].

The success of a critical strain criterion for creep rupture suggests to apply it to creep crack growth as well. We assume that during creep crack growth the strain at some structural distance $x_c$ ahead of the crack tip must always be maintained at a constant, critical value, $\varepsilon_c$. This assumption leads to an (integral) equation for the crack growth rate, $\dot{a}$, as will be shown. The formulation of the equation of motion follows the method used by Riedel [14], whereas Ohji [2] formulates the same physical problem in a discretized manner.

The stress field at a (stationary or moving) crack tip in a power-law viscous material is given by eq. (6). The strain rate follows from the stress through the material law, eq. (4). If the test has been started with a crack of initial length $a_i$ in an initially unstrained material, the strain at a distance $x_c$ ahead of the current tip position is calculated from the strain rate by time integration as

$$\varepsilon_c\left(\frac{x_c}{a-a_i+x_c}\right)^{n/(n+1)} + B\,\tilde{\sigma}_e^n(0)\int_{a_i}^{a}\left(\frac{C^*}{I_n B(a+x_c-a')}\right)^{n/(n+1)}\frac{da'}{\dot{a}(a')} = \varepsilon_c. \qquad (13)$$

Here the time-differential has been replaced by $da'/\dot{a}(a')$ and the equality of the strain ahead of the tip with the critical strain is required by the critical strain criterion. The first term is the strain which has been accumulated during the crack growth initiation time

$$t_i = (I_n B x_c/C^*)^{n/(n+1)}\varepsilon_c/(B\tilde{\sigma}_e^n(0)) \qquad (14)$$

which elapses until the critical strain is accumulated ahead of the still stationary crack. The integral in eq. (13) represents the strain at the



current crack tip, which has been added during the crack growth period. The integral extends over all prior crack tip positions. The linear, Volterra-type integral equation for the unknown quantity $1/\dot{a}(a)$ can either be solved by the Laplace transformation method (which leads to a very complicated analytic expression) or numerically (which only requires the evaluation of a recurrence formula) or by elementary analytic methods for small and large crack growth increments $\Delta a=a-a_i$. The complete numeric solution is given in Ref.[14]. The initial growth rate is found to be $\dot{a}_i=(1+1/n)x_c/t_i$, while for large $\Delta a/x_c$ there results

$$\dot{a} = \frac{\pi\,\tilde{\sigma}_e^n(0)(Bx_c)^{1/(n+1)}}{\varepsilon_c\sin(\pi\alpha)}\left[\frac{C^*(a)}{I_n}\right]^{n/(n+1)}\left[(\Delta a/x_c)^{1/(n+1)} - \frac{\Gamma(\alpha)}{\Gamma(2-\alpha)\,\Gamma(2\alpha-1)}\right], \qquad (15)$$

where $\alpha=n/(n+1)$ and $\Gamma$ is the complete Gamma-function. The expresssion involving the Gamma-functions has the numerical values 0.85, 0.95 and 1 for $n$ = 4, 7.5 and $\infty$, respectively. The results will be discussed together with experimental observations below.

## Comparison with Experiments

Figure 1 shows experimental results of Wagner and Riedel [15] on 1%Cr1/2%Mo steel at 535°C. The crack growth rate is found to be an almost unique function of $C^*$ whereas the stress intensity factor gives no reasonable correlation. Further, the slope of the $\dot{a}(C^*)$ relation in the doubly logarithmic plot agrees with $n/(n+1)$, which is predicted by eq. (15). The temperature dependence is found to be small in the range 450 to 600°C, in accord with eq. (15): the activation energy of power-law creep enters into eq. (15) through the material parameter $B$ of Norton's creep law. If this activation energy is, say 270 kJ/mole, the activation energy of the term $B^{1/(n+1)}$ is only 30 kJ/mole if $n$=8. Finally the dependence of $\dot{a}$ on $\Delta a$ appears to be weak, both experimentally and theoretically, except for the initial stages of crack growth. It should be noted that such a moderate dependence is not necessarily obtained if a crack growth criterion is used which is different from a critical strain criterion. For example, Riedel [14] shows that a stronger dependence of the form $\dot{a}\sim\Delta a^{n/(n+1)}$ obtains if local failure is controlled by unconstrained diffusive cavity growth. The experimental results quoted above are representative for other studies on various steels and superalloys at higher temperatures, e.g. [1-7].

