ENERGY-RELEASE RATES OF DIFFUSIONAL CRACK GROWTH

Tze-jer Chuang

National Bureau of Standards, Washington, D.C., USA

This paper aims to investigate energy release rates (G) that accompan with diffusive crack growth. Crack-like cavities at grain boundaries are frequently observed in creep ruptured specimens of crystalline solids. Their growth can be attributed to a mechanism involving coupled crack surface and grain-boundary self-diffusion. Chuang et al.[1] reviewed the subject of diffusive cavitation along grain interfaces and gave the conditions under which the growth of crack-like cavities prevails. In general, creep cavities favor crack-like (slit) shapes when the ratios of applied stress to capillary stress and grain-boundary diffusivity to surface diffusivity are high (say >>1) and when the service time approaches the later stage in the growth phase. Under these circumstances, the crack travels in a steady-state fashion at a moderate velocity along the grain boundary. It is then appropriate to treat the crack as a semi-infinite, growing at a constant speed in an infinite elastic bicrystal under plane-strain conditions. This case has been considered by Chuang[2] who solved the coupled problem of diffusion and elastic deformation leading to a specific kinetic law for subcritical crack growth. The present paper attempts to perform the analysis of G thoroughly.

DERIVATION OF THE ENERGY-RELEASE RATE

The conventional energy-release rate G is defined by $v G \equiv -dP/dt = \dot{W} - (d/dt)\int_V w \, dv$ for a class of elastic-brittle cracks without diffusion. Here v is crack growth rate, P is the total potential energy, and $w=w(x,y)$ is the strain energy density. For a sake of consistency we extend this definition to the case of diffusional crack growth by writing

$vG = \dot{W}-\dot{F}_V$ since the time rate of strain energy is identical to that of the Helmholtz free energy for isothermal processes. Here F_V represents the total bulk free energy stored in the interior of the body; it relates to F by $F=F_V+F_S$ where F_S is the free surface energy associated with cavities and interfaces. Thus we have $vG = \dot{F}_S+\dot{W}_{dis}$, since $F=W-W_{dis}$ from the first law of thermodynamics. This is justified for the special case of diffusive crack growth in an elastic-brittle material since under creep conditions where the sustained loads are relatively low, the dissipative work generated from dislocation motion can be justifiably nelgected. Accordingly, the total dissipative work in diffusional crack growth is predominantly produced by matter transport notably along the high diffusivity paths at internal void surfaces and grain interfaces. This means that the contribution to W_{dis} by bulk diffusion could also be ignored unless the temperature approaches the melting point. Thus $\dot{W}_{dis} \approx \dot{W}^S_{dis} + \dot{W}^{g.b}_{dis}$ and because $\dot{W}_{dis} \equiv TS = \int \underset{\sim}{J} \cdot (-\nabla\mu)\, dV$ the expression for G becomes

$$v G = \dot{F}_s + \dot{W}^s_{dis} + \dot{W}^{g.b}_{dis} \tag{1}$$

where $\dot{W}^S_{dis} = 2\int \underset{\sim}{J}_s \cdot (-\nabla\mu_s)\, ds$ and $\dot{W}^{g.b}_{dis} = \int \underset{\sim}{J}_b \cdot (-\nabla\mu_b)\, dx$ are the dissipative work generated per unit time at the crack surfaces and along the grain boundary (g.b)., respectively.

As in the case of Griffith cracks, $\dot{F}_S$ due to steady state crack growth along a grain boundary is simply

$$\dot{F}_s = v\,(2\gamma_s - \gamma_b) = v\, G_{Gr} \tag{2}$$

where γ_s and γ_b are the free energy per unit area of crack surface and g.b., respectively; G_{Gr} is the Griffith energy.

