DISLOCATION-SHIELDING ANALYSIS OF A BLUNT-NOTCHED BRITTLE CRACK EMBEDDED IN A DUCTILE MATERIAL

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A fracture model of a blunt-notched brittle crack embedded in a plastically deformed ductile medium is developed. An elastic enclave separates the notched tip from the plastic zone that is generated by the dislocations created within this zone. Effects of the notch-root radius and material parameters on the fracture toughness are predicted. The predicted fracture toughness is consistent with experimental observations.

1. Introduction

Fracture initiation and propagation from blunt notches have been the object of numerous theoretical and experimental studies [1-4]. So far, all the proposed fracture criteria are based on the critical stress or strain, either at a point or acting over a characteristic distance related to metallurgical microstructures such as grain size and inclusion spacing.

Recently Thomson [5] and Weertman [6] have developed elastic enclave models and predicted the apparent fracture toughness of materials. The dislocation shielded sharp crack models have since been examined by others [7-11].

The purpose of this paper is to extend the general idea of a dislocation shielded sharp crack to investigate the fracture behavior of a notched brittle crack embedded in a plastically deformed ductile medium. In the following sections, first the stress distribution and the intrinsic toughness of the core notched crack are derived; then the stress distribution in the plastic zone is examined; finally a self-consistent method is used to derive the apparent fracture toughness as a function of the notch root radius and material parameters.

2. Stress Distribution of a Blunt-Notched Brittle Crack Shielded by a Plastic Zone

Following the suggestion of Thomson [5], the dislocation density in the vicinity of the notch root is low enough such that an elastic enclave zone always exists between the notch tip and the plastic zone.

2.1 The elastic enclave zone. The stress distribution at the notch region is estimated by Inglis stress solution of an elliptic crack [12] in an infinitely wide plate. The notch root radius, ρ, is related to the elliptical notch of minor axis b and major axis a by $\rho=b^2/a$, provided the notch is sharp but the tip radius is finite. The stress normal to the crack plane, σ, is

$$\sigma = \frac{2K_t}{\sqrt{\pi}} \frac{(\rho+r)}{(\rho+2r)^{3/2}} \quad (1)$$

where r is the distance measured from the notched tip and K_t is the stress intensity factor at the crack tip, defined by

$$K_t = \sigma_A (\pi a)^{1/2} \quad (2)$$

Here σ_A is the stress applied to the notch. From Eqn. [1] the maximum notch tip stress, defined by $\sigma_{max} = (\sigma)_{r=o}$, is

$$\sigma_{max} = 2K_t/\sqrt{\pi\rho} \quad (3)$$

The intrinsic stress intensity factor at fracture is determined when σ_{max} reaches the theoretical strengths for materials, $\sigma_{th} = \alpha(E\gamma/a_o)^{1/2}$. Then from Eqn. [3] and $\alpha^2 = \frac{8}{\pi}$, we have

$$K_y^{*2} = 2E\gamma\rho/a_o \quad (4)$$

where a_o is the lattice parameter, E is Young's modulus, and γ is a the surface energy. In Orowan's estimation of σ_{th} [13], $\alpha=1$, however, Eqn. [4] predicts the intrinsic toughness of a blunt-notched core crack as a function of the notch root radius and material parameters. When σ goes to the limit, i.e., $\rho=a_o$, Eqn. [4] reduces to the Griffith result [14].

2.2 The plastic zone. The stress σ and strain ε fields generated in the plastic zone are estimated by a notch in a strain hardening material of the form

$$\sigma = \sigma_o(\varepsilon/\varepsilon_o)^n, \quad \varepsilon>\varepsilon_o \quad (5)$$

where n is the work hardening coefficient, ε_o is the yield strain, and σ_o is the yield stress.

The strain distribution is written as

$$\varepsilon = \varepsilon_{max} f(n,r/\rho) \quad (6)$$

where ε_{max} is the maximum notch tip strain and f is a dimensionless function of n and the normalized distance r/ρ. Here f is a positive number and increases to unity as r/ρ reduces to zero. Rice [15] found

$$\varepsilon_{max} = C_n\varepsilon_o [B_n J/\sigma_o\varepsilon_o\rho]^{\frac{1}{1+n}} \quad (7)$$

$$B_n = \frac{(n+1/2)\ (n+3/2)\ \Gamma\ (n+1/2)}{\Gamma\ (1/2)\ \ \Gamma\ \ (n+1)} \quad (8)$$

where J is the well known J-integral and $\Gamma(...)$ is the gamma function. It can be shown $B_1 = 15/8$, $C_1 = (\frac{32}{15\pi})^{1/2}$ and $f(1,r/\rho) = \rho^{1/2} (\rho+r)/(\rho+2r)^{3/2}$, provided Eqns. [1], [4] and [5-8] are combined properly at the limit n=1. In the calculation of C_1 and the determination of $f(1,r/\rho)$, we have used the relation between J and K,

$$J = \frac{K^2}{E} \quad (9)$$

3. Fracture Toughness

The fracture toughness is obtained in a self-consistent way: the fracture stress, σ_f, generated at the inner elastic-plastic boundary, R_c, by the elastic core crack must be the same as the stress generated at R_c obtained from the combined external stress and the dislocation field, provided the stress distribution at the core crack region is dominated by the K field. From Eqn. [1], we have

$$\sigma_f (R_c) = \frac{2K_t^*}{\sqrt{\pi}} (\rho+R_c)/(\rho+2R_c)^{3/2} \quad (10)$$

