# STUDY ON THE DISTRIBUTION RULE OF FATIGUE RESIDUAL STRENGTH

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## ABSTRACT

In this paper, the theories of fracture mechanics and fatigue are used for studying the fatigue residual strength. The probabilistic analysis method is introduced for analyzing this problem. Through the experiment of seven series of specimen, we get the P-R-S-N curves of the connecting rod of 18CrNiWA steel. The analytic method presented is also suitable for other materials and components.

## INTRODUCTION

In 1977, Talreja and Weibull[1] has proposed a new method on the analysis of fatigue failure based on residual strength. Later on further experiments and analysis on this subject are presented[2,3]. Unlike the more common cumulative damage methods, which are usually based on fatigue life data, the proposed method is based on tensile strength data, which is loaded in tension after prespecified numbers of cycles.

Based on this method, the fatigue residual strength of connecting rod have been studied and some useful results was acquired.

## ANALYSIS

1) Assume a visible crack is arised and it will be propagation. The relation between residual strength and crack length given by Griffith-Irwin is as follows

$$R = \alpha K_c a^{-\frac{1}{2}} \qquad (1)$$



where $K_c$ is the fracture toughness of material; a is the crack length and $\alpha$ is a constant.

The crack propagation rate can be described by Paris' formula

$$\frac{da}{dN} = C \cdot \Delta K^n \qquad (2)$$

It can be rewritten as

$$\frac{da}{dN} = \beta a^{n/2} \qquad (3)$$

where $\beta$ is an experimental constant.

Taking derivative of R and a with respect to the variable N in both sides of Equation (1) and substituting the Equation (3) into it, we have

$$\frac{dR}{dN} = -\gamma\left(\frac{1}{R}\right)^{n-3} \qquad (4)$$

where

$$\gamma = \frac{1}{2}\beta(K_c\alpha)^{n-2} \qquad (5)$$

2) For the 18CrNiWA forging steel, the parameter n is determined as follows: 1.523, 3.11, 1.711, 1.947, 1.948, and 1.982.

The average of n is equal to 2 approximately. Then integrating the Equation (4), the following result will be obtained

$$R = S \exp\left[\frac{\beta}{2}(N_f - N)\right] \qquad (6)$$

where S is applied stress; $N_f$ is the number of cyclesto failure.

## EXPERIMENTAL

1) Material and Specimen:

The material used was a 18CrNiWA steel and its chemical composition and mechanical properties are listed as follows:

| Element | C | Cr | Ni | W | Si | Mn | S | P |
|---|---|---|---|---|---|---|---|---|
| Content in % Wt. | 0.20 | 1.53 | 4.08 | 0.98 | 0.17 ~0.37 | 0.25 ~0.55 | ≤ 0.030 | ≤ 0.035 |

Mechanical Properties: $\sigma_s$ = 91.5kgf/mm², $\sigma_b$ = 112.0kgf/mm², $\delta_5$ = 16.5%, $\psi$ = 67.0%, $a_k$ = 16.4 $\frac{\text{kgf-mm}}{\text{cm}^2}$

The specimen configurations and dimensions are shown in Fig. 1.

2) Experimental Procedures

The test program consists of seven series of specimens. Nos.2 and 5 were tested to failure under cyclic stress $S_1$(26.79 kgf/mm²) and $S_2$ (31.25 kgf/mm²), respectively. The specimens in series No. 1 were loaded in tension to determine their initial strength. The specimens in Series Nos. 3 and 4 were subjected to prespecified numbers of cycles ($15\times10^4$ and $22\times10^4$ cycles) at stress $S_1$, after which the unfailed specimens were loaded in tension to determine their residual strength. Similarily, specimens in Series Nos. 6 and 7 at stress $S_2$, with prespecified numbers of cycles of $8\times10^4$ and $11\times10^4$ respectively were investigated.

## RESULTS

The experimental results are listed in Table 1.

Let t be a random variable with a two-parameter Weibull distribution as follows

$$F(t;\eta,m) = 1 - \exp\left[-\left(\frac{t}{\eta}\right)^m\right] \tag{7}$$

If we make the transformation x=lnt, the cumulative distribution for x will be[4]

$$F(x) = 1 - \exp\left[-\exp\left(\frac{x-u}{b}\right)\right] \qquad -\infty < x < \infty \tag{8}$$

where $u=\ln\eta$ and $b=\frac{1}{m}$.