To evaluate $\dot{W}^S_{dis}$ and $\dot{W}^{g.b.}_{dis}$, the formulation of $\underset{\sim}{J}$ and μ must be determined first. This is given in Ref. [3]. Substitution of these expressions in Eqn. (1) and after some tedious mathematical manipulations, the final results are[4]

$$\dot{W}^{s}_{dis} = 2h\gamma_s K_{tip} - (2\gamma_s - \gamma_b) - 2h\,w_{tip} + \int_{-h}^{h} w\,dy \tag{3}$$

and

$$\dot{W}^{g.b}_{dis} = 2h\,w_{tip} + \int_0^\infty w(\partial\delta/\partial x)dx - 2h\gamma_s K_{tip} - \int_0^\infty \sigma_{yy}\left(\frac{\partial\delta}{\partial x}\right)dx \tag{4}$$

where 2h is the crack thickness, σ_{yy} grain boundary normal stress and δ the opening displacement, $\delta(x) = u^{+}(x) - u^{-}(x)$. (See Fig. 1 below)

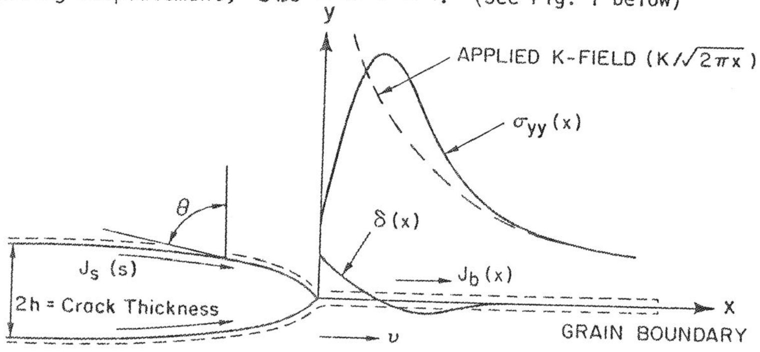

Fig. 1. Diffusive crack growth along a boundary. Normal stress $_{yy}(x)$ and opening displacement (x) distributions are shown schematically.

Substituting Eqns. (2-4) into Eqn. (1), one obtains

$$G = \int_{-h}^{h} w\,dy + \int_0^\infty w\left(\frac{\partial\delta}{\partial x}\right)dx - \int_0^\infty \sigma_{yy}\left(\frac{\partial\delta}{\partial x}\right)dx \tag{5}$$

This is a general expression for Irwin's energy release rate for diffusional cracks in a general elastic solid. We note that in the absence of diffusion, $\delta = 0$ and $G = \int_{-h}^{h} w\,dy$ becomes the strain-energy release rate appearing in conventional fracture mechanics theory. It can be seen that G consists of three terms: (1) the loss of strain energy of material that is removed from the crack surface; (2) the gain of strain energy by stressing the deposited matter at the g.b. to a proper level so as to ensure a coherent fit; and (3) the mechanical work done by normal stresses on opening up the g.b. to allow insertion of the matter that has diffused away from the crack surfaces. It is also of interest to note that an alternative approach from the theory of elasticity can be adopted to derive G and the end results are

identical to Eqn. (5). Further, G can be shown to be exactly identical to the well known J-integral[5] as in the case of linear elastic fracture mechanics, if one takes finite deformation into consideration. The complete proof is given in Ref. [4].

In order to further investigate each individual component contributing to G, the curvature at the crack tip, stress and strain energy distributions have to be determined. (See Eqns. (3) and (4)). Unfortunately, no solutions are available in the general case. We therefore focus our attention to the linear case where solutions are readily available. In this special case the diffusion equations follow Fick's law and take the form of $J \propto \nabla\mu$. Also the stress-strain relations follow Hooke's law, so the strain energy function takes quadratic form in stress. Further, the strain energy contribution to μ (and hence to G) can be ignored even at the crack tip[3]. Actual stress solutions given by Chuang[2] confirm this assertion. Thus the versions of Eqns. (3-5) in the linear case take the following forms

$$\dot{W}^{s}_{dis}/v = 2h\gamma_s K_{tip} - (2\gamma_s - \gamma_b) \tag{6a}$$

$$\dot{W}^{g.b}_{dis}/v = -2h\gamma_s K_{tip} - \int_0^\infty \sigma_{yy}\left(\frac{\partial\delta}{\partial x}\right)dx \tag{6b}$$

and

$$G = J = -\int_0^\infty \sigma_{yy}(\partial\delta/\partial x)\,dx \tag{6c}$$

Chuang and Rice[6] have solved the entire crack tip profile both numerically and in closed form. The results indicate that approximately the following relation hold between K_{tip} and 2h: $K_{tip} = \frac{2}{h}(1 - \gamma_b/2\gamma_s)$ Substituting this into Eqn. (6a) we find $\dot{W}^{s}_{dis} \approx G_{Gr}$ (independent of v).

