A CONTINUOUS MODEL FOR CREEP CRACK GROWTH IN DAMAGED MATERIAL

M. Chrzanowski
Technical University, Cracow, Poland

I. INTRODUCTION

A phenomenon of failure of structural members in creep conditions draws engineers' attention since early works in the thirties (cf e.g.[1]). This is not only because of its practical importances but also due to its specific character. Among many others, two features of creep failure are distinctive:

i) creep fracture occurs at any stress level after finite time,

ii) the main part of fracture process develops without macroscopically observable discontinuities ("cracks"), and often without any acceleration in creep strain accumulation.

The concept of a crack successfuly used in classical fracture mechanics is applicable only in the final part of the process when discontinuities caused by a damage of a material start to lead to failure. Consequently the idea of damage parameter ω, $(0 \leq \omega \leq 1)$ responsible for defects growth and cumulation during the stage of "hidden fracture" was introduced by L. M. Kachanov [2] and Yu. N. Rabotnov[3]. Not to go into physical interpretation of its nature, the damage parameter can be dealt with as a new state variable. It has found a broad application, reviewed in [4], to form a branch of fracture mechanics called Continuous Damage Mechanics (CDM), the term coined by J. Hult in the paper [5] by J. Janson and himself.

In the present paper two possibilities of description of both stages of fracture process i.e. crack formation and propagation are discussed. The first one bases on the assumption that damage parameter concept can be applied on submacroscopic level to describe crack formation along grain boundary (Sec.2). The following crack propagation, taking into account damage has

been studied by J.Janson[6].

In the second approach the equation of fracture front motion proposed by L. M. Kachanov is used. According to this theory creep crack propagation occurs because of damage rate accumulated during period of hidden deterioration (Sec.3).

II. CRACK NUCLEATION DUE TO VOIDS COALESCENCE

Evidence of crack formation in metals under creep conditions along grain boundaries has been documented in numerous works (cf [7]). Following this observation W. D. Nix et al. [8] proposed a mechanism of voids growth due to creep deformation. If we assume that in the vicinity of a void (see Fig.1) damages accumulate according to Kachanov's law:

$$\frac{d\omega}{dt} = A\left(\frac{\sigma_{max}}{1-\omega}\right)^{m} \tag{2.1}$$

where A and m are material constants, t—time, σ_{max}—maximum principal stress, the problem of void coalescence can be reduced to description of the fracture of a sheet of infinite width containing a circular hole and loaded by stress σ_0. It holds until λa is great enough.

After the time t_I damage reaches its critical value $\omega=1$ at point P (Fig.2). Assuming that the fractured zone remains of circular shape with radius a(t) this function determines voids growth. The time t_I can be easily found if we assume that the stress distribution in creep conditions does not differ significantly from elastic one. Using known solution by Kirsch[9] we obtain after integration of Eq. (2.1):

$$t_I = \frac{1}{A(m+1)(3\sigma_0)^m} \tag{2.2}$$

and damage distribution along x axis:

$$\omega(x;t = t_I) = 1-\{1-[\frac{1}{3}+\frac{1}{6}(\frac{a_0}{x})^2+\frac{1}{2}(\frac{a_0}{x})^4]^m\}^{\frac{1}{m+1}} \tag{2.3}$$

For time $t>t_I$ Eq. (2.1) has to be integrated with initial condition (2.3) to give:

$$(1-\omega)^{m+1} = 1-A(m+1)\{[\frac{\sigma_0}{2}(2+\frac{a_0^2}{x^2}+3\frac{a_0^4}{x^4})]^m t_I+\int_{t_I}^{t}[\frac{\sigma_0}{2}(2+\frac{a^2}{x^2}+3\frac{a^4}{x^4})]^m dt\} \tag{2.4}$$

