

## FATIGUE CRACK GROWTH UNDER NON-PROPORTIONAL LOADING

Gao Hua (高桦)

Institute of Mechanics, Academia Sinica, Beijing, China

M. W. Brown, K. J. Miller

Mechanical Engineering Department, University of Sheffield, U.K.

### INTRODUCTION

Fracture Mechanics has frequently been used to analyse the growth of fatigue cracks and a large number of papers on the Linear Elastic Fracture Mechanics (LEFM) behaviour of long cracks have been published for uniaxial loading conditions. However only a few papers on biaxial proportional loading, and one or two paper eg [1] concerning non-proportional loading are available albeit that in many engineering situations, flaws may not only experience mixed mode crack growth conditions but also different and continuously varying multiaxial stress states, which cause non-proportional loading.

This paper details the last category which is now assuming greater importance because of: (1) the way different types of crack tip plastic zone develop, (2) the effects of non-proportional loading on fatigue threshold, and (3) the correlation between crack growth and cyclic plasticity.

### DIFFERENT TYPES OF CRACK TIP PLASTIC ZONES

With regard to the case of an inclined crack in a plate subjected to biaxial cyclic loads (Fig. 1), if the two loads are out-of-phase, or if the load amplitudes are in-phase but the stress state ( $\lambda = \sigma_x / \sigma_y$ ) is different for the mean and cyclic components, (i.e.  $\lambda_m \neq \lambda_c$ ) then non-proportional loading will arise. For proportional loading, the crack tip maximum plastic zone direction always remains constant (Fig. 2). In the case of non-pro-

portional loading, since the crack tip stress intensity factor ratio  $K_{II}/K_I$  changes continuously during a cycle then the maximum plastic zone direction as well as the direction of both the crack tip maximum tensile and shear strains change during a cycle. Hence the slip systems operating in the crack tip neighbourhood have to change correspondingly.

Fig. 2 shows the monotonic plastic zones corresponding to the applied loads at various times in the cycle. It therefore follows that the build up of dislocations caused by the crack tip reversed plastic deformation will be distributed over different slip systems, and will be no longer concentrated as in the proportional loading case. In other words, for non-proportional loading the intensity of plastic strain accumulation in the maximum shear direction ahead of the crack path must be lower than that for proportional loading, if the values of  $\Delta K_I$  and  $\Delta K_{II}$  are equivalent in each case.

### THE EFFECT OF NON-PROPORTIONAL LOADING ON THRESHOLD

Experiments on 316 stainless steel show that for both proportional and non-proportional mixed mode loading there is a threshold, below which no crack growth was observed. Above this threshold, an initial shear mode of crack growth was noted. For specimens that used a narrow slot to simulate a crack, a mode-I branch crack appeared after a short period of shear mode coplanar growth. However, in previous experiments [2] with pre-fatigue cracked specimens, because of crack surface sliding friction induced by the mode-II displacements and roughness of the fracture surface the shear cracks were non-propagating, and a branch crack was formed only when stress intensity factors were above an upper bound threshold.

For any fixed ratio of  $\Delta K_{II}/\Delta K_I$ , fatigue thresholds obtained under non-proportional loading show higher values than those for proportional loading [3] (see Table 1). With proportional and non-proportional loads,

Table 1. The Influence of Non-proportional Loading on Fatigue Thresholds

$\lambda_c$	$\lambda_m$	$\Delta K_{II}/\Delta K_I$	$\Delta K_I$	$\Delta K_{II}$
			(MN.m <sup>-3/2</sup> )	
-0.5	-0.5	3	1.24	3.75
-0.5	+0.5	3	1.51	4.53
-0.1716	-0.1716	$\infty$	0	3.88
-0.1716	+0.25	$\infty$	0	4.47

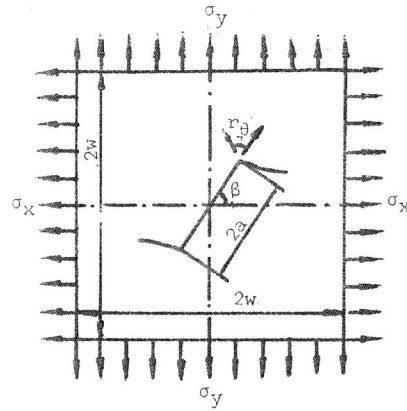


Fig.1 Inclined crack and branch crack in biaxial loaded plate of uniform thickness B.