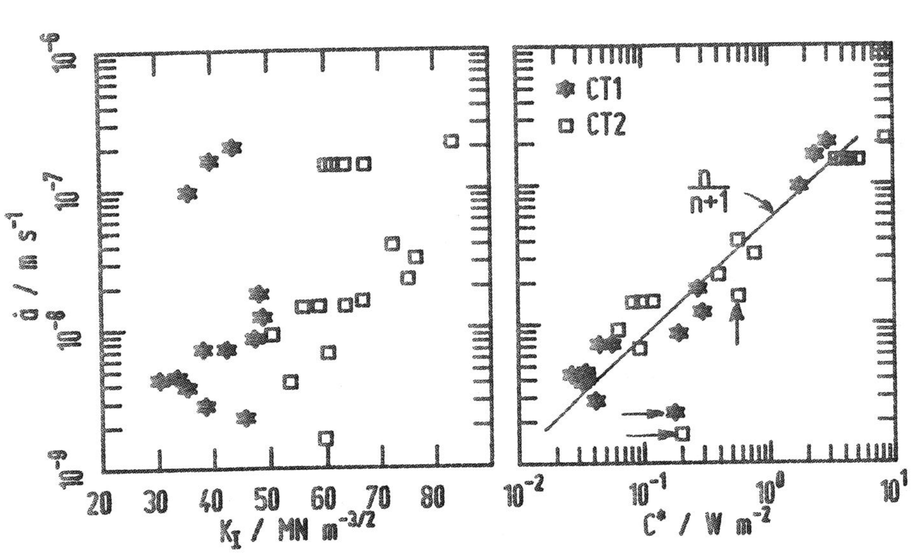


Fig. 1. Crack growth rates in a 1Cr1/2Mo steel at 535$^0$C vs. $K_I$ and $C^*$. Stars: CT1-, squares: CT2-specimens. Arrows: Measurement near beginning of test. Temperature dependence: factor 5 between 450 and 600$^0$C.

## II LIMITATIONS TO $C^*$ SET BY BLUNTING

Just as in time-independent fracture mechanics, crack-tip blunting may be a limiting factor for the applicability of the respective fracture parameters. We define the crack-tip opening displacement, $\delta$, where two lines drawn through the apex of the crack profile inclined by $\pm 30^0$ to the crack plane, intersect the crack profile. For a stationary crack, the displacement associated with the HRR-field is [16]

$$\delta = \left\{ 2 \frac{|\tilde{u}_\theta(\pi)|^{(n+1)/n}}{I_n (\tan 30^0)^{1/n}} \right\} B^{1/n} \, C^* t^{(n+1)/n}, \qquad (16)$$

where the numerical factor in parentheses is approximately equal to unity. If the time is taken as the initiation time for crack growth given in eq. (14), there results $\delta_i = 450\ \varepsilon_c^{(n+1)/n} x_c$ if n=7. The critical strain at the crack tip, $\varepsilon_c$, can be indirectly estimated by fitting the theoretical crack growth rate (eq. 15) to experimental data. Usually $\varepsilon_c$ is found in this way to be of the order of one per cent in the highly triaxial field ahead of a crack [15]. The structural distance $x_c$ is expected to be of the order of the cavity spacing (typically 3µm) so that the crack opening



displacement at crack growth initiation is of the order of 10 µm.

By analogy with elastic-plastic fracture mechanics (ASTM E-813), the condition for valid $C^*$-testing in the presence of blunting is

$$(a,\ W-a) \geq 2M\delta_i, \qquad (17)$$

where the numerical factor M depends on the specimen geometry and on the stress exponent n. For CT-specimens, M is typically of the order 25, while for center-cracked panels in plane-strain tension, M = 200 is representative. If $\delta_i$ is as small as estimated above (10µm), blunting is not a serious limitation for $C^*$. After crack growth initiation, the opening of the crack at the original crack tip position continues but the current crack tip becomes sharper. This is observed experimentally and predicted theoretically [14]. Therefore it appears that crack tip blunting becomes even less important when the crack grows.