On the boundary S of fractured zone is:

$$\omega(x = a,t)|_S = 1$$

and consequently:

$$\frac{d\omega}{dt}\Big|_S = \frac{\partial\omega}{\partial t}\Big|_S + \frac{\partial\omega}{\partial v}\frac{dv}{dt}\Big|_S = 0 \tag{2.5}$$

where v is a normal to the boundary of failed zone S. Using (2.4) to calculate derivatives in (2.5) and taking into account that on S is $v \equiv x = a$, we obtain equation for a(t):

$$\frac{da}{dt} = \frac{6}{2m}\left[\left(2+\frac{a_0^2}{a^2}+3\frac{a_0^4}{a^4}\right)^{m-1} \cdot \left(\frac{a_0^2}{a^2}+6\frac{a_0^4}{a^4}\right)t_I + 7.6^{m-1}\int_{t_I}^{t}\frac{dt}{a}\right]^{-1} \tag{2.6}$$

The details of integration of this equation are discussed in the paper[10]. Fig.3 shows the solution of Eq. (2.6) for m=4.

III. UNIFORM DESCRIPTION OF CRACK NUCLEATION AND GROWTH

Among assumptions made in previous section one certainly is based on very rough approximation: It is that of circular shape of fractured surface S. More realistic is assumption that in point P forms a sharp crack causing singularity in stress distribution at x=a. But then a normal v on fracture front S cannot be defined, and derivatives in Eq. (2.5) with respect to v remain unknown.

Let us calculate derivative dv/dt from (2.5) using (2.1) and its integral:

$$\omega = 1-\left[1-A(m+1)\int_0^t \sigma_{max}^m(\tau)d\tau\right]^{\frac{1}{m+1}} \tag{3.1}$$

to obtain:

$$\frac{dv}{dt} = -\frac{\sigma_{max}^m|_S}{\left[\frac{\partial}{\partial v}\int_0^t \sigma_{max}^m(\tau)d\tau\right]\Big|_S} \tag{3.2}$$

If we assume that the stress at crack tip has a finite value (cf Fig.4) then stress gradient in (3.2) can be estimated as in [11]:

$$\frac{\partial\sigma_{max}}{\partial x}\Big|_{x=1} = k(\sigma_c-\sigma_m) \tag{3.3}$$

where k is a material constant, and σ_m is mean stress calculated when stress concentration is neglected.

Eq. (3.2) which holds for x=1 takes now the form:

$$\frac{dl}{dt} = -\frac{1}{km\int(1-\sigma_m/\sigma_c)d\tau} \tag{3.4}$$

After integration with respect to time we obtain:

$$\frac{d^2l}{dt^2} - km(1-\sigma_m/\sigma_c)\left(\frac{dl}{dt}\right)^2 = 0 \tag{3.5}$$

For $0 \le t \le t_I$ is l=0 and $\sigma_m = \sigma_0$, hence $t_I = [A(m+1)\sigma_0^m]^{-1}$. Initial conditions for Eq. (3.5) are:

For $t = t_I$ $l = 0$

and

$$\frac{dl}{dt}\Big|_{t=t_I} = -\frac{1}{km(1-\sigma_0/\sigma_c)t_I} \tag{3.6}$$

One can observe that must be k<0 since $\sigma_0<\sigma_c$ and dl/dt should be positive. Eq. (3.5) was used successfuly by E.Dusza and author [12,13] to describe creep crack propagation under various loading conditions, and can be easily applied to the problem discussed in Sec. 2. A strip of width B under constant stress σ_0 may be used as a model of creep crack growth along grain boundary, if we neglect interaction of voids (Fig.5). Numerical integration of Eq. (3.6) with mean stress

$$\sigma_m = \frac{\sigma_0}{1-1/2\lambda} \tag{3.7}$$

yields curves of creep crack length versus time, which shape depends on material constants m and K, and on time t_I. Fig.6 illustrates typical result for m=4.

The same model can be used for calculation of a time corresponding to the crack length equal to the grain boundary length L, as well as—on macroscopic level—for time of crack propagation throughout width of structural members, as done in [12].

IV. FINAL REMARKS

The application of fracture mechanics under creep conditions needs non-zero initial length of a crack. It can be calculated in the frame of CDM by considering damage growth around voids distributed along grain boundary, as shown in Sec.2. Two material characteristics are then necessary to perform effective calculations i.e. initial radius of voids a_0 and their spacing λ. Due to assumptions made, the rate of voids growth has no acceleration preceding thier coalescence, what is not experimentally confirmed.

