MULTIAXIAL CUMULATIVE DAMAGE IN LOW CYCLE FATIGUE

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The problem of how damage accumulates in fatigue under variable loading conditions has received a great deal of attention, but interest has been focussed primarily on changing stress amplitude and mean stress in simply loaded specimens e.g. [1]. The most widely used hypothesis is due to Palmgren and Miner, namely,

$$\sum_i n_i/N_{fi} = D \qquad (1)$$

Here n_i is the number of cycles applied at a stress level σ_i that corresponds to an endurance N_{fi}, and failure occurs when the summation of damage, D, is equal to unity. However when only two stress levels are employed, if the loading sequence is from low to high stress, then $D > 1$; conversely for high to low, $D < 1$. Furthermore overloads can introduce localised residual stress fields which also influence D. For example tensile overloads produce compressive crack tip stresses and retard subsequent propagation.

When the more general problem of biaxial stress fatigue is assessed, one would expect to observe similar effects due to changes of stress amplitude, at least if the modes of crack growth are unchanged [2], [3]. However a new dimension to the cumulative damage problem is introduced by including the second independent variable, i.e. stress biaxiality, since not only stress amplitude but also stress state may be changed [4]. Indeed a third controllable factor is the phase angle between the stress waveforms. If the principal stress axes are changed due to biaxiality or phase, a number of mutations may be postulated.

a) *Crack rotation* - the direction of crack growth changes to suit the new biaxial strain field although cracks continue to grow in the same mode.

b) *Reinitiation* - a new set of cracks initiate when the previous planes of growth are unfavourable, or when cracks are blunted out by the load change.

c) *Change of mode* - Stage I cracks become Stage II cracks, and vice versa.

d) *Crack interference* - growth is slowed down or suppressed by transverse non-propagating cracks generated by previous loadings.

e) *Stable growth* - propagation continues in the same direction if the stress field created by the crack dominates the far field stresses.

If the level of effective stress is unchanged in two stage multiaxial cumulative damage tests, thereby minimising the sequence effects mentioned previously, then (a), (c) and (e) should show damage summations D close to unity, whereas (b) and (d) will extend life, giving $D > 1$.

Experimental Results

A series of tension-torsion two stage cumulative damage tests have been conducted on a strain controlled multiaxial low cycle fatigue machine, fully described elsewhere [5]. Failure was defined by the instant at which the load ranges began to decrease rapidly due to cracking. Baseline data have been published [6] for this heat of 1% Cr-Mo-V steel. New results from 29 cumulative damage tests are shown in Figs. 1-3; excluding the data where cracks clearly initiated from deep honing marks.

Crack directions were measured using a toolmakers microscope at 40X magnification. They are presented in the figures as the mean value plus standard deviation for a number of cracks measured on each specimen. Where solid points are used in Figs. 1-3, 20-50 cracks were measured, for half-shaded points, 10-19 cracks, and for open symbols, less than 10 cracks. Thus the dominant cracking modes can be distinguished. The figures show that cracks may be categorised as either Stage I or Stage II within reasonable scatter bands in almost every case. Cracks that clearly formed on polishing marks were excluded from this analysis.

In Figs. 1 and 2 the tests were conducted with a fixed strain amplitude and biaxiality $\lambda = \Delta\gamma/\Delta\varepsilon$, where $\Delta\gamma$ and $\Delta\varepsilon$ are the torsional and axial strain ranges respectively. After n_1 cycles, the sense of twist for the torsional strain was reversed, rotating the principal axes of strain through an angle θ due to the 180° phase change. Clearly in this case N_{f1} and N_{f2} are identical, as the material was isotropic, and the sequential effects on D are avoided.

In Fig. 3, the results were obtained by switching from one strain state to another after n_1 cycles. The two strain amplitudes were selected to correspond to the same point on the cyclic stress strain curve to avoid the problem of generating residual stresses on the changeover. The stable stress response was well characterised by the cyclic stress strain curve

$$\tau_{max} = 643\ (\tfrac{1}{2}\gamma_{max})^{0.127}\ \text{MPa} \qquad (2)$$

which was derived from the constant amplitude test results.

