

## TIME DEPENDENT QUASIFRACTURE NUCLEATION AND PROPAGATION

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The important problem on fracture of solid media has been studied by numerous scientists all over the world. A tremendous amount of literature has been accumulated for many many years. Yet a good deal of work is still needed to elucidate the fundamental understanding of pre-fracture and fracture phenomena in engineering applications. This paper presents a time dependent theory of quasifracture of viscoelastic media prior to fracture development. The theory is particularly suited for studying solid materials. The quasifracture considerations in viscoelastic materials lead naturally into the understanding of the important problems of fracture nucleation and propagation.

### INTRODUCTION

The phenomenon of solid fracture has long been observed and utilized since the beginning of civilization. Despite the importance of this subject the development of fracture theory on the basis of fundamental principles of science has been surprisingly slow. The fact that attentions centered around highly idealized assumptions on

perfectly elastic brittle solid behavior has greatly retarded the normal progress of fracture studies. Recently the intense concern has begun to shift from time independent fracture studies to the time dependent fracture initiation and propagation investigations.

The principles in mechanics and thermodynamics are employed together with the essential deformational characteristics of the solid medium.

In a good many fracture studies it has been assumed that there exists a fracture in the first place. This is apparently not realistic as fracture does not occur suddenly without a cause. A more reasonable approach is to consider the quasifracture behavior of a viscoelastic medium emphasizing the development of a fracture and its time dependency. The fundamental laws of continuum mechanics and the molecular mechanisms of the solid under stressing must be carefully coupled to create an efficient and practical method of approach.

The fundamental laws available for this purpose are equations of motion, strain-displacement relations, boundary and initial conditions of the problem and the constitutive behavior of the viscoelastic solid medium. To correctly assess the constitutive deformational behavior of a solid requires a sufficient knowledge of molecular deformation mechanism and strength behavior of that solid. With the help of the generally accepted concept of the conservation of energy that there will be no increase or decrease of the total energy in a solid system including internal, kinetic, potential, surface and thermal energies of the solid, a proper procedure can be set up for obtaining the time dependent fracture information at least for some simple cases.

## APPROACH AND MODEL

The development of quasifracture under tensile load depends not only on stress but also on time. The phenomenon of quasifracture has been recognized as a prior mechanical behavior in solids to failure resistance. Any prediction in resistance to failure must take the time dependent viscoelastic properties into account. This means that the effect of quasifracture development is highly important. Among many factors it is easy to see that the lengths  $c_i(t)$  of the  $i$ th individual craze at time  $t$  as well as the total number  $N(t)$  of quasifractures developed on the surfaces and in the interior of a stressed medium are influential. Of course both quantities are also dependent upon the magnitude of applied stress and the conditions of environment including such parameters as temperature and heat.

In dealing with quasifracture, like any other physical or mechanics problems, continuum models are utilized. Aside from the regular field and constitutive equations each individual quasifracture must obey during the developing stage of quasifracture the fundamental principle of the global conservation of energy. The rate of work done of external forces and all other energies that enter or leave the material body containing quasifractures per unit time must equal the time rate of change of the internal and kinetic energies plus the energies associated with the formation of quasifractures, for simplicity excluding chemical, electrical, etc. energies except heat. On this basis the global conservation of energy becomes:

$$\dot{W}(t) + \dot{Q}(t) = \dot{E}(t) + \dot{D}(t) + \dot{K}(t) + \sum_i^N \dot{c}_i(t) \quad (i=1,2,3,\dots,N) \quad (1)$$

where  $\dot{W}(t)$  is the rate of mechanical work done by external forces,  
 $\dot{Q}(t)$  is the total heat created per unit time during deformation,  
 $\dot{E}(t)$  is the stored strain energy rate of the (original) solid medium,  
 $\dot{D}(t)$  is the dissipative energy rate of the (original) solid medium,  
 $\dot{K}(t)$  is the time rate of change of the kinetic energy of the (original) solid medium,  
and  $\dot{\zeta}_i(t)$  is the total energy absorption rate by the  $i$ th quasifracture developed in the medium.

Since the energy is balanced for arbitrary small regions, one may consider the local conservation of energy in the neighborhood of a single quasifracture.

