FATIGUE PROPAGATION OF AN INCLINED CRACK IN UNIAXIAL TENSION STRESS FIELD

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ABSTRACT

On the basis of the experimental works on LC4 aluminum alloy center cracked panel (CCP) specimens, the fatigue propagation law of an inclined crack in uniaxial tension stress field is studied, and it is suggested that the fatigue life may be predicted by an equivalent mode I crack.

INTRODUCTION

The fatigue propagation of a crack inclined in an arbitrary angle to the load line is used to be encountered in practice. Iida[1], Pustejovsky[2], and Sih[3] have studied the fatigue propagation of an inclined crack in CCP specimens.

In the present paper based on the experimental results of LC4 aluminum alloy CCP specimens with inclined crack, the fatigue crack propagation law of the inclined crack in uniaxial tension stress field under plane strain condition has been studied, and an approximate method to predict its fatigue life is suggested.

The angles of inclinetion of the inclined crack in CCP specimens in the experimental work were 30°, 45°, 60°, and 90°. The total number of specimens tested is 11. The solid line in Fig. 3 represents a propagated inclined crack. Details of the experiments are given in Ref. [4].

FATIGUE CRACK PROPAGATION LAW UNDER MIXED MODE CONDITION

Let η represents a combined quantity (CQ). It can be generally expressed in the following two forms

$$\eta_1 = a_1(\theta) K_I + a_2(\theta) K_{II} \tag{1}$$

and

$$\eta_2 = a_{11}(\theta) K_I^2 + a_{12}(\theta) K_I K_{II} + a_{22}(\theta) K_{II}^2 \tag{2}$$

where, η_1 is the 1st order CQ, and η_2 the 2nd order CQ.

The mixed mode criteria applied in the present paper belong to these two CQ's respectively. The criteria of max. principal stress (σ_1) and max. circumferential tensile stress (σ_θ) belong to the 1st order CQ, while the criteria of strain energy density factor (S), the energy release rate (G) and the plastic range size factor (r_p) belong to the 2nd order CQ.

Treating the experimental data with these five criteria and plotting the results in ds/dN versus $\Delta\eta$ plots (e.g. Fig. 1) the data points could be regressed linearly. The slopes m_η, the intercepts C_η, the correlation coefficients r and the remainder standard deviations σ of such lines are listed in Table 1.

From the r values in Table 1, it is evident that for all of the five criteria, log (ds/dN) vs. log($\Delta\eta$) are of linear relationship. Dispersions of distribution of the experimental data using different criterion are nearly the same. From this , it may be concluded that there is no significant difference in describing the fatigue propagation law by different criterion. Therefore, this law could be expressed by a unified form

$$ds/dN = C_\eta (\Delta\eta)^{m_\eta} \tag{3}$$

PREDICTION OF FATIGUE LIFE

Integrating eq. (3), the fatigue life is obtained as

$$N = \int_{s_1}^{s_2} ds / C_\eta (\Delta\eta)^{m_\eta} \tag{4}$$

Due to the difficulty in the determination of the exact values of SIF, K_I and K_{II}, it is necessary to make some assumptions in the integration of eq. (4). Two hypothetic cracks are suggested, namely, the equivalent mode I crack (EIC), and the imaginary tangent crack (ITC), i.e. the inclined crack parallel to the tangent of the propagating path at the current tip.

Both are defined in Fig. 3. The calculated values of mode I SIF of the EIC, K_I^{EIC}, and mode II SIF of the ITC, K_{II}^{ITC} are approximately the same as the exact K_I and K_{II} respectively at the current tip for all of the testing specimens with various inclined cracks.

The values of K of a hypothetic crack are

$$K_I = -Y\sigma(\pi a)^{\frac{1}{2}} \sin^2\beta, \text{ and } K_{II} = Y\sigma(\pi a)^{\frac{1}{2}} \sin\beta\cos\beta \qquad (5)$$

here, a is the hypothetic crack length and $Y = \sec\sqrt{(\pi a_e/W)}$.

According to different ways to estimate K values, two models are suggested for predicting fatigue life.

