# MODELS FOR PREDICTING FATIGUE CRACK GROWTH UNDER SPECTRUM LOADING

Zhang Yongkui (张永奎)
Shanxi University, China
Gu Mingda (顾明达) Yan Minggao (颜鸣皋)
Institute of Aeronautical Materials, China

## ABSTRACT

In this paper, the fatigue life prediction under spectrum loads in structural materials is discussed. Two recently developed models, the Willenborg-Chang model and the modified Maarse model were selected and found to be improved by some modifications on the calculation of plastic zone size and effective overload retardation zone size. A comparison of the analytical results with the experiment data indicated that the present modified models are rather simple and efficacious for life prediction under spectrum loading.

## INTRODUCTION

At present there are essentially two different approaches for the life prediction under spectrum loads in the damage-tolerance design of aircraft structures: the root-mean-square (RMS) method and the cycle-by-cycle method combined with some retardation model. The former is only applicable to flight spectrum loading with random sequence and the latter appears to be applicable to the life prediction where the load sequence effect should be properly taken into consideration.

Thus, a number of retardation models[1-7] have been proposed, in which the Willenborg-Chang model[8] seems to make some new progress in life prediction under random spectrum loads. Recently, some investigations of the crack growth retardation behavior[9,10] and an evaluation of the ability to predict life for four current retardation models[11] have been conducted. Two modified models were also proposed for life prediction under simple spectrum loading[12].

In this paper, the selection and modifications of these models for life prediction under complex spectrum loads with a view to engineering application are described.

## SELECTION AND MODIFICATION OF RETARDATION MODEL

In view of engineering application, it may be suggested that the selection of retardation model should be based upon the following criterions: a) the model chosen should be capable of accounting the effect of overload retardation, compressive load acceleration and the coupling effect, b) the model should be formulated based on the LEFM concept and c) the material constants could be acquired from constant amplitude cyclic tests. Hence, the Willenborg-Chang model[8] and the modified Maarse model[12] were selected to fulfil these requirements.

1. The Willenborg-Chang Model

The formulae used in the Willenborg-Chang model can be expressed as

$$\frac{da}{dN} = \begin{cases} C[(1-R_{eff})^{m-1}(\Delta K)_{eff}]^n & R_{eff}>0 \\ C[(K_{max})_{eff}]^n & R_{eff}=0 \\ C[(1-R_{eff})^q(K_{max})_{eff}]^n & R_{eff}<0 \end{cases} \qquad (1)$$

where C, m and n are constants determined from constant amplitude cyclic tests at various positive stress ratios and q is the acceleration index derived from that at negative stress ratios. The $(\Delta K)_{eff}$ and $R_{eff}$ are the effective stress intensity range and effective stress ratios respectively.

$$(K_{max})_{eff} = K_{max} - \phi[K_{01}(1-\frac{\Delta a}{Z_{01}})^{\frac{1}{2}} - K_{max}]$$

$$(K_{min})_{eff} = K_{min} - \phi[K_{01}(1-\frac{\Delta a}{Z_{01}})^{\frac{1}{2}} - K_{max}] \qquad (2)$$

$$\phi = (1-\frac{K_{th}}{K_{max}})/(S_{so}-1)$$

where K, $K_{01}$ and $K_{th}$ are the stress intensity factors corresponding to the applied load, maximum overload and threshold respectively, $\Delta a$ is the crack growth length after overload, $S_{so}$ is the overload shut-off ratio and $Z_{ol}$ is the overload plastic zone size which can be expressed as follows.

$$Z_{01} = \frac{1}{\alpha\pi}(\frac{K_{max}}{\sigma_y})^2 \qquad \begin{matrix} \alpha = 2 \text{ for plane stress} \\ \alpha = 6 \text{ for plane strain} \end{matrix} \qquad (3)$$