## III. LIMITATIONS TO THE $C^*$-APPROACH SET BY ELASTIC STRAINS - APPLICABILITY OF $K_I$

In alloys that are relatively creep resistant and exhibit relatively rapid creep crack growth at a given temperature, crack extension may occur under predominantly elastic conditions. Predominantly elastic deformation, of course, invalidates the $C^*$-approach, which is based on viscous material behavior, and it makes the stress intensity factor $K_I$ the proper load parameter as one may expect. The ranges of validity of the two parameters have been delimited from each other by Riedel an Rice [16]. They consider a stationary crack in an elastic/nonlinear viscous material which is described by the material law

$$\dot{\varepsilon}_{ij} = (1+\nu)\ \dot{\sigma}'_{ij}/E + (1-2\nu)\ \dot{\sigma}_{kk}\delta_{ij}/\ (3E) + (3/2)\ B\ \sigma_e^{n-1}\sigma'_{ij}. \qquad (18)$$

Here E is Young's elasticity modulus, $\nu$ is Poisson's ratio, $\delta_{ij}$ is the unit tensor and the last term describes Norton-type power-law creep.

If a load is applied suddenly to an elastic/nonlinear viscous body, the instantaneous response is elastic. For short times after load application, large creep strains have developed in an only small though growing zone around the crack tip, while further away the straining is still predominantly elastic. Therefore the development of creep strains and simultaneous stress relaxation near the crack tip can be analyzed mathematically by prescribing the elastic singular field, $\sigma_{ij} \propto K_I/\sqrt{r}$ as the remote boundary condition for the processes in the near-tip zone. Thus, in the short-time limit, the linear elastic stress intensity factor $K_I$ determines the near-tip fields and therefore crack growth behavior. It is related to the applied load by

$$K_I = \sqrt{\pi a}\,\sigma_{net} f(a/W) \tag{19}$$

where the geometry-dependent, dimensionless function f(a/W) is tabulated for many specimen geometries; for example by Rooke and Cartwright [17].

The relaxation of stress near the crack tip is analytically described by the time-dependent, HRR-type asymptotic stress field

$$\sigma_{ij} = \alpha \left( \frac{K_I^2(1-\nu^2)/E}{(n+1)I_n Brt} \right)^{1/(n+1)} \tilde{\sigma}_{ij}(\theta) \tag{20}$$

where the dimensionless factor is $\alpha$=1 within a few per cent accuracy [16]. Eq. (20) explicitly shows the dependence of the short-time stress field on $K_I$ rather than on any other combination of crack length, specimen geometry and load.

If, somewhat arbitrarily, a creep zone is defined as the zone within which the equivalent creep strain exceeds the equivalent elastic strain, the size of this zone is found to increase for short times according to

$$r_{cr} = K_I^2\ (EBt)^{2/(n-1)} F_{cr}(\theta) \tag{21}$$

Finite element calculations [18] show that the dimensionless shape function $F_{cr}(\theta)$ has maximum values of 0.10 for plane strain and 0.20 for plane stress if n=5. The shape resembles the plastic zone in elastic/ time-independent plastic materials.

After long times the creep zone has spread across the whole ligament, the specimen creeps extensively and elastic strains become negligible compared with the ever increasing creep strains. Therefore, in the long-time limit, the elastic/nonlinear viscous material behaves as if it were purely nonlinear viscous and the stress fields are exactly those discussed in Section I.



The applicability of $K_I$ ends, and that of $C^*$ begins, when the creep zone becomes comparable in size with the crack length and ligament width or, in other words, when the stress field approaches its long-time limit. Equating the short-time and long-time stress fields (eqs. 20 and 6) gives the characteristic time

$$t_1 = K_I^2(1-\nu^2)/[E(n+1)C^*] \tag{22}$$

for the transition from the initial, elastically dominated to the final, creep dominated response of the cracked specimen. Although this transition occurs gradually, finite element analyses of cracked specimens show that the stress field can be described with good accuracy by using the short-time solutions up to the time $t_1$ and the long-time (nonlinear viscous) solutions thereafter [18].