1) Model A

Take K_I^{EIC} and K_{II}^{ITC} as the approximate values of K_I and K_{II}.

Assume that a crack grows step by step. The calculation starts from some point A (Fig. 2). The direction of propagation is determined from the hypothetic crack which is the imginary line OA, and by different mixed mode criteria. The transverse length of each step is L. From the geometric relationship in Fig. 2, the half lengths of the hypothetic cracks OM' and DM are easy to obtain. Then from eqs. (5) and (4), the number of cycles N for each step of crack propagation (from A to B) can be readily calculated. The result for specimen with β=45° is shown in Fig. 5 by curves corresponding to each mixed mode criterion. In this calculation, the values of C_n and m_n used are those from the CCP specimen with a transverse crack (β=90°). The errors of the predicted life by the model A for specimens with various angles of inclination are within the range of -32% to +21%.

2) Model B

Fig. 4 is the results from the finite element analysis of a CCP specimen with β=45°. From these results the general tendency of the variation of K_I and K_{II}, while the crack propagates, is evident, i.e. K_{II} could be neglected except at the initial stage of crack propagation. The calculation shows that the errors induced by neglecting K_{II} are less than 2% for all of the CQ's. Therefore, the fatigue life prediction of an inclined crack in CCP specimen may be simplified to a mode I problem. Using an imaginary mode I crack, the eq. (4) is reduced to

$$N = [1/C_n\ (A_n \Delta\sigma^j \pi^{j/2})^{m_n}] \int_{a_1}^{a_2} da_e/(Y\sqrt{a_e})^{jm_n} \qquad (6)$$

here, j = 1 for the 1st order CQ, and j = 2 for the 2nd order CQ, and A is a constant.

The fatigue life prediction is obtained from integration of eq. (6) by model B, and the results are shown in Fig. 5 by a dashed line. The errors are within the range of -20% to +25%.

The fatigue life prediction for the titanium CCP specimen with β=45° in Ref. [2] is also made by using model A with S-criterion, and model B, the results are shown in Table 2, and the life prediction of the same specimen by Sih in Ref. [3] is also given in the same Table. From this it is evident that the models A and B give better prediction than Sih's approach.

CONCLUSION

Under uniaxial tension loading

1) All of the five mixed mode criteria could be used to describe the fatigue crack propagation law, and their propagation rate expression is given in an unified form

$$ds/dN = C_n(\Delta n)^{m_n}$$

2) The fatigue life prediction of an inclined crack in tension panel could be made by using a projected mode I crack.

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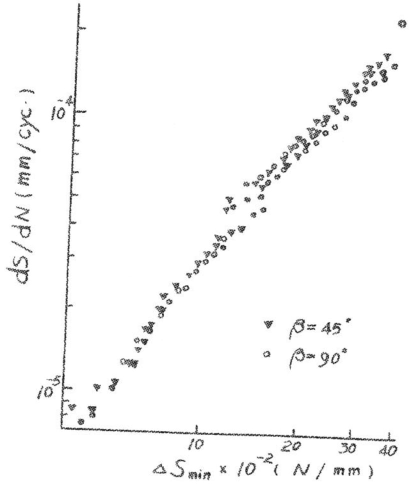

Fig. 1 Crack propagation rate

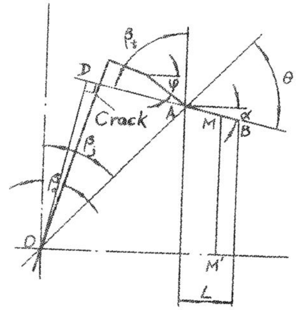

Fig. 2 Step by step crack propagation

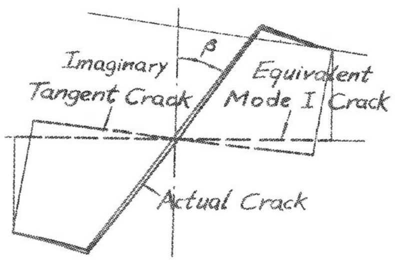

Fig. 3 Imaginary and equivalent cracks

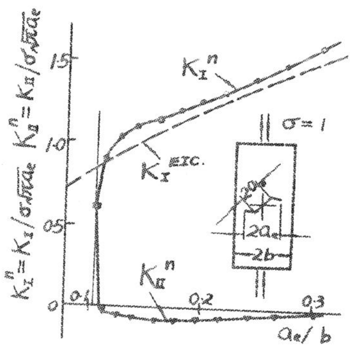

Fig. 4 Normalized stress intensity factors (β=45°)