The effective overload retardation zone size $(Z_{01})_{eff}$ denotes the reduction of overload retardation effect by compressive overload immediately after a tensile overload, i.e.

$$(Z_{01})_{eff} = (1+R_{eff})Z_{ol} \qquad -1<R_{eff}<0 \qquad (4)$$

2. The Modified Maarse Model

The formula used in the modified Maarse model can be expressed by[12],

$$\frac{da}{dN} = C^*[U(\Delta K)]^n \qquad (5)$$

in which,
$$C^* = C\left[\frac{1-R}{1-\frac{P_{op}}{P_{max}}}\right]^n \qquad (6)$$

$$U = \frac{P_{max}-P_{op}}{P_{max}(1-R)} \qquad (7)$$

where C and n are material constants derived from Paris' Law and $P_{op}$ is the crack opening load.

The ellipse equation of the modified model can be described by the following expression:

$$\left(\frac{X-\frac{1}{2\alpha}}{\frac{1}{2\alpha}}\right)^2 + \left(\frac{Y}{0.234}\right)^2 = 1 \qquad (8)$$

in which, $\alpha = \frac{6}{1+2S}$ (9)

where,
$$S = \frac{\Delta K-\Delta K_{th}}{(1-R)K_c} = \frac{K_{max}-\frac{\Delta K_{th}}{1-R}}{K_c} \qquad (10)$$

which may be regarded as a fraction of the plane stress region occupied in the fracture surface[11] and $K_c$ is the fracture toughness.

3. Modifications of the Willenborg-Chang Model

Based upon and analysis of the mode of fatigue fractures, the stress conditions in FCG process may be considered as a successive transition from a plane strain to a mixed mode and finally to a plane stress state[11]. Hence, the coefficient $\alpha$ in eq. (3) of the Willenborg-Chang model can be substituted by a variable $\alpha$ expressed by eqs. (9) and (10).

Refer to Ref. [10], the maximum plastic strain $\varepsilon_3$, for a tensile-compressive overload is about half of that for a single tensile overload, that is to say, the retardation effect cannot be elliminated fully at $R_{eff}=-1$. Thus, eq. 4 of the Willenborg-Chang model should be modified as follows

$$(Z_{01})_{eff}=(1+\gamma R_{eff})Z_{ol} \tag{11}$$

For aluminium alloys, we may assume

$$\gamma=\begin{cases}0 & \text{for } R_{eff}\geq 0\\ 1/2 & \text{for } -1\leq R_{eff}<0\end{cases}$$

## RESULTS AND DISCUSSION

In order to verify the above mentioned two modified models proposed, two types of spectrum have been studied:

(a) The random cycle-by-cycle type of spectrum for a multimission fighter[8] are shown in Figs. 1 and 2. The material investigated was a 2219-T851 Al alloy using CCT specimens.

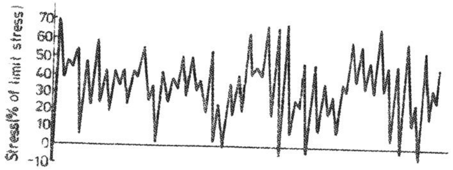


Fig. 1 A-A flight spectrum

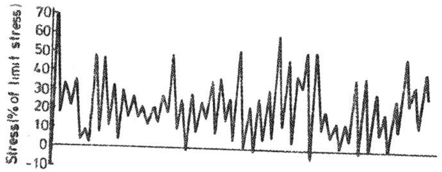


Fig. 2 A-G flight spectrum

(b) Two simplified test spectrums, the variable mean value and the two-wave method[13], are illustrated in Figs. 3 and 4. The material used was a LY12-CZ (comparable to 2024-T4) alloy with CCT specimens.

The mechanical properties and other related parameters used in these calculations for two aluminium alloys are listed in Table 1.