Therefore it is recommended to calculate the characteristic time when creep crack growth tests are done. If the expected or actually measured test duration exceeds $t_1$, then $C^*$ should be employed. In the opposite case, $K_I$ is the appropriate load parameter. Brittle materials with high creep resistance and high crack growth rate, and large specimens tend to fail under $K_I$-controlled conditions.

## Experiments on $K_I$-Controlled Crack Growth

An example for $K_I$-controlled creep crack growth is the nickel-base alloy Nimonic 80A at 650°C. Wagner and Riedel [15] give $B = 3.10^{34} Pa^{-n}/s$, n = 13 and E = 170 GPa so that for a typical load on a CT1-specimen of 30 kN, the characterisitic time is found to be $t_1$ = 600 years, while failure by creep crack growth occurs after a few weeks. Correspondingly, the observed crack growth rates correlate better with $K_I$ than they do with $C^*$. On the other hand, for the tests on ferritic steel quoted in Section I, eq. (22) gives $t_1$ typically in the range of minutes to hours, while the tests lasted a few weeks. Hence, the good correlation of $\dot{a}$ with $C^*$ is not surprising.

Another effect of elastic straining is that, other than in purely viscous materials, the near- tip fields at growing cracks become different from those at stationary cracks [19]. It is unlikely, however, that this particular effect limits the applicability of $C^*$ in practical cases except, maybe, for high growth rates.

## IV. LIMITATIONS TO THE $C^*$-APPROACH SET BY INSTANTANEOUS PLASTICITY-APPLICABILITY OF J

In relatively short-term tests the load level may be so high that the specimen becomes fully plastic immediately upon load application. We assume that the response of the material can be described by a sum of instantaneous plastic strain rate and creep strain rate according to

$$\dot{\varepsilon}_{ij} = (3/2)\, B_o\, \sigma_e^{1/N-2}\, \dot{\sigma}_e\, \sigma'_{ij} + (3/2)\, B\, \sigma_e^{n-1}\, \sigma'_{ij} \qquad (23)$$

where N is the hardening exponent, $B_o = \sigma_y^{1-1/N}/E$ if $\dot{\sigma}_e > o$ (loading), $B_o = o$ if $\dot{\sigma}_e \leq o$ (unloading), and $\sigma_y$ is yield stress.

In such a material, the J-integral determines the short-time solutions whereas the long-time response is determined by $C^*$. For strong strain hardening, i.e. if $1/N > n$, the transition can be treated similarly as in the elastic-nonlinear viscous case and the characteristic transition time is found to be [20]

$$t_2 = J/[(n+1)\, C^*] \qquad (24)$$

Both, C* and J, can be calculated using eq. (7); B and n must be replaced by $B_o$ and $1/N$ in the case of J.

Here we mention in passing that transient, primary creep may also limit the validity of the $C^*$-approach. The limitations of $C^*$ with respect to transient material response have been discussed in Refs. [20,21] for several constitutive models. Procedures for creep crack growth testing have been proposed for pronouncedly transient material response.

### An Experimental Example for J-Controlled Creep Crack Growth

Saxena et al [22] have done creep crack growth tests on AISI 316 stainless steel at 594°C at relatively high load levels so that the specimens became fully plastic directly upon load application. They also give the material parameters $B = 3\cdot10^{21}\text{Pa}^{-8}/\text{s}$, $n = 8$, $B_o = 3\cdot10^{5}\text{Pa}^{-1/N}$ and $N = 0.53$. They did their tests on single-edge-notched tension specimens. For a crack length $a = 18.8$mm, $a/W = .37$ and for a load corresponding to $\sigma_{net} = 192$ MPa, the transition time is found to be $t_2 = 200$h. Typical test durations in this study were less than, or of the same order as $t_2$ so that the specimens spent at least most their life time in the J-controlled, short-time limit. In accord with this, Saxena et al [22] report poor correlation of the crack growth rate with $C^*$ but good correlation with J.



The dominance of short-time plasticity in these experiments has also be proven experimentally. The deflection rate of the load line during crack growth has been compared with the deflection rate of specimens that were identical except that they contained a rounded notch rather than a growing crack. It turned out that the deflection rate was by a factor of 3 to 13 smaller in the notched specimens than in the cracked specimens. This means that the response of the cracked specimen was determined primarily by short-time plasticity due to the increasing plastic compliance during crack growth. Then J rather than $C^*$ is likely to be the appropriate load parameter. The measurement of $C^*$ using eq. (9) is then meaningless since $\dot{\Delta}$ in this equation is meant to originate from steady-state creep deformation rather than from crack extension.