It is well known that a quasifracture usually develops into a planar configuration perpendicular to the direction of applied stress. It normally occurs easily first on the surface of a solid instead of its interior at a place where a flaw exists. Thus starting from the flaw along the surface of the solid, an elliptical quasifracture front propagates into the medium.

Referring to a rectangular coordinate system ( $o$ - $xyz$ ), an idealized symmetrical quasifracture section is given in Fig. 1. As shown bounded by an elliptical quasifracture front oriented molecular domains containing for example bundles of fibers for polymers and crystalline aggregates for metals and voids or dislocations are formed. If  $xz$  plane is the surface of the solid then the major axis of the quasifracture region is represented by  $c(t)$  as a function of time  $t$  while the minor axis is  $p(t)$  at time  $t$  showing the central maximum penetration of

the quasifracture front into the material body. The oriented domains are load bearing in spite of voids and dislocations. Each domain is assumed to be round with a diameter  $\delta(x,y,z,t)$  which may change as a function of time. The distance between the  $xy$ -plane and the top quasifracture surface is the opening displacement designated by  $w(x,y,o,t)$  which reduced to  $w(x,o,o,t)$  on the surface of the solid at time  $t$ . On the envelope surface of the quasifracture, the stress component in  $z$ -direction is shown as  $\sigma_c(x,y,z,t)$ . The general state of stress, strain and displacement fields are respectively  $\sigma_{\alpha\beta}(x,y,z,t)$ ,  $\epsilon_{\alpha\beta}(x,y,z,t)$  and  $u_\alpha(x,y,z,t)$  with  $\alpha$  and  $\beta$  as tensor subscripts in the rectangular coordinate system  $(x,y,z)$ .

Each oriented domain sustains a stress  $\sigma_d(x,y,z,t)$  which may be different from the quasifracture envelope surface stress  $\sigma_c(x,y,z,t)$ . The former represents the true domain stress while the latter is an average engineering stress taking domains and voids altogether into consideration. Thus the ratio  $V_f(x,y,z,t) = \sigma_c(x,y,z,t) / \sigma_d(x,y,z,t)$  is the volume fraction of domains within a quasifracture. The actual domain distribution density function  $\rho(x,y,z,t)$  is defined as the number of load bearing domains passing through a unit area in the quasifracture plane at time  $t$ .

In general the nucleation of quasifracture requires the increase of the internal energy including stored and dissipative energies to a critical level. Beyond which the original structural configuration of the medium is no longer stable and a new phase must take place. As a result highly oriented molecular domains and voids as well as new surfaces are created. Referring to Fig. 2 in the reference frame  $(0-X_1X_2X_3)$  an undeformed or unoriented material system is considered.

Under a load  $P(t)$  the current frame of reference  $(\sigma-x_1x_2x_3)$  may be used to calculate the strength and degree of molecular orientation of the microstructural system. Using random variables  $(\theta, \phi)$ , any microelement  $dp$  will oriente to its new direction as indicated in the current reference frame as  $dp$  [1,2]. Depending upon the nature of the medium the variation of the modulus may be determined as a function of orientation strain  $\epsilon$  as shown in Fig. 3. The modulus of elasticity  $E_{ZZ}$  in the direction of orientation increases for some linear polymeric model solids, while in the transverse direction the moduli  $E_{XX}$  and  $E_{YY}$  decrease. The oriented molecular domains may be amorphous or crystalline or both. They can carry load. Only when they cease to bear load, fracture will be initiated or nucleated. In time these microfractures may eventually develop into a macrofracture.

#### QUASIFRACTURE NUCLEATION AND PROPAGATION

Based upon thermodynamic and topological considerations the necessary conditions for the nucleation of a quasifracture in a viscoelastic medium are to achieve a critical free volume under a specific state of stress  $\sigma_0$ . As shown in Fig. 4, the fracture processes start from the quasifracture nucleation [3]. Essentially the mechanism of nucleation is viewed as the expansion of the free volume in a local region of a material system such that molecular rearrangements become possible. Of course this process takes time and is greatly affected by stressing. At a stress level  $1\sigma_0$ , the length of quasifracture is not observable until time  $1t_c$ . At a higher stress  $2\sigma_0$ , the time required for seeing the quasifracture is  $2t_c$ . At a still higher stress  $3\sigma_0$ , the