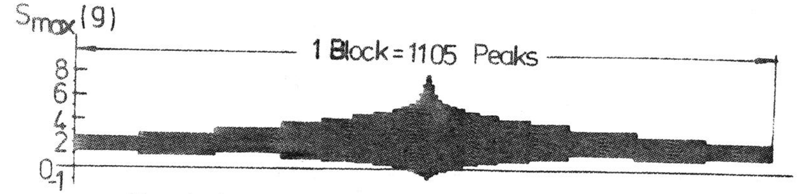


Fig. 3 The variable mean value test spectrum

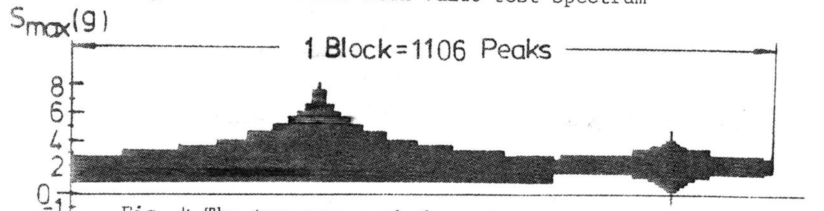


Fig. 4 The two-wave method spectrum

Table 1 Mechanical properties and Parameters Used in Calculation for two Aluminium Alloys

| Materials | Thickness (mm) | $\sigma_y$ (MPa) | $K_c$ (MPa$\sqrt{m}$) | $K_{th}$ (MPa$\sqrt{m}$) | $S_{so}$ | $c\times10^{-8}$ | n | m | q |
|---|---|---|---|---|---|---|---|---|---|
| 2219-T851 | 6.5 | 245 | 59.14 | 2.27 | 3.0 | 2.125 | 3.64 | 0.6 | 0.3 |
| LY12-CZ | 2.5 | 351 | 99.24 | 2.08 | 2.4 | 30.73 | 3.19 | - | - |

Tabls 2 and 3 list the experiment data and the lives predicted by the present modified willenborg-Chang Model and modified Maarse model respectively. It can be seen that the ability to predict life of these modified models appear to be more promising than the original ones, in which the life prediction by the modified Maarse model is fairly consistant with the experimental results under spectrum tests.

## CONCLUSION

The ability to predict life under spectrum loadings of the Maarse model and the willenborg-chang model could be improved by some modifications on the calculation of the plastic zone size and the effective overload retardation zone size. Since all the material constants used in present modified models can be derived from constant amplitude cyclic tests, these models are relatively easy to be manipulated and feasible to life prediction under complex spectrum loadings for engineering applications.

Table 2 Comparison of the Experiment Data and Lives Calculated by Models for 2219-T851 Alloy.

| Test No. | Types of Mission | $\sigma_{lim}$ (MPa) | $a_i$-$a_f$ (mm) | $N_e$ (cycle) | $N_{cal}$ (cycle) | $\frac{N_{cal}-N_e}{N_e}$ % | Method* |
|---|---|---|---|---|---|---|---|
| | | | | | 213,110 | +84.19 | (1) |
| M-81 | A-A | 138 | 4.1-12.7 | 115,700 | 168,720 | +45.83 | (2) |
| | | | | | 167,985 | +45.19 | (3) |
| | | | | | 74,055 | +26.41 | (1) |
| M-82 | A-A | 207 | 3.8-Fail. | 58,585 | 53,312 | -9.00 | (2) |
| | | | | | 53,997 | -7.83 | (3) |
| | | | | | 25,944 | +39.39 | (1) |
| M-83 | A-A | 276 | 3.8-Fail. | 18,612 | 17,309 | -7.00 | (2) |
| | | | | | 18,000 | -3.29 | (3) |
| | | | | | 496,284 | +84.56 | (1) |
| M-84 | A-G | 138 | 4.0-Fail. | 268,908 | 368,662 | +37.10 | (2) |
| | | | | | 367,835 | +36.79 | (3) |
| | | | | | 131,868 | +37.88 | (1) |
| M-85 | A-G | 207 | 3.7-Fail. | 95,642 | 91,816 | -4.00 | (2) |
| | | | | | 91,961 | -3.85 | (3) |
| | | | | | 45,034 | +23.83 | (1) |
| M-86 | A-G | 276 | 3.9-Fail. | 36,367 | 29,093 | -20.00 | (2) |
| | | | | | 29,992 | -17.53 | (3) |

* (1) Generalized Willenborg Model[8]; (2) Willenborg-Chang Model[8]; (3) Present Modified Model.