The estimators are termed best linear invariant estimators (BLIE).

The shape parameter m and the scale parameter of seven series of specimen are listed in Table 2. In comparison with the BLIE method, the best linear unbiased estimator is used, (i.e. BLUE method) its results are also listed in Table 2.

Table 2. The Results of Parameter Estimation

| Series Number | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| BLIE | m | 33.67 | 1.226 | 2.543 | 6.292 | 3.615 | 1.705 | 1.746 |
| | $\eta(\times10^4)$ | 107.12 | 37.46 | 70.20 | 40.44 | 19.99 | 50.7 | 39.57 |
| BLUE | m | 33.22 | 1.227 | 2.571 | 2.488 | 3.614 | 1.71 | 1.746 |
| | $\eta(\times10^4)$ | 109.08 | 37.42 | 71.85 | 40.26 | 19.8 | 50.67 | 39.57 |



Table 1. Experimental Results (Units of Stress and Strength, kgf/mm²)

Initial Tensile Strength; $S=S_1=26.79$ $S_{max}=26.79$, $S_{min}=-41.85$; $S=S_2=31.25$ $S_{max}=31.25$, $S_{min}=-48.83$; $S_m=-8.79$ $S_a=40.04$

| Series 1 | | Series 2 | | Series 3 | | Series 4 | | Series 5 | | Series 6 | | Series 7 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| No. | Roi | No. | Nfi $(\times10^4)$ | No. | After $15\times10^4$ cycles, $R_i$ | No. | After $22\times10^4$ cycles, $R_i$ | No. | Nfi $(\times10^4)$ | No. | After $8\times10^4$ cycles, $R_i$ | No. | After $11\times10^4$ cycles, $R_i$ |
| 1–1 | 96.65 | 2–1 | 17.0 | 3–1 | 18.03 | 4–1 | 13.23 | 5–1 | 11.6 | 6–1 | 18.17 | 7–1 | 5.26 |
| 1–2 | 101.54 | 2–2 | 18.0 | 3–2 | 34.38 | 4–2 | 21.53 | 5–1 | 12.2 | 6–2 | 20.54 | 7–2 | 17.16 |
| 1–3 | 105.04 | 2–3 | 18.0 | 3–3 | 43.05 | 4–3 | 23.25 | 5–2 | 12.2 | 6–3 | 20.55 | 7–3 | 17.45 |
| 1–4 | 105.36 | 2–4 | 19.8 | 3–4 | 43.30 | 4–4 | 24.38 | 5–3 | 12.2 | 6–4 | 22.67 | 7–4 | 20.06 |
| 1–5 | 106.38 | 2–5 | 20.4 | 3–5 | 72.85 | 4–5 | 24.70 | 5–4 | 12.5 | 6–5 | 37.42 | 7–5 | 25.69 |
| 1–6 | 108.50 | 2–6 | 24.7 | 3–6 | 85.71 | 4–6 | 32.00 | 5–5 | 13.2 | 6–6 | 41.47 | 7–6 | 25.95 |
| 1–7 | 108.95 | 2–7 | 22.2 | 3–7 | 86.24 | 4–7 | 34.65 | 5–6 | 14.7 | 6–7 | 48.42 | 7–7 | 45.69 |
| 1–8 | 109.40 | 2–8 | 22.4 | 3–8 | 90.98 | 4–8 | 39.84 | 5–7 | 16.0 | 6–8 | 68.93 | 7–8 | 49.00 |
| | | 2–9 | 22.8 | 3–9 | 92.98 | 4–9 | 45.78 | 5–8 | 16.5 | 6–9 | 77.91 | 7–9 | 49.25 |
| | | 2–10 | 25.6 | | | 4–10 | 46.62 | 5–9 | 17.5 | 6–10 | 86.83 | 7–10 | 54.67 |
| | | 2–11 | 26.5 | | | 4–11 | 52.87 | 5–10 | 18.8 | | | 7–11 | 68.39 |
| | | 2–12 | 27.3 | | | 4–12 | 68.71 | 5–11 | 19.5 | | | | |
| | | 2–13 | 56.0 | | | | | 5–12 | 20.3 | | | | |
| | | 2–14 | 228.0 | | | | | 5–13 | 22.6 | | | | |
| | | | | | | | | 5–14 | 25.8 | | | | |
| | | | | | | | | 5–15 | 26.6 | | | | |
| | | | | | | | | 5–16 | 27.0 | | | | |