nucleation of a quasifracture takes even less time ( $3t_c$ ). Once a quasifracture is developed it will continue to propagate until a real fracture occurs. Corresponding to  $1\sigma_0$  the time to fracture is  $1t_d$ . At this time the domain structure at the center of a quasifracture begins to fracture. As shown in Fig. 1, the length of this fracture is  $a(t)$  while that of the quasifracture is  $c(t)$ . In reality  $c(t)$  is always greater than  $a(t)$ . For many solids under stress usually many quasifractures nucleate and propagate until fracture develops. As the magnitude of the applied stress increases, say  $3\sigma_0$ , at some stage of the propagation of fracture, fracture nucleates at  $3t_d$  as shown. Similarly corresponding to  $2\sigma_0$ , fracture nucleates and propagates at  $2t_d$ .

#### SIMPLIFICATIONS AND RESULTS

Returning back to the model with multi-quasifractures mathematically  $\dot{\zeta}_i(t)$  can be calculated in the following manner for any  $i$ th surface elliptical quasifracture:

$$\dot{\zeta}_i(t) = 4 \int_0^{c_i(t)} \int_0^{b_i(x,t)} \int_0^{\sigma_c(x,y,z,t)} w(x,y,z,t) dydx, \quad (2)$$

$$\text{where } b_i(x,t) = p_i(t) [1 - x^2/c_i^2(t)]^{1/2}, \quad (3)$$

$$\text{and } z = w(x,y,0,t). \quad (4)$$

This absorbed energy rate governs the development of quasifracture. Applying the energy balance again for the  $i$ th craze only and after suitable mathematical manipulation and simplifications, an implicit nonlinear differential equation for the surface only is obtained by

omitting both y and z coordinates in the expression:

$$\int_0^{c_i(t)} \left\{ \sigma_c(x,t) \cdot \dot{w}(x,t) - \frac{\partial}{\partial t} \left[ V_f(x,t) \xi(x,t) \right] w(x,t) \right\} dx = \eta \dot{c}_i(t) \quad (5)$$

where  $\xi$  is an energy quantity associated with molecular deformation process while  $\eta$  is one related to the initiation of the oriented domains near the very tips of the quasifracture.

This means that if a two dimensional model quasifracture is considered the mathematical problem is greatly simplified. Now referring to Fig. 5, a two-dimensional quasifracture. The principal interest is to determine  $c(t)$ . Under certain assumptions, this has already been analyzed [4], for a single quasifracture. Fig. 6 shows the increase of the length of a single quasifracture as a function of time. However, when a multi-quasifracture system is considered the nature of the length increase per each quasifracture behaves quite differently.

Based upon the measured value of the number of quasifractures on the surface of viscoelastic solid samples and the geometry of individual quasifracture, Equation (5) has been solved through computer aided numerical methods. The calculated result of the quasifracture length propagation has been found to agree fairly well with the experimental measurements for polystyrene as shown in Figs. 7 and 8. It was also interesting to note that individual quasifractures were essentially developed to resist fracture under tensile loading. A calculation of the energy quantities associated with quasifracturing development has shown that the strain energy absorbed by each quasifracture is about several hundred times that of the unoriented original medium at the

inception of nucleation. During the propagation period of the quasifracture, a ratio of approximately fifty to one has been obtained.

Finally the time dependent fracture nucleation and propagation is illustrated by showing in Fig. 9 the quasifracture and fracture transition. The top diagram illustrates the opening  $w$  and the envelope stress  $\sigma_c$  of a quasifracture under a simple tension  $\sigma_0$ . When an actual fracture is nucleated, the oriented domains at the center of the quasifracture break. This is shown in the middle of the figure identifying the fracture nucleation. Subsequently this breaking process continues. The quasifracture opens up to fracture as the envelope stress  $\sigma_c$  shifts to the right, thus the fracture propagation continues also to the right as illustrated in the bottom diagram of the figure.