From Eqns. [5], [6], and [7], we obtain

$$\sigma_f (R_c)/\sigma_o = (B_n J/\sigma_o \varepsilon_o \rho)^{n/1+n} [c_n f(\rho/R_c)]^n \qquad (11)$$

By combining Eqns. [10] and [11] to eliminate $\sigma_f(R_c)$, we have the fracture toughness predicted by the expression

$$J = \frac{\sigma_o \varepsilon \rho}{B_n[c_n f(n,R_c/\rho)]^{1+n}} \left[\frac{8}{\pi} \frac{\rho}{a_o} \frac{\gamma E}{\sigma_o^2} \frac{(\rho + R_c)}{(\rho+2R_c)^3}\right]^{\frac{1+n}{2n}} \qquad (12)$$

Using Eqn. [9], [12] becomes

$$K^2 = \frac{\sigma_o(1-\nu^2)\rho}{B_n[C_n f(n,R_c/\rho)]^{1+n}} \left[\frac{8}{\pi} \frac{\rho}{a_o} \frac{\gamma E}{\sigma_o^2} \frac{(\rho+R_c)}{(\rho+2R_c)^3}\right]^{\frac{1+n}{2n}} \qquad (13)$$

4. Discussion

The results show that the equation for the apparent fracture toughness has the correct quantitative dependence on the physical parameters, n, γ, σ_o, and ρ. Equation [13] indicates K^2 is increased with the larger notch root radius. This is consistent with experimental observations [1]. The fracture toughness is proportional to the surface energy of the solid raised to a power characterized by the work hardening coefficient. Likewise, increasing the yield stress decreases the toughness. As n=1, using the results for B_1, C_1 and $f(r/\rho,1)$, presented in section 2.2, Eqn. [13] reduces to intrinsic toughness, shown in Eqn. [4].

We emphasize that Eqn. [13] is quite general by showing when ρ goes to the sharp crack limit, i.e., $R_c >> \rho = a_o$, (13) shall reduce to the fracture toughness of a sharp crack embedded in a ductile material, found by Thomson [5].

The well known HRR singularity [16,17] has the strain distribution

$$\varepsilon = \varepsilon_o (EJ/\sigma_o^2 r)^{\frac{1}{1+n}} \alpha_n \qquad (14)$$

where α_n is a constant.

Comparing Eqns. [6] and [14], we have

$$f(r/\rho,n) = \frac{\alpha_n}{C_n B_n^{\frac{1}{1+n}}} (\rho/r)^{\frac{1}{1+n}} \qquad (15)$$

Substituting [15] into [13] and setting $a_o = \rho << R_c$, we then have Thomson's result for fracture toughness

$$K^2 = \frac{\sigma_o^2}{\alpha_n} \left[\frac{\gamma E}{\pi \sigma_o^2}\right]^{\frac{1+n}{2n}} \frac{1}{R_c^{(1-n)/2n}} \qquad (16)$$

REFERENCES

[1] Wilshaw, T.R., C.A. Rau and A.S. Tetelman, Eng. Fract. Mech. 1, 191 (1960).

[2] Rice, J.R. and M.A. Johnson, Inelastic Behavior of Solids, M.F. Kanninen, W.F. Adler, A.R. Rosenfield and R.J. Jaffe, eds. (McGraw-Hill, N.Y.), 641 (1970).

[3] Richie, R.O., J.F. Knott and J.R. Rice, J. Mech. Phys. Solids 21, 395 (1973).

[4] Onyewuenyi, O.A. and J.P. Hirth, Metall. Trans. A. 13A, 2209 (1982).

[5] Thomson, R., J. Matls. Sci. 13, 128 (1978).

[6] Weertman, J., Acta Met. 26, 1731 (1978).

[7] Thomson, R. and J.E. Sinclair, Acta Met. 30, 1325 (1982).

[8] Majumdar, B. and S. Burns, Acta Met. 29, 579 (1981).

[9] Weertman, H., I.-H. Lin and R. Thomson, Accepted by Acta Met.

[10] Chang, S. and S.M. Ohr, J. Appl. Phys. 52, 7174 (1981).

[11] Majumdar, B and S. Burns, Int. J. Fract. Mech., in press.

[12] Inglis, E., Trans. Inst. Nav. Arch. 55, 219 (1913).

[13] Orowan, E., Rep. Prog. Phys. 12, 185 (1949).

[14] Griffith, A.A., Phil. Trans. Roy. Soc. A221, 163 (1920).

[15] Rice, J.R., J. Appl. Mech. 35, 379 (1968).

[16] Hutchinson, J.R., J. Mech. Phys. Solids 16, 13 (1968).

[17] Rice, J.R. and G.F. Rosengren, J. Mech. Phys. Solids 16, 1 (1968).