Discussion

The strain state $\lambda = 1.5$ was chosen for the first test series to give a rotation of 45° to the principal strain axes, assuming a Poisson's ratio of 0.5 and a principal axis inclination of $\phi = \tfrac{1}{2}\tan^{-1}(2\lambda/3)$ shown in Fig. 4. For this special case, the initial Stage II crack plane, normal to the maximum principal strain axis, is turned to align with a Stage I direction during the second loading, as shown in Fig. 1. Similarly one of the initial Stage I shear planes becomes the final Stage II orientation. This is ideally suited to mutation (c) above, and the dominance of measured cracks on these planes appears to substantiate this model for higher n_1/N_{f1} fractions, giving damage summations D close to unity. But when $n_1/N_{f1} < 0.3$, cracks initiated in the first loading presumably become non-propagating, being blunted out during the phase change, and new cracks must be initiated to give $n_2/N_{f2} \simeq 1$ and extended endurances.

The anomalous behaviour for the high strain range results can be attributed to mutation (d) in the case of Stage II cracks, since the initial shear cracks at 158° lie transverse to the path of Stage II cracks, providing efficient crack arresting features. Mutation (d) can only be effective at higher strain ranges where the density of cracks is great.

In Fig. 2, the strain state $\lambda = 4$ gives a reduced principal axis rotation of 21°. This makes mutation (c) unfavourable as Stage I cracks prefer to transfer to the nearest shear plane following a phase change, since this only involves a 21° alteration in crack path. Thus mutation (a) is dominant, possibly shifting to (e) for longer cracks where n_1/N_{f1} is high. But here also for $n_1/N_{f1} < 0.3$, reinitiation gives damage summations above unity. Almost no Stage II cracks were observed.

In Fig. 3, tests are presented where the strain state λ was altered, while keeping the same relative phase. This produces a small θ value of 12°, favouring mutation (a) again, and probably (e) for longer cracks. One would not expect reinitiation when the rotation θ is small. There is unfortunately some scatter, possibly because the two tests falling below the Palmgren-Miner line showed cracking from honing marks. There were however many other smaller cracks to be seen.

Each of the five postulated crack mutations appears to play a role in damage accumulation. Since $D \geq 1$, the Palmgren-Miner rule can be used safely for design where there are no initial high stress cycles giving a high to low sequential effect. However this may be excessively conservative when the sequence is low to high, combined with mutations (b) and (d), for example in refs [7] and [8], where n_2/N_{f2} can exceed 2.

Conclusions

Five types of behaviour have been postulated for fatigue cracks in situations where principal axes rotate during cumulative-damage mechanical-fatigue tests. When effective stress levels are constant in multiaxial fatigue, endurance either increases or remains unchanged, making the Palmgren-Miner equation a useful design rule.

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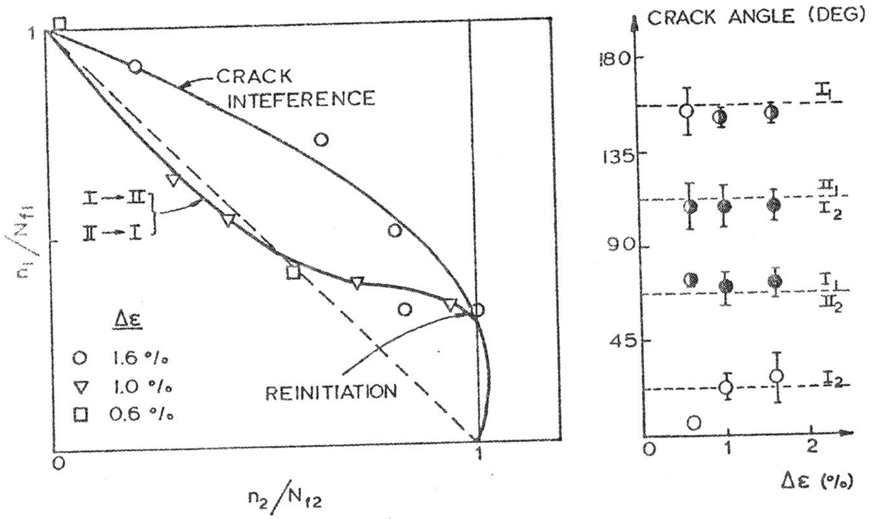