#### REFERENCES

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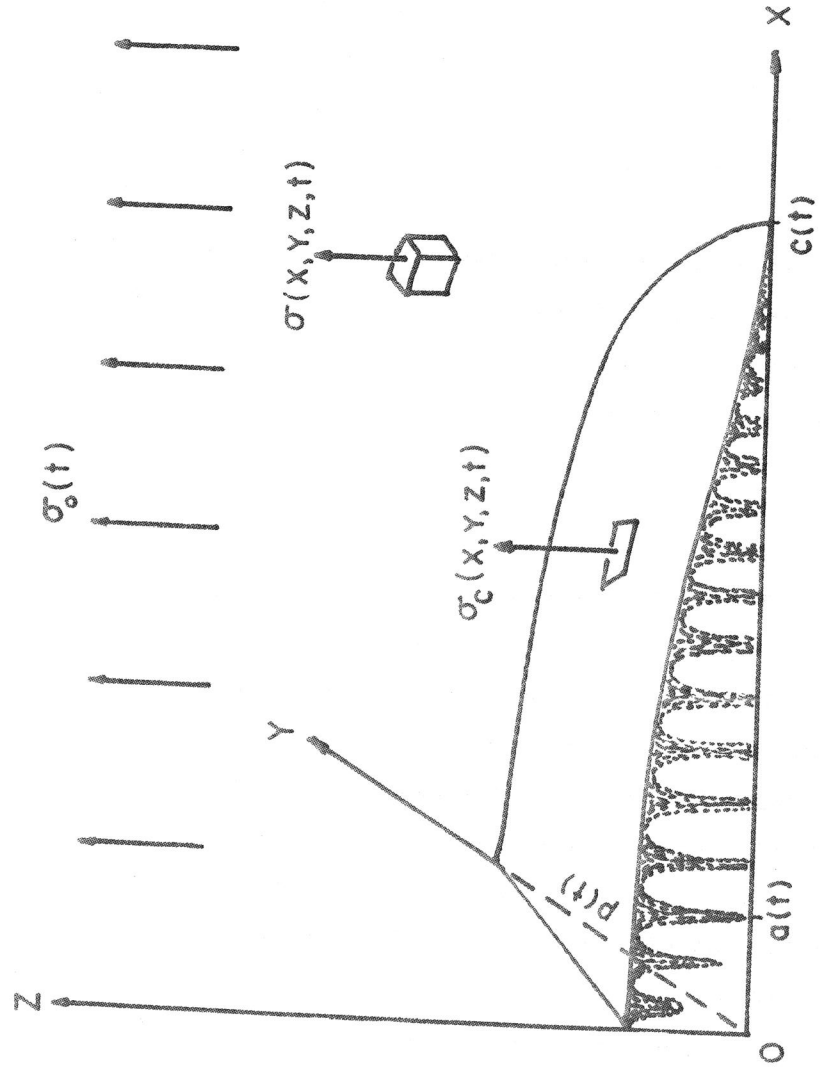