## V. LIMITATIONS TO $C^*$ SET BY CAVITATION

Since the effect of grain boundary cavitation on the stress field is not included in the nonlinear viscous description of the material, the $C^*$-approach might be invalidated by profuse cavitation of the whole ligament. It will be argued, however, that $C^*$ need not necessarily be abandoned if grain boundary cavitation occurs by the creep-constrained void growth mechanism proposed by Dyson [13].

In the limit of fully developed polycrystalline constraint the cavitating grain facets transmit practically no tractions and therefore behave like microcracks. Hutchinson [23] shows that macroscopically (on a scale large compared with the grain size), such a cavitating material responds in a compressible, power-law viscous manner. Obviously, $C^*$ is then path-independent. It can be determined from eqs. (7-9), but the geometrical functions $g_1$, $g_2$ and $g_3$ are now different from what is known for incompressible, non-cavitating material behavior. However, the error is not expected to be great if the form $C^* = g_2 \sigma_{net} \dot{\Delta}$ is employed with $g_2$ for incompressible material. This conjecture should be checked by finite element analysis using Hutchinson's constitutive equation for a cavitating viscous solid.

If cavitation is confined to a small zone around the crack tip ("small-scale cavitation") while the bulk of the specimen obeys Norton's creep law (eq. 4), $C^*$ is also a valid parameter, since a small cavitational zone is completely encompassed and controlled by the HRR-field (eq. 6). Further, Hutchinson [23] surmises, and this is supported by experience with related problems [16], that the $C^*$-integral is approximately path-independent throughout the whole range of distances from the crack tip including the near-tip field where cavitation is prevalent and the far field where cavitation is negligible. This conjecture needs numerical verification.

In summary, the foregoing arguments suggest that $C^*$ might be an appropriate load parameter for the whole range from small-scale cavitation to extensive cavitation across the whole ligament. Of course, this can only be considered as a preliminary conclusion since Hutchinson's model [23] is an idealization which describes the limiting case of creep-constrained cavity growth but disregards other possibilities.

## VI. CONCLUSIONS

The theoretical basis for $C^*$-testing has been described and the relevant formulas for its use have been compiled. The observed dependences of the crack growth rate $\dot{a}$ on $C^*$, on temperature and on the crack growth increment, $\Delta a$, have been explained on the basis of a critical strain criterion for crack growth. Limitations to the $C^*$-approach that may arise from several sources have been discussed:

(1) The calculation of $\dot{a}$ based on local failure criteria near the crack tip shows that, even if $C^*$ determines the crack-tip fields, the crack growth rate may depend also on $\Delta a$ so that the dependence on $C^*$ is not unique. Fortunately, it turns out that the dependence on $\Delta a$ is moderate if the crack grows subject to a critical strain criterion.

(2) The limitation to $C^*$ by crack tip blunting has been formulated. A numerical estimate suggests that blunting is not a serious limitation except for very ductile materials and for center-crack geometries in plane-strain tension.

(3) Predominantly elastic specimen response requires the use of $K_I$ and offsets the applicability of $C^*$. Quantitatively, a decision between $K_I$ and $C^*$ can be made by comparing the test duration with the characterisitic time $t_1$ for the transition from the $K_I$-dominated limit to the $C^*$-dominated limit. The condition for the applicability of $K_I$ can also be expressed by the requirement that the calculated creep zone size must be small compared with the crack length and ligament width. In many practical cases, when gross plastic deformation of the specimen accompanies crack growth, it is immediately obvious that $K_I$ cannot be applied.

(4) In terms of the characteristic time $t_2$, the validity of $C^*$ has been delimited against that of J in cases where the specimen becomes fully plastic immediately upon load application. As for the effects of primary creep the reader is referred to Refs. [20,21].

(5) If grain boundary cavitation occurs by the creep-constrained mechanism, cavitation might have a surprisingly small effect in terms of setting limitations to the $C^*$-approach.



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