The better agreement with experimental results gives the approach discussed in Sec.3, which does not need any initial crack, and thus may be proposed as unique description of both stages of fracture process: Crack formation and propagation. Its application needs another material constant k which has no simple physical interpretation. A comparison of obtained results with any other criterion of creep crack propagation may enlight better its nature.

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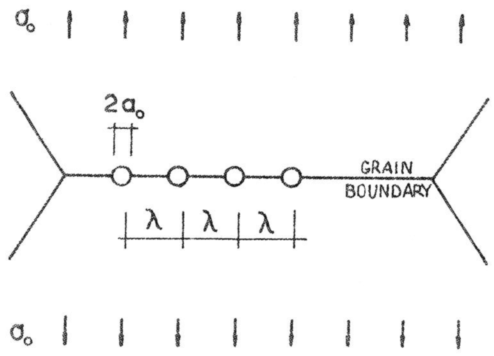

Fig. 1

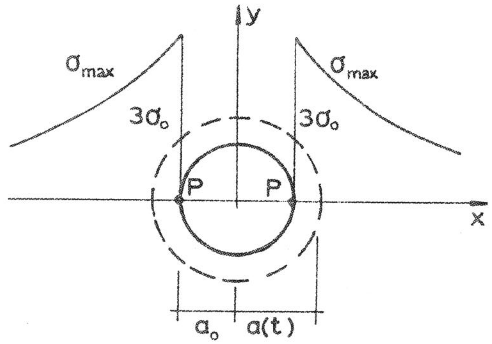

Fig. 2

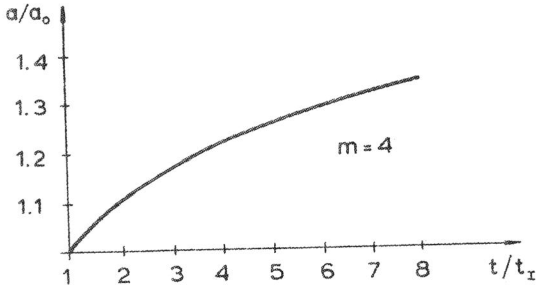

Fig. 3

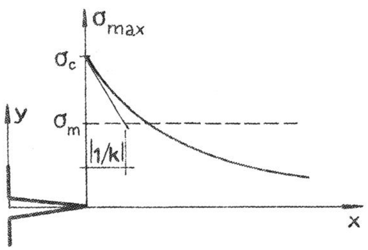

Fig. 4

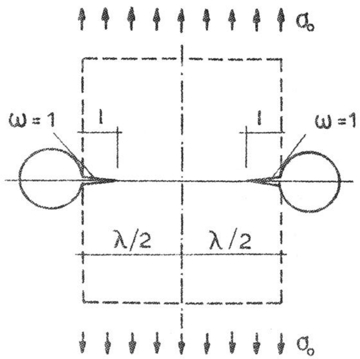

Fig.5

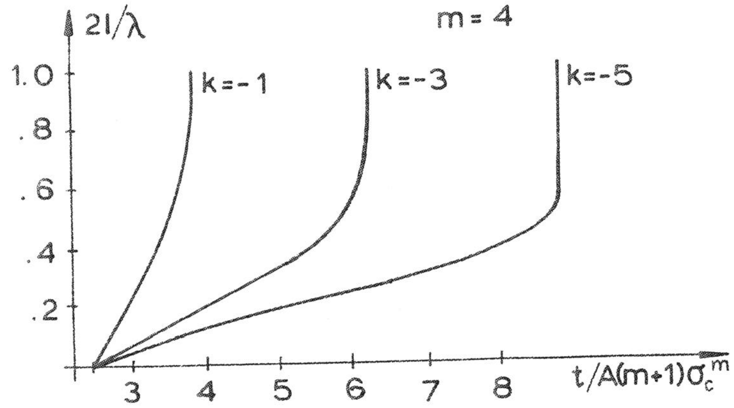

Fig. 6