Fig. 1 Three-Dimensional Quasifracture Propagation in Solids

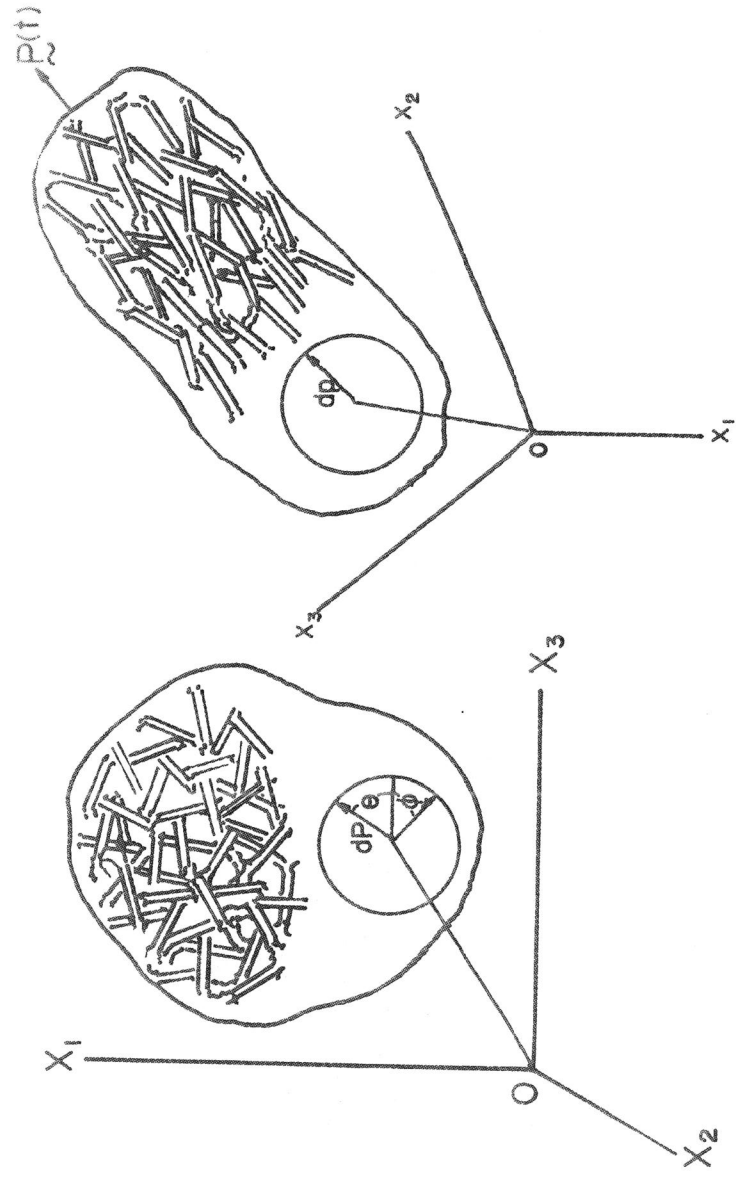


Fig. 2 Molecular Deformation and Orientation of Microstructures