$$\sigma_x = \lambda \sigma_c \sin \omega t + \lambda \sigma_m$$

$$\sigma_y = \sigma_c \sin \omega t + \sigma_m$$

$$t = 2n\pi \quad t = \frac{(2n+1)}{2}\pi \quad t = (2n+1)\pi \quad n = 0, 1, 2, \dots$$

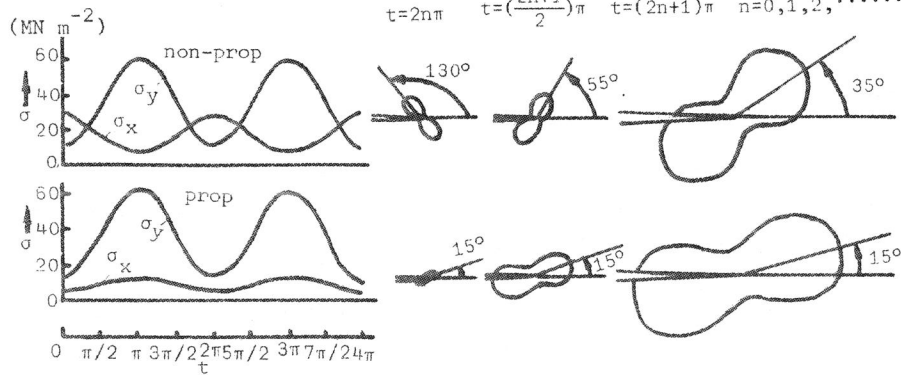


Fig. 2 Proportional and non-proportional loading, and the corresponding crack tip plastic zone.

branch cracks always appeared in the direction corresponding to the maximum normal stress range,  $\Delta\sigma_\theta$ , in the crack tip stress field. In other words, the branched crack propagation path is well predicted by a maximum  $\Delta K_I$  criterion.

As the crack extends, the effect of changing maximum shear direction under non-proportional loading becomes less important because the cracks always tend to the pure mode I direction. Immediately after the crack branches, the crack growth rate under non-rpoportional loading may be lower than for proportional loading but as the branch crack extends the crack growth rates for proportional, non-proportional and pure mode-I loading become very similar. As shown by Fig.3, all the experimental results are eventually within a narrow band. In Fig. 3 the crack tip cyclic plastic zone size  $r_p$  was estimated from both stress intensity factor ranges and

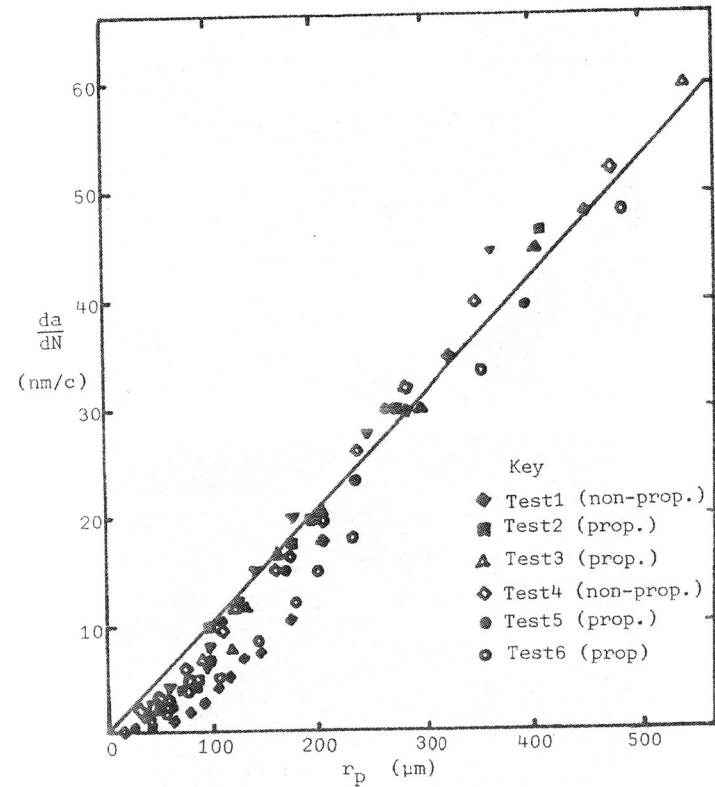


Fig. 3 Relationship between crack growth rate and crack tip cyclic plastic zone size.