Consequentely the following equations were obtained. Under the cyclic stress $S_1$=26.79 kgf/mm²:

$$P = 0.9: \quad R = 26.97 \exp\left[\frac{1}{2} \times 3.59 \times 10^{-6} (73.9627 \times 10^4 - N)\right] \tag{9}$$

$$P = 0.5: \quad R = 26.79 \exp\left[\frac{1}{2} \times 10.47 \times 10^{-6} (27.7801 \times 10^4 - N)\right] \tag{10}$$

$$P = 0.1: \quad R = 26.79 \exp\left[\frac{1}{2} \times 3.3737 \times 10^{-6} (5.9759 \times 10^4 - N)\right] \tag{11}$$

Under the cyclic stress $S_2$=31.25 kgf/mm²:

$$P = 0.9: \quad R = 31.25 \exp\left[\frac{1}{2} \times 1.17 \times 10^{-5} (25.1733 \times 10^4 - N\right] \tag{12}$$

$$P = 0.5: \quad R = 31.25 \exp\left[\frac{1}{2} \times 1.045 \times 10^{-5} (18.0626 \times 10^4 - N\right] \tag{13}$$

$$P = 0.1: \quad R = 31.25 \exp\left[\frac{1}{2} \times 1.7142 \times 10^{-5} (10.7265 \times 10^4 - N\right] \tag{14}$$

The P-R-N curve under $S=S_1$ and $S_2$ are shown in Fig. 2 and 3 respectively and R-S-N curve at p=0.5 is illustrated in Fig. 4.

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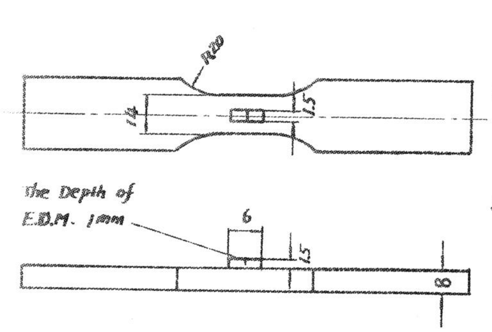


Fig. 1 The specimen residual strength

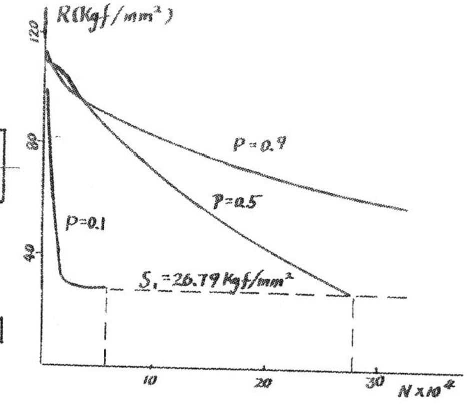


Fig. 2 The P-R-N curve under $S_1$=26.79 kgf/mm²

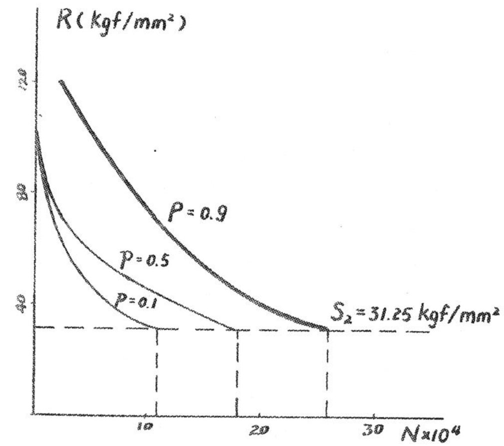


Fig.3 The P-R-N curve under $S_2$=31.25 kgf/mm²

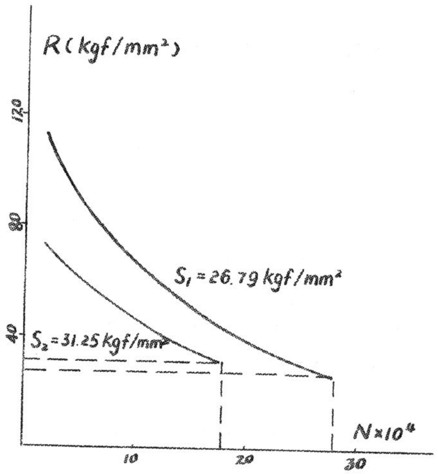


Fig.4 The R-S-N curve at p=0.5