To compute $\dot{W}^{g.b.}_{dis}$ according to Eqn. (6b), the σ_{yy} and δ along the g.b. must be found in addition to K_{tip}. The control equations were derived from the requirements that matter be conserved and Fick's law be satisfied along g.b., and that the stress and strain fields in the interior satisfy the equilibrium and compatibility conditions, respectively, and also satisfy boundary conditions at the crack plane and at the outer boundaries where the sustained loads are prescribed. The solutions are given in Ref. [2]. It was shown there that a g.b. parameter L exists, where L has a dimension of length and is a function of material properties, T and v, such that the size of the diffusion zone is confined to within 4L ahead of the moving crack tip wherein the influence of diffusion is significant. The resulting small diffusion zone implies that the applied K is able to control the crack growth behavior. Mathematically, this is analogous to the small cohesive zone in the Dugdale-Barenblatt model or small-scale yielding zone in the linear elastic fracture mechanics. Hence there is a one-to-one correlation between K and v for $v > v_{min}$, where v_{min} is a material constant, thus

$$G/G_{Gr} = 0.714 \left[(v/v_{min})^{1/2} + (v/v_{min})^{-1/2} \right]^2 \qquad (7)$$

The main features of the energy release rate that occurs during diffusive growth of a sharp crack can be unveiled if we plot nondimensional G vs from Eqn. (7) as illustrated in Fig. 2 where the three components of G are shown for an arbitrary velocity. For example, at the threshold point where $v = v_{min}$ an amount of energy 0.85 G_{Gr} goes to disspative work in the grain boundary whereas 1.0 G_{Gr} each is spent on $\dot{F}_s/v$ as well as on surface diffusion. For a higher crack velocity corresponding to a higher applied load, the first part increases following the curve while the last two parts remain fixed. However, when $v < v_{min}$, there is insufficient energy available to move the crack tip and, as a result, the crack will cease to grow.

CONCLUSION

The energy release rates of diffusional crack growth were derived in a general elastic solid. The strain energy contribution is included in the formulation. This expression is valid even if the material is nonlinear, and consisted of Griffith energy and heat generation in the mass transport process.

Field solutions in linear case indicate that heat generated in surface diffusion is $\approx 1.0 G_{Gr}$ (regardless of v) and that in g.b. diffusion is increasing with increasing v from a minimum of 0.85 G_{Gr}.

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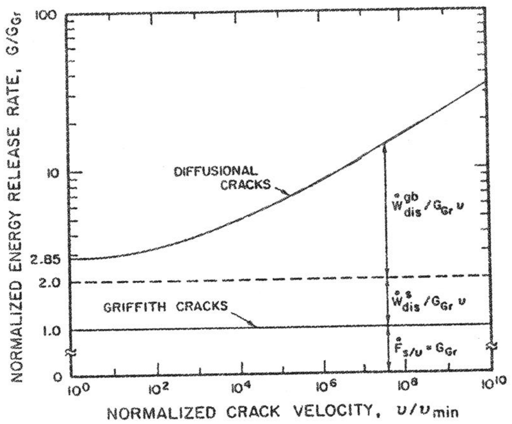

Fig. 2. Plot of normalized energy release rate vs. normalized crack growth rate (Eqn. (7)).

REFERENCES

[1] Chuang, T.-J., K.I. Kagawa, J.R. Rice and L.B. Sills, Overview No. 2: Non-Equilibrium Models for Diffusive Cativation of Grain Interfaces, Acta Met. 27, 265-284 (1979).

[2] Chuang, T.-J., A Diffusive Crack-Growth Model for Creep Fracture, J. Am. Cer. Soc. 65, 93-103 (1982).

[3] Rice, J.R., and T.-J. Chuang, Energy Variations in Diffusive Cavity Growth, J. Am. Cer. Soc. 64, 46-53 (1981).

[4] Chuang, T.-J., NBS IR 82-2628 (1982).

[5] Rice, J.R., A Path-Independent Integral and the Approximate Analysis of Strain Concentration by Notches and Cracks, J. Appl. Mech. 35, 379-386 (1968).

[6] Chuang, T.-J. and J.R. Rice, The Shape of Intergranular Creep Cracks Growing by Surface Diffusion, Acta Met. 21, 1625-1628 (1973).