T-stress ranges (irrespective of the mean stress components), with the Von Mises Yield Criterion [4].

As the crack branch extends and stress intensity factor ranges become higher, many slip systems of different grains at various orientations can operate in the crack tip region. While the distribution of slip bands in the crack tip zone of a polycrystalline material is quite random, the plastic deformation at the crack tip may be considered as typical of continuum plasticity. The crack growth mechanism, in this case, is dependent on a plastic decohesion process [5], in which the crack tip is successively blunted and resharpened in each cycle, and fractographs do not depict crystallographic features (unlike near threshold growth, Fig. 4a), but rather a flat and featureless topography (Fig. 4b).

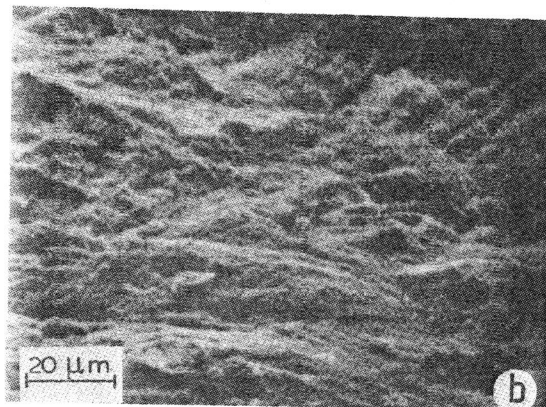
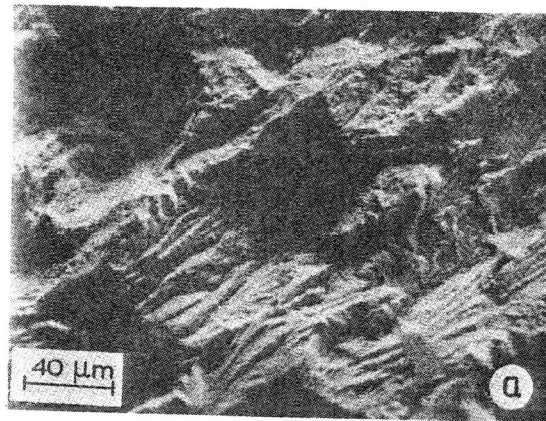


Fig.4. (a) Fractograph of near threshold growth  
(b) Fractograph of above threshold growth

Fig. 3 also shows a linear relationship between crack growth rate and branched crack tip cyclic plastic zone size. This is in agreement with dimensional analysis [6] which predicts that, if there are no size effects arising from specimen geometry or microstructural features, the crack extension in each cycle should be proportional to the crack tip plastic zone size.

#### CONCLUSIONS

- (1) Under mixed mode non-proportional loading, fatigue thresholds are higher than for proportional loading because crack tip plasticity is diffuse.
- (2) For both proportional and non-proportional loading, the crack propagation path is well predicted by the  $\Delta K_{I_{max}}$  criterion.
- (3) At the very beginning of crack branching, the crack growth rate in non-proportional loading may be lower than that of proportional loading, but as the crack grows the propagation rates become very similar, and the cracks tend to the pure mode-I direction and growth rate.

#### REFERENCES

- [1] Hourlier, F., d'Hondt, H., Truchon, M. and Pineau, A. Fatigue crack path behaviour under polymodal fatigue. ASTM Symposium on Biaxial/Multi-axial Fatigue. Dec. 1982, San Francisco.
- [2] Gao Hua, Brown, M.W. and Miller, K.J. Mixed-mode fatigue thresholds. Fatigue of Eng. Mat. and Struct., 5 (1982) 1.
- [3] Gao Hua, Brown, M.W. and Miller, K.J. Fatigue crack growth under proportional and non-proportional loading. Sheffield University Report (1983).
- [4] Gao Hua, Alagok, N., Brown, M.W. and Miller, K.J. Growth of fatigue cracks under combined mode I and mode II loads. ASTM Symposium on Biaxial/Multi-axial Fatigue, Dec. 1982, San Francisco.
- [5] Laird, C. and Smith, G.C. Crack propagation in high stress fatigue. Phil. Mag., 7 (1962) 847.
- [6] Brown, M. W., Liu, H. W., Kfoury, A.P. and Miller, K.J. An analysis of fatigue crack growth under yielding conditions. Proc. ICF 5, 2 (1981) 891.