FIG. 1 Damage accumulation for high angle principal axis rotation, $\theta = 45^\circ$, with $\lambda = 1.5$

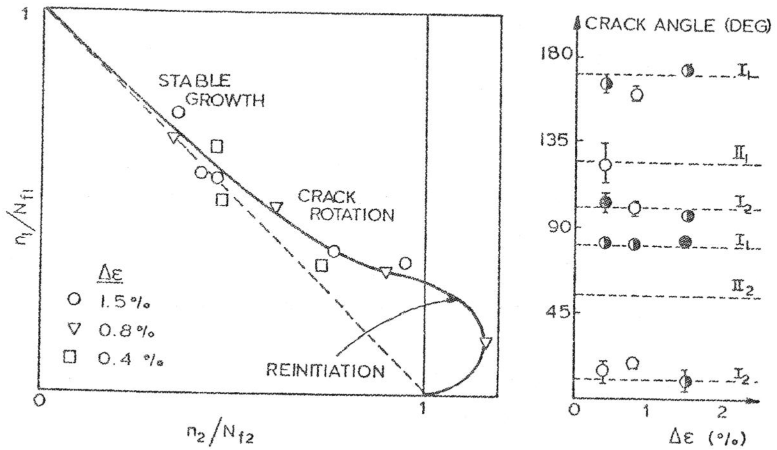

FIG. 2 Damage accumulation for $\theta = 21^\circ$, with $\lambda = 4$

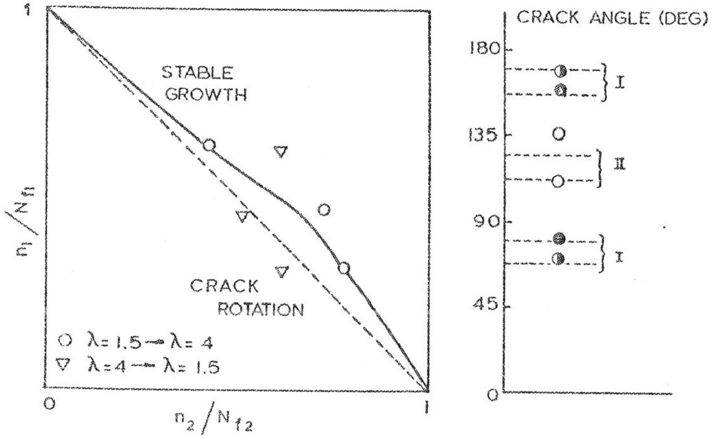

FIG. 3 Damage accumulation for $\theta=12^\circ$ with $\lambda=1.5(\Delta\varepsilon=1.6\%)$, $\lambda=4(\Delta\varepsilon=0.8\%)$

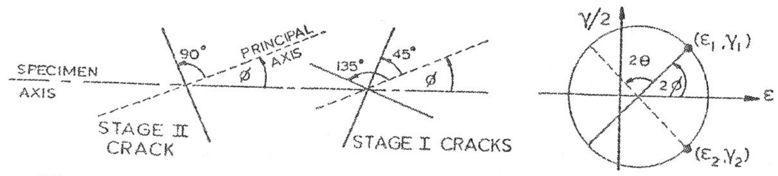

FIG. 4 Crack orientations for $\lambda = 1.5$ and Mohr's circle of strain