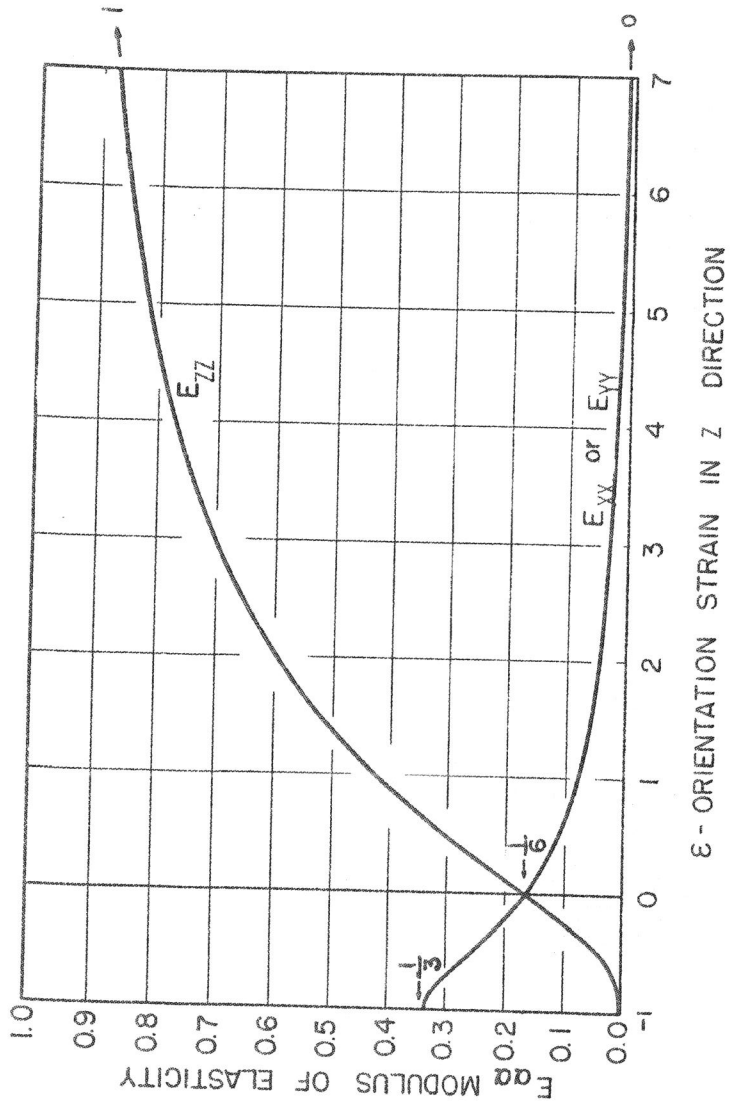


Fig. 3 Variation of Modulus of Elasticity vs. Orientation Strain

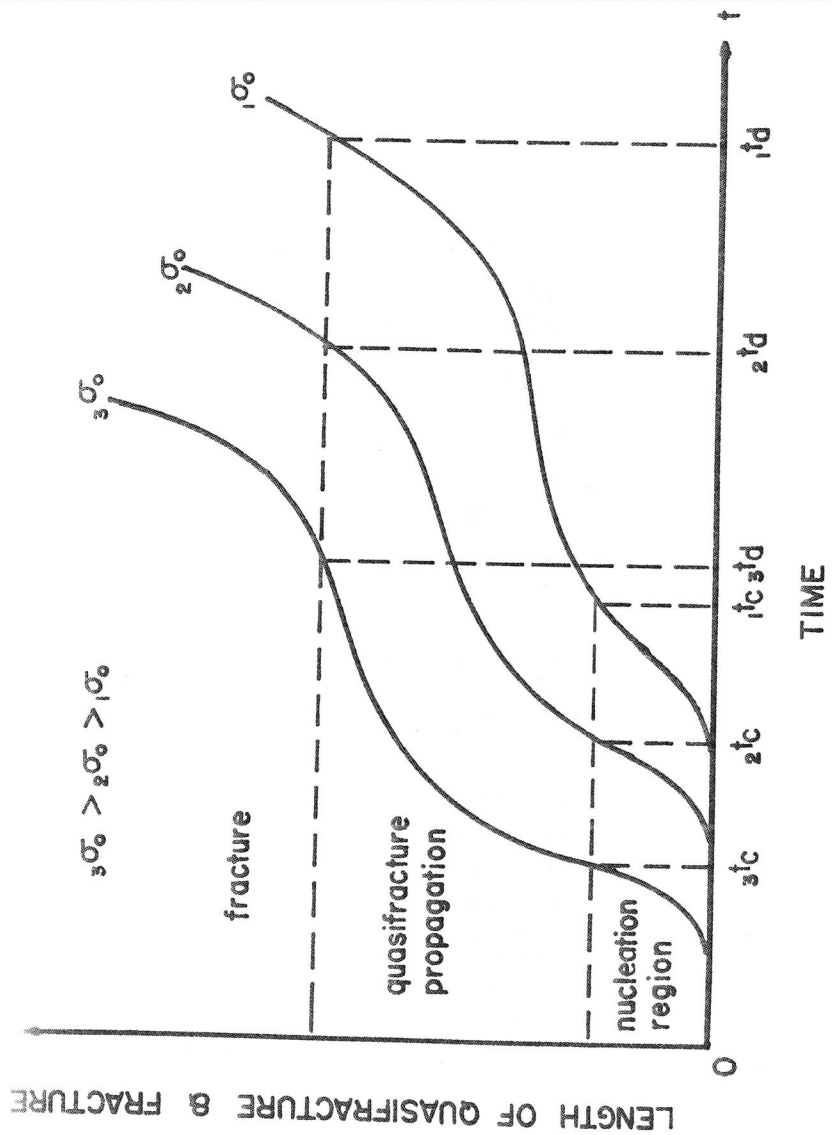