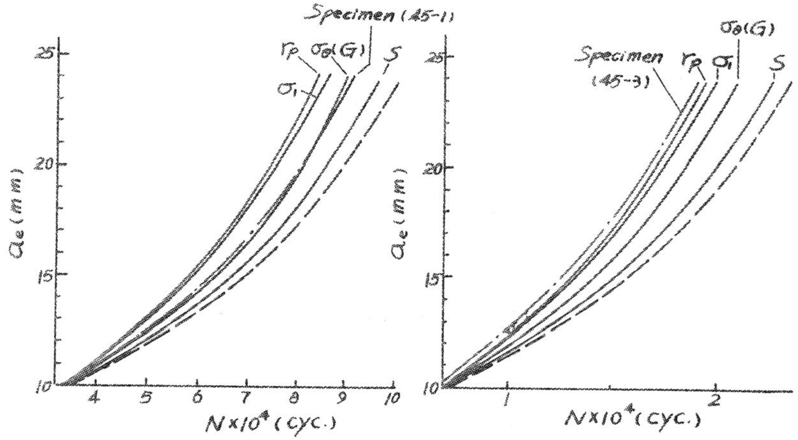

Fig. 5 Predicted fatigue life (β=45°)

Table 1 Fitting Parameters

Criteria		C_q	m_q	r	σ
β=30°	σ_1	6.610×10^{-9}	2.2017	0.9364	0.0751
	σ_θ	1.167×10^{-8}	2.2025	0.9364	0.0751
	G	1.912×10^{-4}	1.1010	0.9364	0.0751
	r_p	5.900×10^{-2}	1.1015	0.9362	0.0752
	S	3.095×10^{-3}	1.0987	0.9367	0.0749
β=45°	σ_1	3.245×10^{-10}	2.8848	0.9962	0.0319
	σ_θ	6.899×10^{-10}	2.8834	0.9962	0.0319
	G	2.267×10^{-4}	1.4421	0.9962	0.0321
	r_p	4.150×10^{-1}	1.4445	0.9962	0.0318
	S	8.817×10^{-3}	1.4396	0.9961	0.0313
β=60°	σ_1	2.260×10^{-9}	2.4277	0.9791	0.0618
	σ_θ	4.260×10^{-9}	2.4281	0.9785	0.0628
	G	1.885×10^{-4}	1.2140	0.9785	0.0628
	r_p	6.500×10^{-2}	1.2119	0.9785	0.0627
	S	6.543×10^{-3}	1.2161	0.9784	0.0629
β=90°	K	1.039×10^{-10}	2.7438	0.9886	0.0525
	σ_1	6.331×10^{-10}	2.7431	0.9886	0.0525
	σ_θ	1.298×10^{-9}	2.7431	0.9886	0.0525
	G	2.299×10^{-4}	1.3715	0.9886	0.0525
	r_p	2.900×10^{-1}	1.3717	0.9886	0.0525
	S	7.386×10^{-3}	1.3715	0.9886	0.0525

Table 2 Predicted Fatigue Life for the Titanium CCP Specimen

Specimen		β=45°		
Cyclic Stress (MPa)		17.84 – 178.36		
Starting Point a_e (mm)		6.9		
Number of Cycles from Starting Point	Sih	11300		
	A	9660		
	B	9730		
	Exp.	10000		
Model for Prediction		Δa_e (mm)	ΔN (cyc.)	Error (%)
The First Point of Prediction	Sih	1.1	4000	92
	A	1.2	2130	2.5
	B	1.2	2260	8.6
	Exp.	1.2	2080	
The Second Point of Prediction	Sih	3.9	7800	31
	A	4.2	5930	-0.5
	B	4.1	6630	11
	Exp.	4.1	5960	