Table 3 Comparison of Experiment Data and Calculated lives by Various Models in LY12-CZ Al Alloy.

| Types of spectrums | $S_{max}$ (MPa) | $C_i$-$C_f$ (mm) | $N_e$ (cycle) | $N_{cal}$ (cycle) | $\frac{N_{cal}-N_e}{N_e}$ % | Method |
|---|---|---|---|---|---|---|
| | | | | 12,700 | -27.33 | L.A.M.* |
| Variable mean value | 221 | 5.0-8.5 | 17,475 | 13,823 | -20.87 | Matsuoka |
| | | | | 18,196 | +4.13 | Mod. Maarse |
| | | | | 13,353 | -12.51 | L.A.M.* |
| Two-wave method | 238 | 4.8-8.0 | 15,263 | 13,679 | -10.38 | Matsuoka |
| | | | | 16,387 | +7.36 | Mod. Maarse |

* L.A.M. — The linear accumulation method.

## REFERENCES


[1] Wheeler, O.E., Spectrum loading and growth. Trans. ASME, J. Bas, Engng., 94 (1972), 181.

[2] Willenborg, J.D., Engle, R.M. and Wood, H.A., A crack growth retardation model using and effective stress concept. Tech. Rep. AFFDLTM 71-1 FBR (1971).

[3] Elber, W., The siginificance of fatigue closure. ASTM STP 486 (1971), 230.

[4] Maarse,J., Crack closure related to fatigue crack propagation. Fracture, ICF-4, 2 (1977), 1025.

[5] Matsuoka, S. and Tanaka, K., Influence of stress ratio at baseline loading on delayed retardation phenomenon of fatigue crack growth in A553 steel and A5083 aluminium alloy, Engng. Fract. Mech., 11 (1979), 703.

[6] He Qingzhi, A new model for predicting the overload retardation effect in fatigue propagation, J. Beijing Inst. Aeronaut. & Astronaut., 80-1 (1980).

[7] Fatigue crack growth under spectrum loads, ASTM STP 595 (1976).

[8] Chang, J.B., Engle, R.M. and Szamassi, M., An improved methodology for predicting random spectrum load interaction effect on fatigue crack growth, Fracture, ICM-5, 5 (1981), 2615.

[9] Gu Mingda, Zhang Shijie and Yan Minggao, Effect of overload on fatigue crack growth in LY12 aluminium alloy. HKCL (Institute of Aeronautical Materials, Beijing,) Tech. Rep., 79-4 (1979).

[10] Ouyang Jie and Yan Minggao, The effect of overload on fatigue crack propagation and its microscopic morphology, Acta Metall. Sinica, 3/2 (1982), 38.

[11] Gu Mingda, Zhang Yongkui and Yan Minggao, An evaluation of overload models on the retardation behavior in a Ti-6Al-4V alloy, Fatigue Engng. Mats. & Struct. (to press).

[12] Zhang Yongkui, Gu Mingda and Yan Minggao, Modification of Willenborg and Maarse models and application to the life prediction for a Ti-6Al-4V alloy, Acta Aeronaut. et Astronaut. Sinica, 3/2 (1982), 91.

[13] Gu Mingda, Zhang Shijie and Wu Dejun, A comparison of spectrum tests for service life prediction. HKCL, (Institute of Aeronautical Materials Beijing,) Tech. Rep. 80-028 (1980).