Fig. 4 Quasifracture Nucleation and Propagation vs. Time





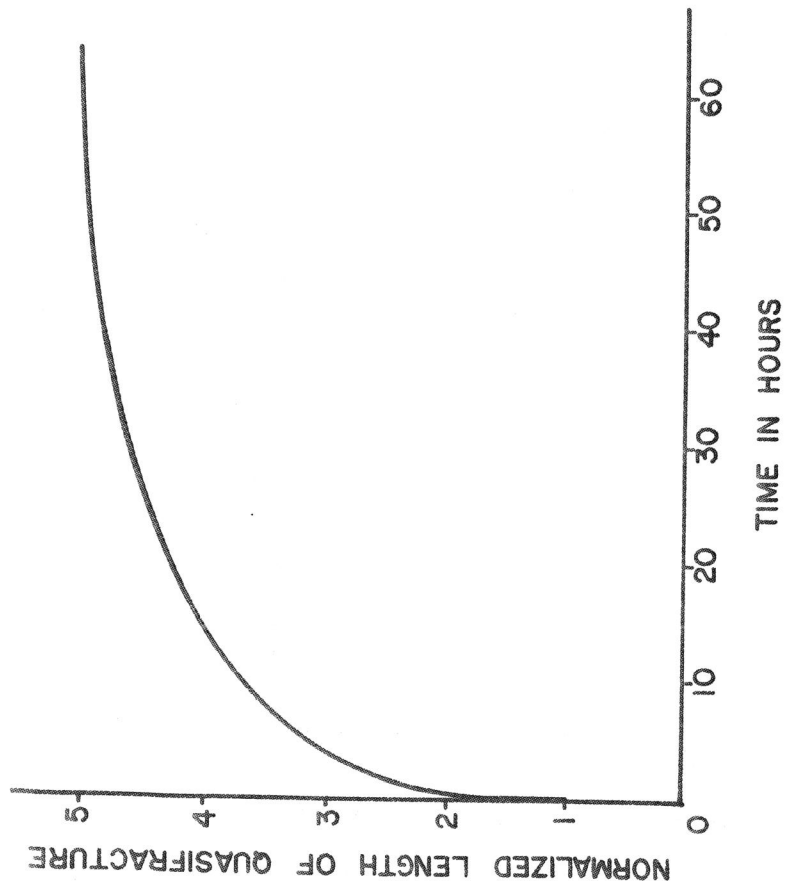


Fig. 7 Length Propagation in a Multi-Quasifracture System

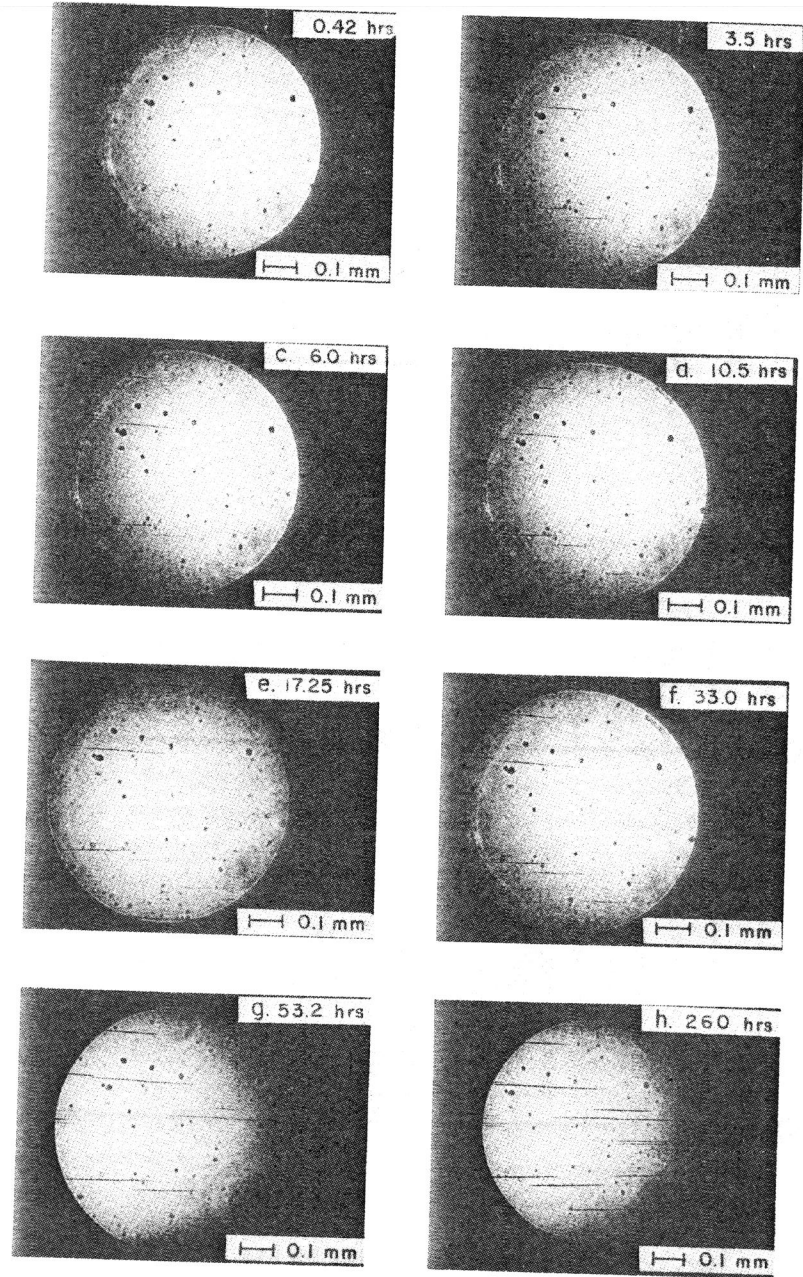


Fig. 8 Propagation of Quasifractures in Creep of Polystyrene

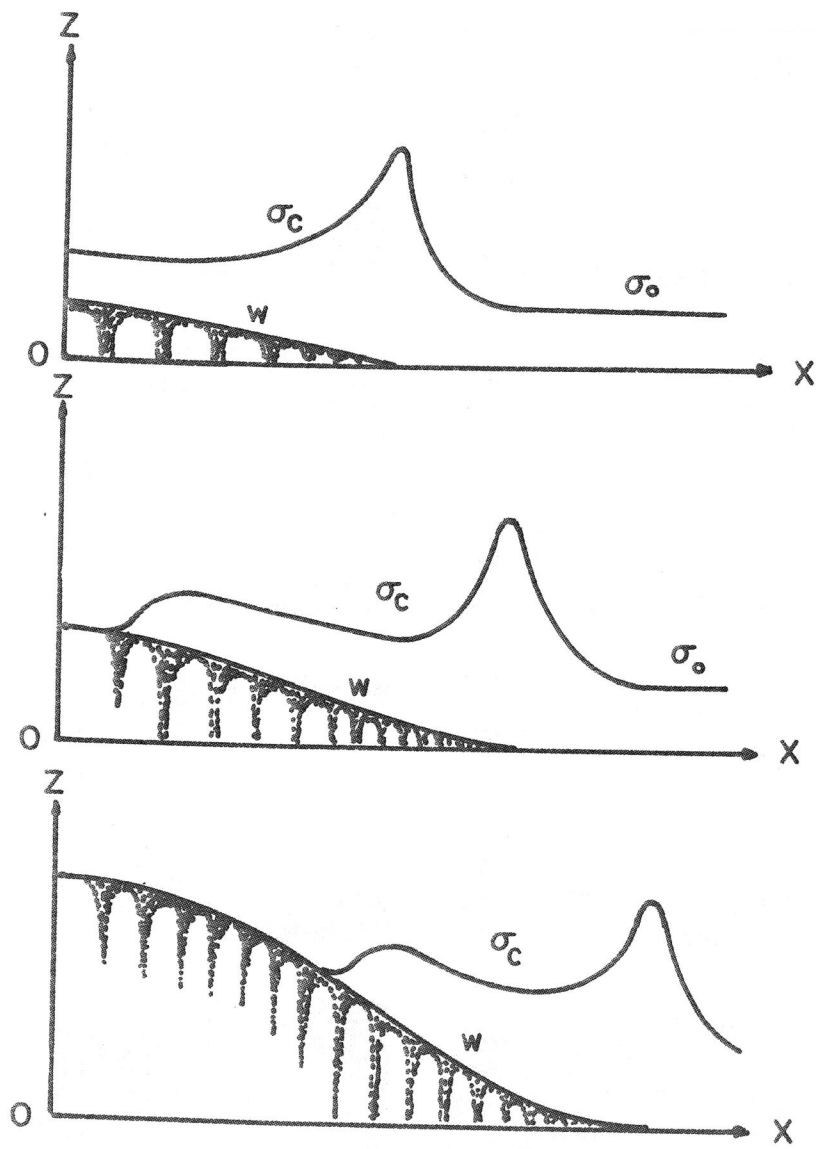


Fig. 9 Quasifracture and Fracture Transition